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α_s from the ALPHA collaboration (part II)

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In this second part we continue the overview of the recent lattice determination of α_s by the ALPHA collaboration. Starting from the result for $\Lambda_{\overline{\rm MS}}^{N_f=3}/\mu_0$ discussed in the first part [1], we first present a precise non-perturbative determination of the Λ-parameter of $N_f = 3$ QCD. Using perturbative decoupling to match the $N_f = 3$ and $N_f = 5$ theories we then extract a precise value for α_s. The final result: $\alpha_s(m_Z) = 0.11852(84)$, reaches subpercent accuracy.

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Introduction

The extraction of α_s we present is based on the determination of $\Lambda_{\overline{\rm MS}}^{N_f=5}$, the Λ-parameter of $N_f = 5$ flavour QCD in the $\overline{\rm MS}$ scheme. The latter is obtained from a non-perturbative determination of $\Lambda_{\overline{\rm MS}}^{N_f=3}$, combined with a perturbative estimate for the ratio $\Lambda_{\overline{\rm MS}}^{N_f=5}/\Lambda_{\overline{\rm MS}}^{N_f=3}$. Our strategy can be summarized into the following equation [2]:

$$\Lambda_{\overline{\rm MS}}^{N_f=5} = \left[\frac{\Lambda_{\overline{\rm MS}}^{N_f=5}}{\Lambda_{\overline{\rm MS}}^{N_f=3}}\right]_{\rm PT} \times \Lambda_{\overline{\rm MS}}^{N_f=3} \quad \text{where} \quad \Lambda_{\overline{\rm MS}}^{N_f=3} = \frac{\Lambda_{\overline{\rm MS}}^{N_f=3}}{\mu_0} \times \frac{\mu_0}{\mu_{\rm had}} \times \frac{\mu_{\rm had}}{f_{\pi K}} \times f_{\pi K}. \tag{1}$$

In the rest of this contribution, we will briefly review the computation of the different factors entering this expression. For a more complete discussion, we refer the reader to the original reference [2], and to the more extended reviews [3, 4, 5].

We begin our presentation from the non-perturbative determination of $\Lambda_{\overline{\rm MS}}^{N_f=3}$ and the different ratios that compose it. The first ingredient appearing in Eq. (1) is the value of $\Lambda_{\overline{\rm MS}}^{N_f=3}$ in units of the technical scale μ_0. This computation is discussed in detail in the first part of this overview [1], which we advise the reader to consult. Here we only quote the final result: $\Lambda_{\overline{\rm MS}}^{N_f=3}/\mu_0 = 0.0791(19)$ [6, 7], and recall that the scale $\mu_0 \approx 4\,{\rm GeV}$ is implicitly defined by the value of the Schrödinger functional (SF) coupling: $\bar{g}^2_{\rm SF}(\mu_0) = 2.012$. It is also worth recalling that this ratio has been obtained by studying the non-perturbative running of the SF coupling in the wide energy range $\mu \approx 4-70\,{\rm GeV}$. With this result at hand, the value of $\Lambda_{\overline{\rm MS}}^{N_f=3}$ in physical units can be obtained by expressing the technical scale μ_0 in terms of some experimentally accessible quantity. We consider a particular combination of the pion and kaon decay constants, f_π and f_K, given by: $f_{\pi K} = \frac{2}{3}(f_K + \frac{1}{2}f_\pi)$; the reasons for this particular choice will be given later in the text. Meson decay constants are typically used to set the physical scale of the lattice theory as they can be accurately determined both phenomenologically and on the lattice[1]. A direct computation of $\mu_0/f_{\pi K}$, on the other hand, is not really feasible if one wants the systematic uncertainties associated with finite-volume and discretization effects comfortably under control. The large energy separation between μ_0 and $f_{\pi K} = \mathscr{O}(100\,{\rm MeV})$ would indeed require us to simulate rather large lattice resolutions, L/a, for today's standards; here and in the following we denote by L the physical extent of the lattice in all four space-time directions and by a its spacing. The solution to this problem is to rely, as we did for the determination of $\Lambda_{\overline{\rm MS}}^{N_f=3}/\mu_0$, on a *step-scaling strategy* (cf. Ref. [1]). More precisely, by studying the non-perturbative running of a finite-volume coupling, we can relate the scale μ_0 to a lower, finite-volume scale, $\mu_{\rm had} = \mathscr{O}(100\,{\rm MeV})$, and in a second step connect $\mu_{\rm had}$ with $f_{\pi K}$ (cf. Eq. (1)).

The gradient flow coupling and its running to low energy

The obvious strategy we could follow at this point would be to continue the non-perturbative running of the SF coupling started at high-energy down to lower energies. On the other hand,

[1]A more natural and conceptually clean quantity to consider would be the proton mass. (The masses of the QCD stable mesons are normally used to fix the value of the bare quark masses appearing in the lattice Lagrangian.) The extraction of the decay constants from experimental decay rates is indeed not theoretically straightforward and also relies on the knowledge of CKM matrix elements. Measuring the proton mass precisely on the lattice, however, is at present very challenging.

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a precise determination of the running of the SF coupling at low energy is impeded by a few technical reasons (see e.g. refs. [6, 8]). The main issue is that the statistical variance of the SF coupling as measured in Monte Carlo lattice simulations is such that: $\mathrm{var}(\bar{g}^2_{\mathrm{SF}}(\mu))/\bar{g}^4_{\mathrm{SF}}(\mu) = c(a\mu)\,\bar{g}^4_{\mathrm{SF}}(\mu) + \mathscr{O}(\bar{g}^6_{\mathrm{SF}}(\mu))$. This implies that it quickly becomes computationally expensive to measure this coupling precisely at low energy where the coupling becomes large. In addition, $\mathrm{var}(\bar{g}^2_{\mathrm{SF}})$ is large in general, and increases as the continuum limit of the lattice theory is approached due to: $c(a\mu) \overset{a\to 0}{\propto} (a\mu)^{-1}$. For these reasons, it is more convenient to consider a different family of finite-volume couplings for the low-energy end of the running. A particularly compelling family to study is given by couplings defined in terms of the Yang–Mills gradient flow (GF) [9]. The latter is specified by the equations:

$$\begin{aligned} \partial_t B_\mu(t,x) &= D_\nu G_{\nu\mu}(t,x), \qquad & G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \\ B_\mu(0,x) &= A_\mu(x), & D_\mu &= \partial_\mu + [B_\mu, \cdot\,], \end{aligned} \tag{2}$$

where A_μ is the QCD gauge potential, and $t \geq 0$ is the *flow time* which parametrizes the evolution of the flow field B_μ along the gradient flow. Gauge invariant fields made out of the flow field B_μ have the remarkable property of being renormalized once the bare parameters of the theory are [10]. This allows us to define a finite-volume GF coupling as [11, 12]:

$$\bar{g}^2_{\mathrm{GF}}(\mu) = \mathscr{N}^{-1} t^2 \langle E_{\mathrm{sp}}(t,x)\rangle_{\mathrm{SF}}\Big|^{\sqrt{8t}=0.3\times L}_{x_0=L/2}, \quad E_{\mathrm{sp}}(t,x) = \frac{1}{4} G^a_{kl}(t,x) G^a_{kl}(t,x), \quad \mu = L^{-1}, \tag{3}$$

where $\langle\cdot\rangle_{\mathrm{SF}}$ stands for the (Euclidean) path-integral expectation value in the presence of SF boundary conditions and $\mathscr{N}$ is a constant; we refer the reader to the given references for more details. Here we just note that in order for the GF coupling to depend on a single scale, L, we express the flow time t in terms of L through the condition $\sqrt{8t}/L = 0.3$. The nice property of the GF coupling is that $\mathrm{var}(\bar{g}^2_{\mathrm{GF}})$ is finite as $a \to 0$, and typically small. In addition, in first approximation, one has that: $\mathrm{var}(\bar{g}^2_{\mathrm{GF}})\,/\,\bar{g}^4_{\mathrm{GF}} \propto$ const., which, as anticipated, makes this coupling well-suited for low-energy studies.

In order to start computing the running of the GF coupling to low energy, we first need to know its value at the reference scale μ_0. This can be obtained through a non-perturbative matching of the SF and GF couplings. The latter is easily achieved by measuring the two couplings for the very same set of bare lattice parameters for which $\bar{g}^2_{\mathrm{SF}}(\mu_0) = 2.012$. Combining this matching with a change of scale by a factor of 2, we obtain: $\bar{g}^2_{\mathrm{GF}}(\mu_0/2) = 2.6723(64)$ [12]. The running to low energy can now proceed in similar fashion to the computation at high energy. In particular, we introduce the step-scaling function (SSF) of the GF coupling and its lattice approximant (cf. Ref. [1]):

$$\sigma(u) = \lim_{a/L\to 0} \Sigma(u, a/L), \qquad \Sigma(u, a/L) = \bar{g}^2_{\mathrm{GF}}(\mu/2)\big|_{u=\bar{g}^2_{\mathrm{GF}}(\mu),\,\overline{m}(\mu)=0}, \qquad \mu = L^{-1}. \tag{4}$$

The SSF encodes the change in the coupling for a finite variation of the energy scale. On the lattice, it is thus a more natural quantity to consider than the β-function. Once the continuum SSF is known, however, the non-perturbative β-function can be determined by noticing that:

$$\ln\frac{\mu_2}{\mu_1} = \int_{\bar{g}_{\mathrm{GF}}(\mu_1)}^{\bar{g}_{\mathrm{GF}}(\mu_2)} \frac{\mathrm{d}g}{\beta(g)} \quad \Rightarrow \quad \log 2 = -\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{\mathrm{d}g}{\beta(g)} \quad \text{where} \quad u = \bar{g}^2_{\mathrm{GF}}(\mu). \tag{5}$$

The left panel of Fig. 1 shows the continuum extrapolations of the lattice SSF for values of the GF coupling $\bar{g}^2_{\rm GF} \approx 2-6.5$, and for the lattice resolutions, $L/a = 8,12,16$. As one can see from the figure, discretization errors are significant, particularly so at large values of the coupling (higher sets of points in the plot). Cautious continuum extrapolations are hence needed [12]. Nonetheless, the good statistical precision of the GF coupling allows us to obtain precise continuum results.

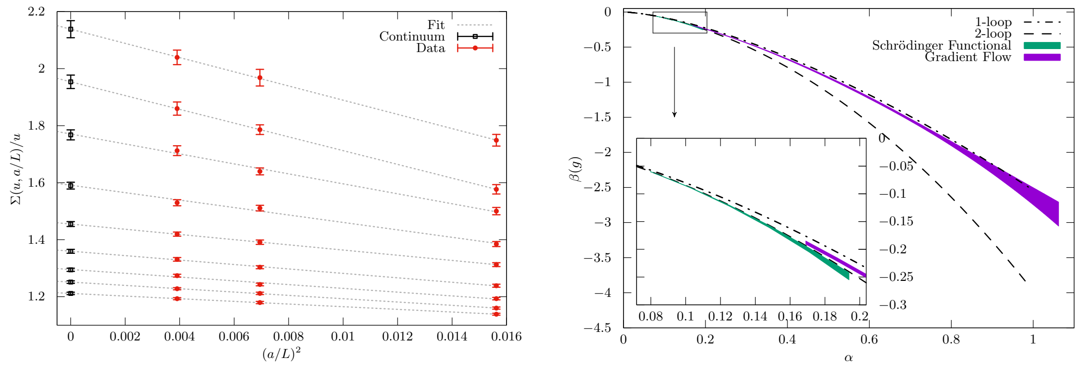

Figure 1: Left: Continuum extrapolations of the lattice SSF of $\bar{g}^2_{\rm GF}$. The lattice data is in red while the black points are the continuum extrapolated results (see Ref. [12] for more details). Right: Non-perturbative β-function of the GF coupling. For comparison the LO and NLO perturbative results are shown, as well as the results for the non-perturbative β-function of the SF coupling at high energy [12]. In this plot: $\alpha = g^2/(4\pi)$, with g^2 the coupling in the given scheme.

Using these results and Eq. (5) the non-perturbative β-function of the GF coupling can be computed; this is shown in the right panel of Fig. 1, together with the LO and NLO perturbative predictions, and the non-perturbative β-function in the SF scheme. It is interesting to observe the peculiar behaviour of the non-perturbative GF β-function which lies very close to the LO perturbative result even at large values of the coupling, where $\alpha \approx 1$. Note however that the deviation from LO perturbation theory is statistically significant for the most part of the coupling range [12]. Only at values of $\alpha \approx 0.2$ the non-perturbative results start to approach the NLO prediction.

Once the β-function is known, we can compute the ratio of any two scales associated with two values of the coupling (cf. Eq. (5)). If we define the technical scale $\mu_{\rm had}$ through the relatively large value of the GF coupling: $\bar{g}^2_{\rm GF}(\mu_{\rm had}) = 11.31$, integrating the non-perturbative β-function we find [12]:

$$\frac{\mu_0}{\mu_{\rm had}} = 21.86(42) \quad \Rightarrow \quad \frac{\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}}}{\mu_{\rm had}} = 1.729(57). \tag{6}$$

Matching to hadronic physics and $\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}}$

Having bridged the gap between the high- and low-energy sectors of QCD, all that is left to do to determine $\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}}$ is to relate the technical scale $\mu_{\rm had}$ with some experimentally accessible quantity. Rather than establishing this relation directly, it is convenient to introduce an intermediate reference scale, $\mu^*_{\rm ref}$, so that:

$$\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}} = \frac{\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}}}{\mu_{\rm had}} \times \frac{\mu_{\rm had}}{\mu^*_{\rm ref}} \times \frac{\mu^*_{\rm ref}}{f_{\pi K}} \times f_{\pi K}. \tag{7}$$

For the scale $\mu^*_{\rm ref}$ we must choose a quantity that can be measured very precisely and easily in lattice simulations. The problem of computing $\mu_{\rm had}/f_{\pi K}$ is thus divided into computing the two ratios $\mu^*_{\rm ref}/f_{\pi K}$ and $\mu_{\rm had}/\mu^*_{\rm ref}$, for which we can consider different strategies in order to achieve the most accurate result. A quantity that satisfies many desirable properties in this respect is given by $\mu^*_{\rm ref} = 1/\sqrt{8t_0^*}$, where t_0^* is a specific flow time (cf. Eq. (2)), implicitly defined by the equation [9, 13, 2]:

$$0.3 = (t_0^*)^2 \langle E(t_0^*,x)\rangle|_{m_{u,d,s}=m_{\rm av,phys}}\,, \qquad E(t,x) = \frac{1}{4}G^a_{\mu\nu}(t,x)G^a_{\mu\nu}(t,x). \tag{8}$$

Note that the expectation value appearing in this equation is that of the theory in infinite space-time, i.e., with $L=\infty$. Moreover, it is evaluated at the SU(3) flavour-symmetric point where all quark masses are set equal to the physical average quark mass. As anticipated, $\mu^*_{\rm ref}$ can be determined very accurately in lattice QCD and with modest computational effort. This is also aided by the fact that it is measured at unphysical values of the quark masses which can be simulated with modest effort, differently from the physical situation which is often reached only through extrapolation. Clearly, $\mu^*_{\rm ref}$ is not measured in experiments, and its value in physical units must thus be fixed by relating it to some experimentally accessible quantity; in our case $f_{\pi K}$.

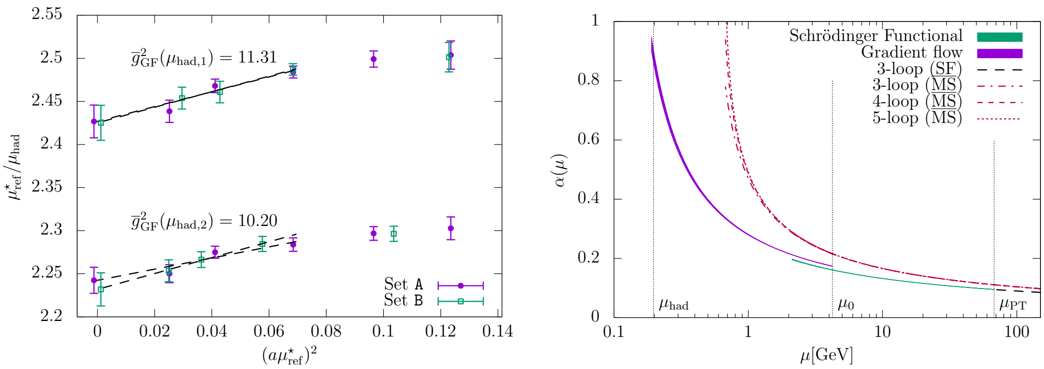

Figure 2: Left: Continuum extrapolations of the ratio $\mu^*_{\rm ref}/\mu_{\rm had}$. Note that as a consistency check of our strategy we considered also a second, larger, value for the technical scale $\mu_{\rm had}$ [2]. The two sets of data, labelled as A,B in the plot, refer to different analysis strategies [2]. Right: Running couplings of $N_{\rm f}=3$ QCD obtained from $\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}}$ by integrating the non-perturbative β-functions [2].

The value of $\mu^*_{\rm ref}$ in physical units was obtained in Ref. [13], to which we refer for any detail. Very briefly, employing an extensive set of state-of-the-art large volume simulations of $N_{\rm f}=3$ QCD [14] and a novel strategy for computing the relevant renormalization constants [15, 16], the precise continuum result: $\mu^*_{\rm ref}/f_{\pi K} = 3.24(4)$, was obtained. The particular combination $f_{\pi K} = \frac{2}{3}(f_K + \frac{1}{2}f_\pi)$ was considered as this showed a very mild quark-mass dependence for the chosen set of simulated quark masses. This allowed for robust and precise extrapolations to the physical quark-mass point; the latter identified by computing $\mu^*_{\rm ref}/m_{\pi,K}$, and taking as inputs the experimental values for the pion and kaon masses, m_π and m_K [17]. Using the PDG value for $f_{\pi K}$ [18], one finally arrives at: $\mu^*_{\rm ref} = 478(7)\,{\rm MeV}$.[2] The ratio $\mu^*_{\rm ref}/\mu_{\rm had}$ can now easily be evaluated using the

[2]Note that the hadronic inputs m_π, m_K, and $f_{\pi K}$, used to fix the bare quark masses and to set the physical scale of the lattice theory should be corrected for electromagnetic and $m_u \neq m_d$ effects [13]. This is necessary since our lattice results do not include QED effects and they assume equal up and down quark masses.

results for $a\mu^*_{\rm ref}$ at several values of the lattice spacing determined in the previous computation [13]. Through a small set of lattice QCD simulations of the SF, $a\mu_{\rm had}$ can indeed be obtained at matching values of the lattice spacing [2] and the ratio $(a\mu^*_{\rm ref})/(a\mu_{\rm had})$ be extrapolated to the continuum. Figure 2 collects these extrapolations, whose final result reads: $\mu^*_{\rm ref}/\mu_{\rm had} = 2.428(18)$ [2]. With this last bit of information at our disposal, we can quote (cf. Eq. (7))[2]:

$$\frac{\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}}}{\mu^*_{\rm ref}} = 0.712(24) \quad \Rightarrow \quad \Lambda^{N_{\rm f}=3}_{\overline{\rm MS}} = 341(12)\,\text{MeV}. \tag{9}$$

From $\Lambda^{N_{\rm f}=3}_{\overline{\rm MS}}$ and the non-perturbative β-functions of the SF and GF couplings, we can reconstruct the non-perturbative running of the couplings over the whole range of energy we covered, which goes from $\mu_{\rm had} \approx 200\,\text{MeV}$ up to $\mu_{\rm PT} = 16\mu_0 \approx 70\,\text{GeV}$. The result is shown in Fig. 2.

Heavy-quark decoupling and α_s

To compute α_s we need $\Lambda^{N_{\rm f}=5}_{\overline{\rm MS}}$. How can we obtain this from our result, Eq. (9)? The first issue we address concerns the determination of the scale $\mu^*_{\rm ref}$, which allows us to express the Λ-parameter in physical units. As described in the previous section, this determination is based on the computation of several low-energy quantities, $\mathscr{Q} = \mu^*_{\rm ref}/f_{\pi K}, \mu^*_{\rm ref}/m_{\pi,K}$, in $N_{\rm f} = 3$ QCD. Can we consider these results, and hence that for $\mu^*_{\rm ref}$, valid for the $N_{\rm f} = 4$ and 5 theories? The decoupling of heavy quarks tells us that for an heavy enough quark we should expect: $\mathscr{Q}_{N_{\rm f}} = \mathscr{Q}_{N_{\rm f}-1} + \mathscr{O}(\Lambda^2/M^2)$, where $\mathscr{Q}_{N_{\rm f}}$ denotes the low-energy quantity computed in the $N_{\rm f}$ theory where one flavour is much heavier than the others and has renormalization-group invariant mass M. Λ stands here for a generic low-energy scale of the theory, and clearly the $N_{\rm f} - 1$ theory is defined only in terms of the lighter quarks (see e.g. Ref. [19]). The $N_{\rm f} = 3$ results can therefore be considered legitimate for $N_{\rm f} = 4$ and hence 5, only if the charm mass M_c is actually large enough for the decoupling relation to be valid, and if the leading $\mathscr{O}(\Lambda^2/M_c^2)$ corrections are negligible within the given precision. Dedicated non-perturbative studies show that the typical $\mathscr{O}(\Lambda^2/M_c^2)$ effects in (dimensionless) low-energy quantities are in fact far below the percent level [20]. As the relevant observables are determined to a precision of $\approx 1\%$, we conclude that, within this precision, $\mu^*_{\rm ref}$ is well-determined from the results of $N_{\rm f} = 3$ QCD.

The second category of heavy quark effects we must discuss are those affecting the running of the coupling. It is well-known that in a massless renormalization scheme like the $\overline{\rm MS}$, the decoupling of heavy quarks is not "automatic". Hence, one typically works with the coupling of the relevant effective theory and matches the couplings of the theories with different flavour content according to: $\alpha^{(N_{\rm l})}_{\overline{\rm MS}}(\mu) = \xi^2(\alpha^{(N_{\rm f})}_{\overline{\rm MS}}, \overline{m}(\mu)/\mu)\, \alpha^{(N_{\rm f})}_{\overline{\rm MS}}(\mu)$, where $\overline{m}$ stands for the (renormalized) $N_{\rm f} - N_{\rm l}$ heavy quark masses and ξ is a computable function (see e.g. [21]). This allows one to write perturbative expansions that naturally contain only the "active" quarks at the energy scales of the processes of interest and avoids the appearance of large logarithms of the heavy quark masses in the computations. This matching between the two effective theories can equivalently be reformulated in terms of a relation between their Λ-parameters: $\Lambda^{N_{\rm l}}_{\overline{\rm MS}}/\Lambda^{N_{\rm f}}_{\overline{\rm MS}} = P_{\rm l,f}(M/\Lambda^{N_{\rm f}}_{\overline{\rm MS}})$. The function $P_{\rm l,f}$ is expected to be more accurately and reliably determined in perturbation theory the larger the invariant masses M of the decoupling quarks are. Thus, the relevant question in this case is how well does perturbation theory describe the function $P_{3,4}$ for values of M corresponding to the charm mass; for the decoupling of the bottom quark the situation is clearly expected to be better. This issue has been recently investigated in detail and the non-perturbative contributions to $P_{l,f}$ studied [21]. The conclusions of this work are that perturbation theory describes $P_{3,4}$ at the charm mass with a precision of at least 1.5% – likely much better. As our determination of $\Lambda_{\overline{\rm MS}}^{N_f=3}$ has a precision of $\approx 3.5\%$ (cf. Eq. (9)), this means that $\Lambda_{\overline{\rm MS}}^{N_f=5}$ can be safely obtained from $\Lambda_{\overline{\rm MS}}^{N_f=3}$ using perturbation theory.

We are now in the position of quoting our results for α_s. Taking as input our non-perturbatively determined $\Lambda_{\overline{\rm MS}}^{N_f=3}$, Eq. (9), the values of the charm and bottom masses $\overline{m}_{\overline{\rm MS}}^c$ and $\overline{m}_{\overline{\rm MS}}^b$ from the PDG [18], and the 4- and 5-loop results for the function ξ [22] and the β-function [23], respectively, perturbative decoupling predicts [2]:

$$\Lambda_{\overline{\rm MS}}^{N_f=3} \to \Lambda_{\overline{\rm MS}}^{N_f=5} = 215(10)(3)\,\text{MeV} \quad \Rightarrow \quad \alpha_{\overline{\rm MS}}^{(N_f=5)}(m_Z) = 0.11852(80)(25). \tag{10}$$

The second error in $\Lambda_{\overline{\rm MS}}^{N_f=5}$, then propagated to α_s, comes from an estimate within perturbation theory of the truncation errors in the perturbative expansion for $\Lambda_{\overline{\rm MS}}^{N_f=5}/\Lambda_{\overline{\rm MS}}^{N_f=3}$ [2]. Our final result for α_s has a precision of $\approx 0.7\%$ and it is well in agreement with the current PDG [18] and FLAG averages [17].

Conclusions

Lattice QCD offers a very powerful framework for determining α_s. By combining finite-volume couplings and a step-scaling strategy, we were able to obtain a subpercent precision determination of α_s where all systematic uncertainties are under control. These include the specific lattice QCD systematics, i.e., discretization and finite-volume effects, as well as the unavoidable uncertainties originating from the use of perturbation theory in extracting α_s. Our result for α_s in based on a determination of $\Lambda_{\overline{\rm MS}}^{N_f=3}$ which relies on perturbation theory only at energy scales of $\mathcal{O}(100\,\text{GeV})$, where we proved it accurate. The strong coupling was then extracted using perturbative decoupling to match the $N_f = 3$ and $N_f = 5$ theories. We argued that non-perturbative corrections to the decoupling relations are not important at our level of precision.

The dominant source of error in our α_s determination comes from $\Lambda_{\overline{\rm MS}}^{N_f=3}/\mu_0$ (cf. Eq. (1)); in other words from the computation of the non-perturbative running of the SF coupling from about 4 to 70 GeV [2]. This error is predominantly statistical and can therefore be straightforwardly reduced. We want to stress that most other lattice determinations of α_s avoid computing the running of the coupling in this energy range by relying on perturbation theory already at a few GeV (see e.g. refs. [17, 24]). In these cases, one ends up dealing with an error which is mostly systematic, and thus much harder to reliably quantify. In the first part of this overview [1], we showed with concrete examples how estimating this sort of error can indeed be very difficult at the level of precision we aim for α_s.

In the near future we expect to be able to reduce our error on $\Lambda_{\overline{\rm MS}}^{N_f=3}$ to about 2%, which would correspond to an error of 0.5% on α_s. To further halve this error, on the other hand, requires several issues to be reconsidered. Non-perturbative decoupling effects might not be negligible anymore, and one might need to include electromagnetic and $m_u \neq m_d$ effects in the lattice computations in order to set the physical scale of the theory to a greater level of accuracy.

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