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The 2019 lattice FLAG α_s average

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We summarise the recent 2019 average of α_s by the FLAG collaboration.

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Introduction

Lattice gauge theory is a non-perturbative formulation of QCD, which allows us to evaluate the Euclidean path integral by a Monte Carlo "simulation" for a few suitably chosen values of

$$L/a,\ T/a,\ g_0,\ \{am_i,\ i = 1 \ldots N_f\}\,. \tag{1}$$

Here L/a is the number of points of the world in each space dimension, T (often bigger than L) is the extent of the time axis, g_0 is the bare coupling of the theory, and am_i are the bare quark masses. Once we obtain the relation between the bare parameters and hadronic low-energy quantities, such as $f_\pi, m_\pi, m_K \ldots$, we can in principle predict all physical quantities in QCD, including α_s.

Methods for the strong coupling.

The general method for extracting $\alpha_{\overline{\rm MS}}$ with lattice QCD is to consider a short-distance, one-scale, observable with an expansion

$$\mathscr{O}(\mu) = c_1 \alpha_{\overline{\rm MS}}(\mu) + c_2 \alpha_{\overline{\rm MS}}(\mu)^2 + \cdots\,, \tag{2}$$

compute $\mathscr{O}(\mu)$ by lattice QCD and determine $\alpha_{\overline{\rm MS}}(\mu)$ from Eq. (2). This requires that we are in a region where perturbation theory is valid, i.e. $\alpha_{\overline{\rm MS}}(\mu)$ is small.

Advantages. An important advantage of taking $\mathscr{O}$ from lattice QCD compared to using experimental data is that one is automatically in the Euclidean region where no hadronisation corrections, duality violations etc. are a concern. Furthermore one has a large freedom to design convenient observables.

Disadvantages. Determining α_s is a two stage process, connecting quantities at two disparate scales, high momentum and the hadronic scale – the latter is where lattice QCD naturally resides. Furthermore, lattice QCD simulations are restricted to $N_f = 3$ or $N_f = 4$ quarks at most, because the b-quark is simply too heavy. One then relies on perturbative matching across the appropriate quark thresholds to determine α_s at the m_Z scale where the number of active flavours is $N_f = 5$. Note that this means that many earlier results for $N_f = 2$ cannot be used, as crossing the strange quark threshold needs a non-perturbative procedure. ($N_f = 0$ results being computationally cheap form a useful testbed for checking different methods.)

The 2019 FLAG review.

The Flavour Lattice Averaging Group (FLAG) formed a working group (R. Horsley, T. Onogi, R.S.) on α_s in 2011 and first included determinations of $\alpha_s(m_Z)$ in its review in 2013 [1]. Updates appeared in 2016 [2] and 2019 [3].

Here we report on this latter work. We first briefly comment on our procedure for determining averages. There are similarities and differences to the PDG approach [4]. The main difference is that FLAG formulates a set of criteria, that computations have to pass in order to enter the average of a given quantity of phenomenological interest [2]. These are based on whether the simulations

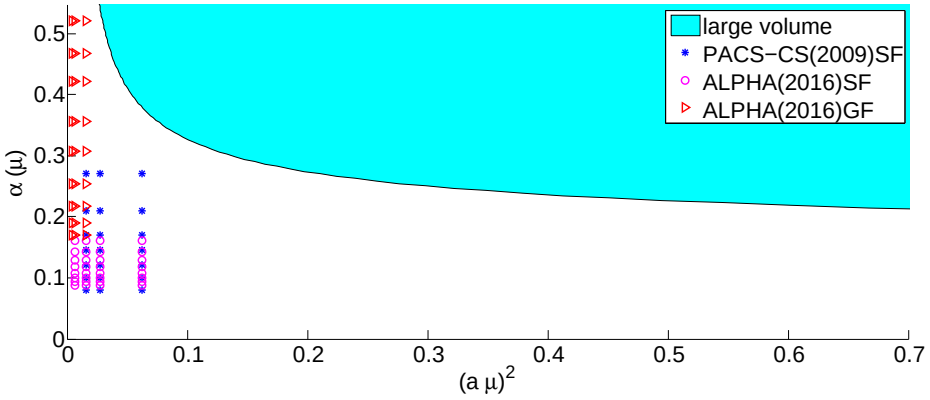

Figure 1: The plane $\alpha^{(3)}_{\overline{\rm MS}}(\mu)$ for $N_f = 3$ against the scale μ in lattice units, where a is the lattice spacing and the blue region corresponds to the rough bound $a > 0.04\,$fm. Note that the continuum limit is approached by extrapolations with $a\mu \ll 1$. The points on the left correspond to actual Monte Carlo simulations in category (III).

cover a range of parameters that allow to achieve a satisfactory control of systematic uncertainties (labeled ★) a reasonable attempt can be made at estimating systematic uncertainties (○), or it is unlikely that systematic uncertainties can be brought under control (■). The appearance of a ■ even in a single source of systematic error of a given lattice result, disqualifies it from inclusion in the global average.

For the computations of α_s, the usual criteria for chiral and infinite volume extrapolations are somewhat relaxed as they do not play a dominant role. Instead criteria on *perturbative behaviour* and *renormalization scale* try to make sure that the computation is at reasonable high μ, the perturbative knowledge is sufficiently good (i.e. the number of known loops, n_l, is sufficiently high) and μ could be varied over some range in order to confirm the perturbative μ-dependence. The general idea is that these criteria try to make sure that the available Monte Carlo data have a few points located sufficiently low in the landscape of Fig. 1, while the continuum limit criterion requires us to not be too far on the right. The precise criteria are given in FLAG 19 [3].

In order to arrive at a final average, we first form pre-averages of computations using one and the same method and after combine them to give a final estimate. We now discuss the different methods, following a certain classification (I-III).

(I) Continuum-limit observables in large volume. Here $\mathcal{O}$ is a finite observable depending on the scale μ. One can then take the continuum limit

$$\mathcal{O}(\mu) \equiv \lim_{a\to 0} \mathcal{O}_{\rm lat}(a,\mu) \quad \text{with } \mu \text{ fixed}. \tag{3}$$

One wants μ to be high such that the expansion Eq. (2) is precise and $a\mu$ small to control the discretization error. However, recall that one is usually in the blue shaded region of Fig. 1 and it is difficult to extrapolate when $\alpha_{\overline{\rm MS}}$ is small, say $\alpha_{\overline{\rm MS}} \leq 0.3$.

There are several different methods. They share the necessity for finding a compromise between large μ and small $a\mu$. In the cases where computations qualify for taking an average (i.e., there is no ■), we perform a weighted average of the different results. According to our judgement

the uncertainties are dominantly systematic. They are due to the truncation error of perturbation theory, whether ordinary higher order or non-perturbative effects. We just estimate the perturbative truncation error and take this as the uncertainty of the pre-range, which is usually somewhat more conservative than the uncertainty estimate in the contributing papers.

The individual methods are (we partially have to simplify here):

(1) Q-$\overline{Q}$ potential: $\mathscr{O}(\mu) = r^2 F_{\rm static}(r)$, $\mu = 2/r$, where $F_{\rm static}(r)$ is the force between static quarks defined by the large-t behaviour of Wilson loops $W(r,t)$. Note that n_l is 3 but $n_l > 3$ terms proportional to $\log\alpha_s$ are also known. Indeed, at fixed order perturbation theory, the basic observable $\mathscr{O}(\mu)$ is infrared divergent. As discussed by N. Brambilla and H. Takaura at this workshop, these divergences can be resummed, leaving terms such as $\alpha_s^4 \log\alpha_s$ in the expansion of $\mathscr{O}(\mu)$.

(2) Vacuum polarization: $\mathscr{O}(\mu) = D(Q^2)$, $\mu^2 = Q^2$, with D the Adler function derived from the V+A two-point function at Euclidean q. This method does not yet enter the average.

(3) Two point HH current: moments of heavy-heavy pseudoscalar-current two-point functions. Heavy quarks of masses around the charm and heavier are used. Different discretizations are available that allow also to compare the continuum-limit moments before the extraction of α_s. There is quite good agreement.

(4) Gluon-ghost vertex: using gauge fixing, the momentum-space vertex is used. This method does not yet enter the average as the continuum limit criterion is not passed.

(5) Dirac eigenvalues: $\mathscr{O}(\mu) = \partial_\lambda \log(\rho(\lambda))/\partial\log(\lambda)$, $\mu = \lambda$ with $\rho(\lambda)$ the spectral density of the massless Dirac operator. This recently introduced method [5] does not yet pass the continuum-limit criterion.

(II) Lattice observables at the cutoff. There is also the possibility to consider lattice observables involving distances of a few lattice spacings, which are not related to a continuum observable. The prominent example is rectangular Wilson loops $W(r,t)$ of extent $r \times t$ with $r = am$ and $t = an$, keeping the integers n,m fixed as one takes the limit $a \to 0$; the loops shrink to size zero in the limit. Such observables have an expansion

$$W(na,ma) \overset{g_0\to 0}{\sim} \sum_{k\geq 0} c^{(k)}_{m,n}\, g_0^{2k} \overset{g_0\to 0}{\sim} \sum_{k\geq 0} \hat{c}^{(k)}_{m,n}\, g^{2k}_{\overline{\rm MS}}(1/a)\,, \tag{4}$$

where in the second step use is made of the relation between the bare coupling and a renormalized coupling at the cutoff scale, $g_0^2 = g^2_{\overline{\rm MS}}(1/a) + \mathscr{O}(g^4)$. The available loop orders are often lower than for continuum perturbation theory. Lattice artefacts can only be separated from perturbative corrections in Eq. (4) by assuming some functional form and fitting to it.

In this category small ($m,n \leq 3$) Wilson loops $\mathscr{O}(\mu) = W(ma,na)\,, \quad \mu = k/a$, and functions thereof (e.g. $\log(W(a,a))$ are often used. The scale factor k is adjusted to have better apparent convergence of PT. Our estimate of perturbative uncertainties is again somewhat conservative [3].

(III) Continuum-limit observables in small volume and step scaling. For finite volume quantities with volume L^4 and some technical requirements, Eq. (2) holds but with

$$\mu = 1/L\,. \tag{5}$$

The advantage is that now μa can easily be taken to $a/L = 1/8 \ldots 1/32$ or smaller. However, a number of steps are needed to connect recursively

$$\mu_0 \to s\mu_0 \to s^2\mu_0 \to \ldots \to s^N\mu_0\,, \tag{6}$$

and in each step a few different lattice spacings a have to be simulated to take the continuum limit. After a few steps, μ becomes very large so that perturbation theory can be applied with confidence and statistical errors dominate the uncertainty. At this workshop, M. Dalla Brida presented a recent precise three-flavour computation with $\mu_0 \approx 200$ MeV and $s^N\mu_0 \approx 100$ GeV . We perform a straight weighted average for mean and error of the two available results for this method.

World average from FLAG. Altogether we have considered 18 computations, of which 9 pass our criteria. These are shown in Fig. 2 and Table 1. For each method, the grey band shows the pre-

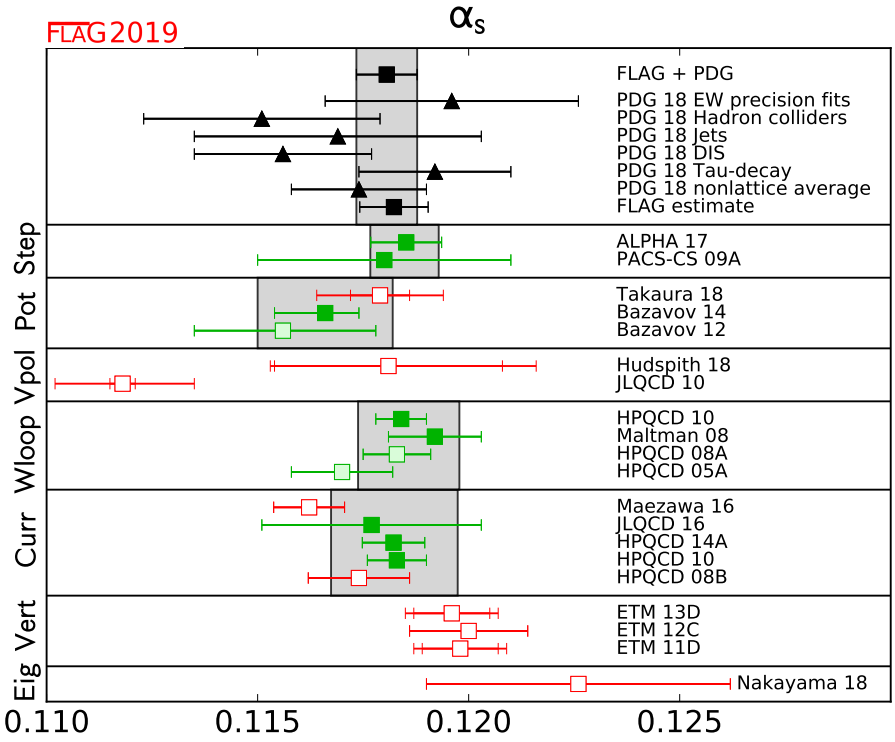

Figure 2: The $\overline{\text{MS}}$ coupling at the Z mass. The PDG 18 [4] entries give the outcome of their analysis from various phenomenological categories including their average. The lattice computations with a filled green box, ■, have no red box, ■, in the previous ratings and therefore qualify for averaging. A □ means the same but the number does not enter an average because it is superseded by a later more complete computation or it was not published at the September 2018 deadline. Computations with □ do not enter the averages because they had at least one ■ before.

average as explained above. We are left with the task to combine those pre-averages. Again we take the central value from their weighted average. However, since the errors of the pre-averages are mostly systematic, we feel that the straight error 0.00057 of the weighted average is too optimistic

Collaboration	Ref.	N_f	publication status	renormalization scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\rm MS}}(m_Z)$	method	n_l
ALPHA 17	[6]	2+1	A	★	★	★	0.11852(84)	step scaling	2
PACS-CS 09A	[7]	2+1	A	★	★	○	0.11800(300)		2
pre-range (average)							0.11848(81)		
Takaura 18	[8, 9]	2+1	P	■	○	○	$0.11790(70)(^{+130}_{-120})$	Q-$\bar{Q}$ potential	3
Bazavov 14	[10]	2+1	A	○	★	○	$0.11660(^{+120}_{-80})$		3
Bazavov 12	[11]	2+1	A	○	○	○	$0.11560(^{+210}_{-220})$		3
pre-range with estimated pert. error							0.11660(160)		
Hudspith 18	[12]	2+1	P	○	○	■	$0.11810(270)(^{+80}_{-220})$	vacuum polarization	3
JLQCD 10	[13]	2+1	A	■	○	■	$0.11180(30)(^{+160}_{-170})$		2
HPQCD 10	[14]	2+1	A	○	★	★	0.11840(60)	Wilson loops	2
Maltman 08	[15]	2+1	A	○	○	★	0.11920(110)		2
pre-range with estimated pert. error							0.11858(120)		
JLQCD 16	[16]	2+1	A	○	○	○	0.11770(260)	HH current, two points	2
Maezawa 16	[17]	2+1	A	○	■	○	0.11622(84)		2
HPQCD 14A	[18]	2+1+1	A	○	★	○	0.11822(74)		2
HPQCD 10	[14]	2+1	A	○	★	○	0.11830(70)		2
HPQCD 08B	[19]	2+1	A	■	■	■	0.11740(120)		2
pre-range with estimated pert. error							0.11824(150)		
ETM 13D	[20]	2+1+1	A	○	○	■	0.11960(40)(80)(60)	gluon-ghost vertex	3
ETM 12C	[21]	2+1+1	A	○	○	■	0.12000(140)		3
ETM 11D	[22]	2+1+1	A	○	○	■	$0.11980(90)(50)(^{+0}_{-50})$		3
Nakayama 18	[5]	2+1	A	★	○	■	0.12260(360)	Dirac eigenvalues	2

Table 1: Results for $\alpha_{\overline{\rm MS}}(m_Z)$ from simulations that use $2+1$ or $2+1+1$ flavours of quarks. A weighted average of the pre-ranges gives 0.11823(57), using the smallest pre-range gives 0.11823(81) and the average size of ranges as an error gives 0.11823(128).

– it would be correct for independent Gaussian distributions. Instead we use the smallest error of the pre-averages. This yields the result

$$\alpha^{(5)}_{\overline{\rm MS}}(m_Z) = 0.11823(81)\,. \qquad \text{Refs. [6, 16, 10, 18, 14, 7, 15].} \tag{7}$$

Further progress

Finally, we collect some lessons that we have learned in our forming of a lattice world average of α_s. The basic problem is simple and has been spelled out often, phrased in varying words. In order to have a precise value with an error that can be estimated by perturbation theory itself, large energy scales μ have to be reached and theory assumptions have to be kept at a minimum. Further progress will be limited if we include processes where non-perturbative contributions have to be fitted or removed by complicated analyses in order to make lower energies accessible. Dealing with non-perturbative physics is always based on assumptions – if only where the expansion in $1/\mu$ applies and lowest-order terms $(1/\mu)^{N_{\min}}$ dominate. We should therefore separate the determination of α_s at high enough μ, simple theory, from tests of perturbation theory, with resummations, studies of higher-twist contributions, etc.

The concept of criteria introduced by FLAG is very useful in this respect, and we advocate to consider such a procedure for phenomenological determinations. One should at least consider a criterion on minimum values of μ, paired with sufficiently high perturbative order. In FLAG these are the "renormalization scale" / "perturbative behaviour" criteria. We also think that the criteria of FLAG should become more strict as time goes on. This is necessary to avoid situations where complicated procedures, involving e.g. separate estimates of perturbative errors (see above), are needed to arrive at a safe range. Finally, it seems that the limit of lattice determinations of α_s is not yet reached; we believe a factor of two reduction in the error is possible with some variation of the developed techniques.

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