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α_s determination from static QCD potential with renormalon subtraction

Hiromasa Takaura*
Department of Physics, Kyushu University, Fukuoka 819-0395, Japan
E-mail: takaura@phys.kyushu-u.ac.jp

In the current α_s determinations based on lattice and perturbation theory, it is generally difficult to take a wide enough matching range. We avoid this problem by improvement of a theoretical calculation: we use the OPE with renormalon subtraction, which is an extended framework of perturbation theory. This allows us to take the matching range widely as $\Lambda_{\rm QCD} r \lesssim 0.6$, where relatively low energy scales are included. We obtain $\alpha_s(m_Z) = 0.1179^{+0.0015}_{-0.0014}$ from a reasonable fit for this wide range.

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Introduction

The strong coupling α_s is a fundamental parameter in the standard model, and its precision has an impact on various studies of the standard model. This parameter is determined by a matching of a theoretical calculation and an experimental or lattice measurement of a QCD observable. Among determinations from various observables, the determinations using lattice data generally have small errors.

In lattice determinations, however, the so-called window problem has been pointed out: it is difficult to take a wide enough matching range. Accurate lattice simulation can be performed at the scale well below its UV cutoff scale a^{-1}, the inverse of the lattice spacing. This lattice result is matched with perturbation theory, where fixed order results are currently accurate at $Q \gtrsim 1$–2 GeV. With the typical lattice spacings available today, it is difficult to take the range satisfying 1–2 GeV $\lesssim Q \ll a^{-1}$ widely.

The step-scaling method is known as a solution to the window problem. This method enlarges the validity range of the lattice simulations. (The latest determination has been performed in Ref. [1].) As an alternative approach, we enlarge the validity range of a theoretical calculation to lower energy so that accurate lattice data (due to $Q \ll a^{-1}$) are available. To this end, we use the operator product expansion (OPE), which is an extended framework of perturbation theory.

Perturbation theory suffers from an inevitable uncertainty known as renormalon uncertainty. It is induced from the divergent behavior of perturbative series where perturbative coefficients typically grow as $\sim \beta_0^n n!$ at large orders. For the static QCD potential $V_{\rm QCD}(r)$, the leading renormalon uncertainty is $\mathcal{O}(\Lambda_{\rm QCD})$, and the next-to-leading one is $\mathcal{O}(\Lambda_{\rm QCD}^3 r^2)$. These errors are not negligible at relatively long distances $\Lambda_{\rm QCD} r \sim 1$, and give limitations of perturbation theory.

In the OPE, which can be regarded as an extension of perturbation theory, renormalon uncertainties are considered to be eliminated. In the following, we focus on the second renormalon uncertainty rather than the first one (which is r-independent and can be eliminated in the QCD force). The OPE of the static QCD potential is performed in the effective field theory, potential non-relativistic QCD (pNRQCD) [2]. It is given in form of multipole expansion as

$$V_{\rm QCD}(r) = V_S(r) + \delta E_{\rm US}(r) + \cdots , \tag{1}$$

where the singlet potential $V_S(r)$ has a Coulomb-type potential and is the leading behavior at short distances ($V_S(r) \sim 1/r$). A power correction in r is added as $\mathcal{O}(r^2)$ ($\delta E_{\rm US}(r) \sim r^2$). Since the second term (and further higher order terms in r) are nonperturbative objects[1], a perturbative expression of $V_S(r)$ coincides with that of $V_{\rm QCD}(r)$. Therefore, V_S contains the renormalon uncertainty of $\mathcal{O}(\Lambda_{\rm QCD}^3 r^2)$. An advantage of the OPE is that this renormalon is cancelled against that of the second term $\delta E_{\rm US}(r)$. This has been shown explicitly in Ref. [2] at the leading-log (LL) level. Then the OPE prediction has smaller error and has wider validity range than perturbation theory.

However, it is difficult to hold this advantage of the OPE in practical calculations. In particular, with a naive perturbative calculation of V_S, one again suffers from the renormalon uncertainty. Consider the case where one adds a power correction term of Ar^2 to the perturbative result V_S. The

[1] Our fit range extends to relatively low energy scale where the ultrasoft scale is not generally perturbative. Hence, the ultrasoft scale is treated as the nonperturbative scale in our analysis.

fitting parameter A [of $\mathcal{O}(\Lambda_{\rm QCD}^3)$] can be extracted from the r^2-term of $V_{\rm QCD}(r) - V_S(r)$. However, since V_S has the error of $\mathcal{O}(\Lambda_{\rm QCD}^3 r^2)$, the nonperturbative effect A has a significant error. (The error is the same size as the nonperturbative effect itself.) Thus, the introduction of the power correction is almost meaningless because its coefficient cannot be determined in practice.

To avoid this feature, we use the OPE while subtracting renormalons in $V_S(r)$. The use of such an OPE allows us to use a wider range as shown below, and thus, it relaxes the window problem. Our fit range is typically taken as 0.6 GeV $\lesssim r^{-1} \lesssim$ 4 GeV. This is significantly wider than previous determinations from the static QCD potential, where typically 1 GeV $\lesssim r^{-1}$ has been used.

Theoretical framework

We explain how we subtract renormalons and how we use the result in the OPE. First, we consider renormalon subtraction from V_S. $V_S(r)$ is given by

$$V_S(r) = -4\pi C_F \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \ \frac{\alpha_V(q)}{q^2} \quad (q = |\vec{q}|)\,, \tag{2}$$

where the potential in momentum space $\alpha_V(q)$ is currently known up to $\mathcal{O}(\alpha_s^4)$ [3]. We apply renormalization group (RG) improvement to $\alpha_V(q)$, i.e., we use the next-to-next-to-next-to-LL (N^{3}LL) result $\alpha_V(q)_{\rm N^3LL}$. Then, the above integral becomes just formal because $\alpha_V(q)_{\rm N^3LL}$ has a singularity at $q \sim \Lambda_{\rm QCD}$ due to the running coupling. In other words, the q-integration is ambiguous and this corresponds to the renormalon uncertainty. In fact, all the known renormalons of the static QCD potential stem from the q integration of the logarithmic terms in $\alpha_V(q)$. In order to render the integral well-defined, we subtract the IR contribution by an IR cutoff scale μ_f:

$$V_S(r;\mu_f) = -4\pi C_F \int_{q>\mu_f} \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{\alpha_V(q)_{\rm N^3LL}}{q^2}\,, \tag{3}$$

where μ_f is taken as $\Lambda_{\rm QCD} \ll \mu_f \ll r^{-1}$. The integral is now well-defined. However, it depends on the artificial cutoff scale. This dependence cannot be removed within perturbation theory. (Note that the cutoff μ_f cannot be sent to zero due to the singularity.) In this sense, this cutoff dependence corresponds to a renormalon uncertainty. On the other hand, a cutoff independent part, which potentially exists, is unambiguously determined within perturbation theory. It corresponds to a renormalon-free part.

To find a renormalon-free part, we separate the cutoff independent part from the cutoff dependent part following Ref. [4]. This is performed by a contour deformation in the complex q-plane. We obtain [4]

$$V_S(r;\mu_f) = V_S^{\rm RF}(r) + \mathcal{O}(\mu_f) + \mathcal{O}(\mu_f^3 r^2)\,, \tag{4}$$

where $V_S^{\rm RF}(r)$ is μ_f independent and renormalon free. It has a Coulomb+linear like form. The cutoff dependence of $\mathcal{O}(\mu_f)$ and $\mathcal{O}(\mu_f^3 r^2)$ correspond to the first and second renormalon uncertainties, respectively.

We list the distinct features of $V_S^{\rm RF}$. First, it has the N^{3}LL accuracy and is accurate especially at short distances. Secondly, unlike the usual RG improvement, it does not have an unphysical singularity at $r^{-1} \sim \Lambda_{\rm QCD}$, generally caused by the running coupling. Thirdly, it is free from the

renormalon uncertainties of $\mathcal{O}(\mu_f)$ and $\mathcal{O}(\mu_f^3 r^2)$. From the last two features, it is expected that $V_S^{\rm RF}$ gives a reasonable prediction even at relatively long distances.

The above perturbative result $V_S^{\rm RF}(r)$ is used as follows in the context of the OPE. Since the IR cutoff scale is introduced to the perturbative calculation, the contribution below μ_f is represented by nonperturbative objects. Then, we introduce the UV cutoff scale to $\delta E_{\rm US}(r)$ as $\delta E_{\rm US}(r;\mu_f)$. In fact, a similar separation of cutoff dependence can be performed for $\delta E_{\rm US}(r;\mu_f)$, where the opposite cutoff dependence of $\mathcal{O}(\mu_f^3 r^2)$ to $V_S(r;\mu_f)$ is found. That is, the cutoff dependence in $V_S^{\rm RF}(r;\mu_f)$ cancels[2] that of $\delta E_{\rm US}^{\rm RF}(r;\mu_f)$ [5]. Thus, we can perform the OPE in a renormalon-free way:

$$V_{\rm QCD}(r) = V_S^{\rm RF}(r) + \delta E_{\rm US}^{\rm RF}(r) + \cdots . \tag{5}$$

This is the OPE calculation used in our α_s determination. $V_S^{\rm RF}(r)$ can be calculated at the N^{3}LL accuracy and has $\Lambda_{\overline{\rm MS}}$ as the only input parameter. We treat $\delta E_{\rm US}^{\rm RF}(r) = A_2 r^2$ where A_2 is a fitting parameter. The difference from the naive OPE is that we subtract the renormalons of $V_S(r)$. This prevents a mixing of the renormalon uncertainty and the nonperturbative effect. Also, it serves to reduce higher order uncertainty of V_S.

α_s determination

The above calculation is matched with lattice result to determine α_s [6]. We use the lattice result of $V_{\rm QCD}(r)$ obtained by the JLQCD collaboration. The simulated lattice sizes are $32^3 \times 64$, $48^3 \times 96$, $64^3 \times 128$, whose lattice spacings are estimated as $a^{-1} = 2.453(4), 3.610(9), 4.496(9)$ GeV, respectively.

In our α_s determination, we perform two analyses. The first one [Analysis (I)] is a step-by-step analysis and the other is a global fit [Analysis (II)]. After examining detailed profiles in Analysis (I), we perform a global fit in Analysis (II), from which our final result is obtained. In this report, we present a consistency check of the OPE, which is a central concern in Analysis (I), and then explain Analysis (II).

We examine consistency of the OPE in Fig. 1, where we compare $V_S^{\rm RF}(r)$ with the lattice continuum limit. Here, we use the PDG value of $\Lambda_{\overline{\rm MS}}$ as an input. According to the OPE, the difference between the lattice result and $V_S^{\rm RF}(r)$, which is shown by the red boxes in the figure, should behave as $\mathcal{O}(r^2)$. Indeed, it is consistent with a quadratic behavior in r up to $\Lambda_{\overline{\rm MS}} r \lesssim 0.8$. Thus, the validity range of the OPE turns out to be $\Lambda_{\overline{\rm MS}} r \lesssim 0.8$. This is significantly larger than that of perturbation theory, $\Lambda_{\overline{\rm MS}} r \lesssim 0.3$.

Now, we explain Analysis (II). This analysis is performed based on the idea that at short distances the OPE prediction should coincide with the lattice result once the discretization errors are removed. Then we assume the continuum limit as

$$V_{\rm latt}^{\rm cont}(r) = V_{{\rm latt},d,i}(r) - \kappa_{d,i}\left(\frac{1}{r} - \left[\frac{1}{r}\right]_{d,i}\right) + f_d \frac{a_i^2}{r^3} - c_{0,d,i}\,, \tag{6}$$

where $V_{{\rm latt},d,i}(r)$ is the original lattice data measured at the i-th lattice ($i = 1,2,3$) and d denotes the direction of $\vec{r}$;[3] the second term is a tree-level correction, where $[1/r]$ denotes the LO result of the

[2] This is confirmed at the LL level.

[3] $d = 1$ and $d = 2$ correspond to the spatial directions $(1,0,0)$ and $(1,1,0)$, respectively.

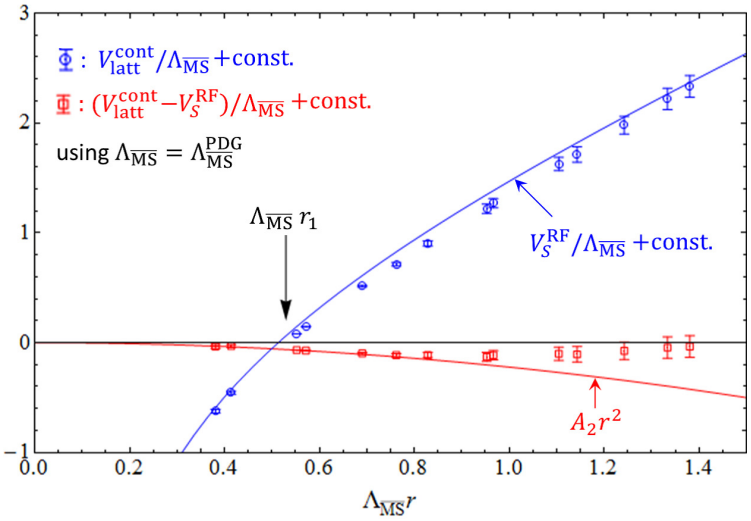

Figure 1: Consistency check of the OPE. The blue date are the lattice continuum limit and the blue line is $V_S^{\rm RF}$. (Both are given in $\Lambda_{\overline{\rm MS}}$ units.) The difference between them are shown by the red data, which are consistent with the quadratic function given by the red line at $\Lambda_{\overline{\rm MS}} r \lesssim 0.8$.

Table 1: Systematic errors in $\alpha_{\rm s}(m_{\rm Z})$ in units of 10^{-4}. See [6] for details.

finite a	Mass	H.o.	Range	Ultrasoft	Fact. scheme	Latt. spacing
± 2	± 0	$^{+12}_{-10}$	± 4	± 2	± 3	± 4

lattice perturbation theory; the third term removes the remaining error of $\mathcal{O}(\alpha_{\rm s}^2 a^2)$; the last term adjusts an r-independent constant. We give the above lattice result in GeV units. This is matched with the OPE prediction in the same units:

$$V_{\rm OPE}(r) = z[V_S^{\rm RF}/\Lambda_{\overline{\rm MS}}](zr) + A_2 r^2\,, \tag{7}$$

where $z = \Lambda_{\overline{\rm MS}}$ GeV. (Note that $V_S(r)$ is originally obtained in $\Lambda_{\overline{\rm MS}}$ units.) In this global fit, we determine 16 parameters in total: $\{z, A_2, \kappa_{d,i}, f_d, c_{0,d,i}\}$. The fit range is $0.07 \le \Lambda_{\overline{\rm MS}}^{\rm PDG} r < 0.6$, which includes not only short but also relatively long distances.

As a result, we obtain $\alpha_{\rm s}(m_{\rm Z}) = 0.1179 \pm 0.0007\,({\rm stat})$, where $\chi^2/{\rm d.o.f.} \approx 8.7/14$ shows the validity of this analysis. After including the systematic errors listed in Table 1, we obtain

$$\alpha_{\rm s}(m_{\rm Z}) = 0.1179 \pm 0.0007\,({\rm stat})^{+0.0014}_{-0.0012}\,({\rm sys}) = 0.1179^{+0.0015}_{-0.0014}\,. \tag{8}$$

which is consistent with the current world average.

Conclusions

Lattice determinations often suffer from the window problem: a matching range cannot be taken sufficiently wide. To avoid this problem, we use the OPE with renormalon subtraction, which is an extended framework of perturbation theory. Such an OPE has the wider validity range than perturbation theory. The fit is performed reasonably for the wide range, which would lead to a reliable value of α_s. The dominant uncertainty in our determination comes from higher order uncertainty. It can be reduced with finer lattice simulations.

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