Nucleon-to-meson transition distribution amplitudes in impact parameter space

Bernard Pire
CPHT, CNRS, École Polytechnique, I. P. Paris, FR-91128 Palaiseau, France
E-mail: bernard.pire@polytechnique.edu

Kirill Semenov-Tian-Shansky[∗]
National Research Centre "Kurchatov Institute": Petersburg Nuclear Physics Institute, RU-188300 Gatchina, Russia
E-mail: cyrstsh@thd.pnpi.spb.ru

Lech Szymanowski
National Centre for Nuclear Research (NCBJ), Pasteura 7, PL-02-093 Warsaw, Poland
E-mail: Lech.Szymanowski@ncbj.gov.pl

Recent analyses of backward meson electroproduction support the validity of a collinear QCD factorization framework for these hard exclusive reactions. This opens a way to the extraction of nucleon-to-meson transition distribution amplitudes (TDAs) from the experimental data. Similarly to the generalized parton distributions, TDAs - after the Fourier transform in the transverse plane - carry valuable information on the transverse location of hadron constituents. We address the properties of integrated nucleon-to-meson TDAs in the impact parameter representation. We argue that the emerging picture provides an intuitive interpretation for the hadron structural information contained in nucleon-to-meson TDAs and allows to study diquark-quark contents of fast moving hadrons in the transverse plane.

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[∗]Speaker.

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1. Introduction

Nucleon-to-meson Transition Distribution Amplitudes (TDAs) [1, 2] occur within the collinear factorized description of a class of hard exclusive reactions with a non-zero baryon number exchange in the cross channel. Prominent examples of such reactions are the backward hard electroproduction of mesons off nucleons [3, 4] and nucleon-antinucleon annihilation into a lepton pair (or a heavy quarkonium) associated with a meson [5, 6]. According to the usual logic of the QCD collinear factorization approach, nucleon-to-meson TDAs are universal non-perturbative objects defined as nucleon-meson matrix elements of a three-quark light-cone operator. Nucleon-to-meson TDAs share common features both with baryon Distribution Amplitudes (DAs) and with Generalized Parton Distributions (GPDs) (see respectively Refs. [7] and [8] for reviews).

Recent experimental studies [9, 10, 11] brought first evidences in favor of the validity of the reaction mechanism involving nucleon-to-meson TDAs for the description of backward pion and ω-electroproduction at JLab kinematical conditions. The perspective to access nucleon-to-meson TDAs experimentally rises a high demand for refining their physical contents. In this paper, relying on the similarity between nucleon-to-meson TDAs and GPDs, we build an intuitive physical picture for TDAs in the impact parameter space.

2. Integrated πN TDAs and quark-diquark picture of the nucleon

For definiteness, throughout this paper, we consider the case of the proton (N^p)-to-π^0 *uud* TDAs. However, our results admit a straightforward generalization for other isospin channels for πN TDAs as well as for more involved cases. The leading twist-3 proton-to-π^0 TDAs are defined through the Fourier transform of N^p-π^0 matrix element of the *uud* trilocal operator on the light cone ($n^2 = 0$)[1]:

$$4(P\cdot n)^3 \int \left[\prod_{i=1}^{3} \frac{d\lambda_i}{2\pi}\right] e^{i(\lambda_1 x_1+\lambda_2 x_2+\lambda_3 x_3)(P\cdot n)} \langle \pi^0(p_\pi)|\, u_\rho(\lambda_1 n) u_\tau(\lambda_2 n) d_\chi(\lambda_3 n)\, |N^p(p_N;s_N)\rangle$$
$$= \delta(x_1+x_2+x_3-2\xi) \sum_{\substack{H=V_{1,2},A_{1,2},\\ T_{1,2,3,4}}} h^H_{\rho\tau,\chi} H^{\pi^0 N^p}(x_1,x_2,x_3,\xi,\Delta^2). \tag{2.1}$$

Here n is the light-cone vector $n^2 = 0$. For simplicity we adopt the light-like gauge $A\cdot n = 0$ and therefore omit the gauge links in the trilocal quark operator. Antisymmetrization in quark color indices (which we do not show explicitly) is assumed. The notations for kinematical variables follow the usual conventions: $P = \frac{p_N+p_\pi}{2}$ is the average momentum, $\Delta = p_\pi - p_N$ is the momentum transfer between the meson and the nucleon; s_N denotes the nucleon polarization variable. The sum in the r.h.s. of (2.1) stands over the 8 independent Dirac structures $h^H_{\rho\tau,\chi}$ relevant at the leading twist-3 accuracy. Each of the 8 proton-to-π^0 leading twist-3 *uud* TDAs are functions of the 3 light-cone momentum fractions x_i, the skewness variable $\xi = -\frac{\Delta\cdot n}{2P\cdot n}$ defined with respect to the longitudinal momentum transfer, momentum transfer squared Δ^2 as well as of the factorization scale.

[1] The explicit form of the parametrization of the leading twist-3 πN TDAs is given in Eq. (10) of Ref. [12]. Here, for simplicity, we assume that the overall normalization factor $i\frac{f_N}{f_\pi M_N}$ is included into the definition of invariant TDAs.

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The support domain of nucleon-to-meson TDAs in the momentum fraction variables x_i has been worked out in Ref. [13]. In the barycentric coordinates defined by the longitudinal momentum constraint $\sum_i x_i = 2\xi$ it is given by the intersection of 3 stripes $-1+\xi \leq x_i \leq 1+\xi$. Similarly to the GPD case, it is natural to single out the Efremov-Radyushkin-Brodsky-Lepage (ERBL)-like region, in which all three quark longitudinal momentum fractions are positive, and two types of Dokshitzer-Gribov-Lipatov-Altarelli- Parisi (DGLAP)-like regions, where either one or two quark longitudinal momentum fractions are positive and the remaining two or one are negative (see Fig. 1). The DGLAP-like-I,II and ERBL-like domains are separated by the cross-over lines $x_i = 0$. Note, that, contrary to the GPD case, the complete support of nucleon-to-meson TDAs depends on ξ (see Fig. 1).

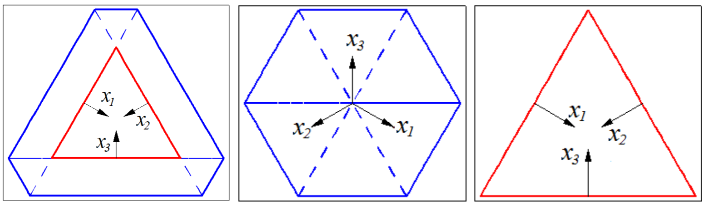

Figure 1: Physical domains for πN TDAs in the barycentric coordinates $\sum_i x_i = 2\xi$. **Left panel:** general case $0 < \xi < 1$. **Central panel:** case $\xi = 0$. **Right panel:** case $\xi = 1$.

It turns out to be convenient to switch to two independent momentum fraction variables instead of x_i that are subject to the constraint $\sum_i x_i = 2\xi$. A natural choice of independent variables is given by the so-called quark-diquark coordinates (w_i, v_i) (there exist 3 equivalent choices $i = 1, 2, 3$ of quark-diquark coordinates, depending on which pair of quark momenta is selected to constitute the momentum of a diquark): $w_i = x_i - \xi$; $v_i = \frac{1}{2}\sum_{k,l=1}^{3} \varepsilon_{ikl} x_k$. Within these coordinates the support of nucleon-to-meson TDAs can be parameterized as

$$-1 \leq w_i \leq 1; \quad -1+|\xi-\xi_i'| \leq v_i \leq 1-|\xi-\xi_i'| \;\; \text{with} \;\; {\xi'}_i \equiv \frac{\xi - w_i}{2}. \tag{2.2}$$

In order to make contact to a quark-diquark picture we would now like to introduce πN TDAs integrated over the momentum fraction variable v_i. For definiteness we choose it to be $v_3 \equiv \frac{x_1 - x_2}{2}$. We employ the following identity for the exponent in (2.1):

$$e^{i(x_1\lambda_1 + x_2\lambda_2 + x_3\lambda_3)P\cdot n} = e^{i(\frac{x_1-x_2}{2}(\lambda_1-\lambda_2) + \frac{x_1+x_2}{2}(\lambda_1+\lambda_2) + x_3\lambda_3)P\cdot n} \equiv e^{i(v_3(\lambda_1-\lambda_2)+\xi_3'(\lambda_1+\lambda_2)+x_3\lambda_3)P\cdot n}. \tag{2.3}$$

Then we integrate the equality (2.1) in v_3 over the interval $(-\infty;\infty)$. In the r.h.s. we may integrate just over the v_3 support (2.2) of πN TDAs. In the l.h.s. the integration of the exponent results in the delta function $\delta(\lambda_1 - \lambda_2)$. This latter delta function can be employed to remove one of the λ-integrals. It brings the arguments of the two u-quark operators to the same point on the light cone $\lambda_1 n = \lambda_2 n \equiv \lambda_D n$ and gives rise to the hadronic matrix element of a bilocal light-cone operator:

$$\widehat{O}_{\rho\tau\chi}^{\{uu\}d}(\lambda_D n, \lambda_D n, \lambda_3 n) = u_\rho(\lambda_D n) u_\tau(\lambda_D n) d_\chi(\lambda_3 n) \equiv \widehat{D}_{\rho\tau}(\lambda_D n) d_\chi(\lambda_3 n). \tag{2.4}$$

It is natural to interpret this operator as the bilocal uu-diquark- d-quark operator on the light-cone. Now we employ the translation invariance of the matrix element to translate the arguments of the bilocal operator (2.4) to the symmetric points $\pm\frac{\lambda}{2}$ on the light-cone introducing $\lambda \equiv \lambda_3 - \lambda_D$ and $\mu \equiv \lambda_3 + \lambda_D$. The integral over μ can be performed producing the momentum conservation δ-function:

$$\int \frac{d\mu}{2\pi} e^{i(x_D+x_3-2\xi)(P\cdot n)2\mu} = \frac{1}{2(P\cdot n)} \delta(x_D+x_3-2\xi); \quad x_D \equiv x_1+x_2 = 2\xi_3'. \tag{2.5}$$

Finally, we get the equality relating the Fourier transform of the nucleon-pion matrix element of the light-cone diquark-quark operator (2.4) to the v_3-integrated nucleon-to-pion TDAs

$$\begin{aligned} & 2(P\cdot n) \int \frac{d\lambda}{4\pi} e^{i(w_3\lambda)(P\cdot n)} \langle \pi(p_\pi)|\widehat{D}^{uu}_{\rho\tau}(-\frac{\lambda}{2}n) d_\chi(\frac{\lambda}{2}n)|N(p_N)\rangle \\ &= \sum_{H=V_{1,2},A_{1,2},T_{1,2,3,4}} h^H_{\rho\tau,\chi} \int_{-1+|\xi-\xi_3'|}^{1-|\xi-\xi_3'|} dv_3 H^{\pi^0 N^p}(w_3, v_3, \xi, \Delta^2). \end{aligned} \tag{2.6}$$

The v_3-integrated TDAs occurring in the r.h.s. of Eq. (2.6) share many common features with GPDs. Namely, they are functions of one longitudinal momentum fraction $w_3 = \frac{x_3-x_D}{2} \in [-1;1]$, of skewness ξ, of invariant momentum transfer Δ^2 and of factorization scale. As a consequence of the Lorentz invariance the Mellin moments of v_3-integrated TDAs possess the usual polynomiality property in ξ. For the v_3-integrated TDAs it is natural to specify the ERBL-region with $w_3 \in [-\xi;\xi]$, DGLAP-I region $w_3 \in [-1;-\xi]$ and DGLAP-II region $w_3 \in [\xi;1]$. We now propose an interpretation of these objects in the impact parameter space. This allows to use v-integrated TDAs as a tool to study the quark-diquark structure of hadrons in the transverse plane. It is worth emphasizing that, contrary to GPDs, the v-integrated TDAs do not possess a comprehensible forward limit in which a probabilistic interpretation [14, 15] applies for GPDs. Thus, similarly to the case of GPDs with non-zero skewness, we get an interpretation in terms of the probability amplitudes.

3. Integrated πN TDAs in impact parameter space

The use of the impact parameter representation for building up a vivid physical picture of hadrons in the transverse plane has been pioneered by M. Burkardt for GPDs in the zero skewness limit in Refs. [14, 15]. The extension of this framework for GPDs with non-zero skewness was proposed in Refs. [16, 17]. Since the general structure of v-integrated nucleon-to-meson TDAs looks similar to GPDs it is natural to adopt for them the impact parameter space representation.

The first step consists in introducing the initial nucleon and final meson states with specified longitudinal momenta localized around a definite position $\mathbf{b}$ in the transverse plane:

$$|p_N^+, \mathbf{b}; s_N\rangle = \int \frac{d^2\mathbf{p}_N}{16\pi^3} e^{-i\mathbf{p}_N\cdot\mathbf{b}} |p_N^+, \mathbf{p}_N; s_N\rangle; \quad \langle p_\pi^+, \mathbf{b}| = \int \frac{d^2\mathbf{p}_\pi}{16\pi^3} e^{i\mathbf{p}_\pi\cdot\mathbf{b}} \langle p_\pi^+, \mathbf{p}_\pi|. \tag{3.1}$$

Rigorous treatment requires forming wave packets with precisely localized states (3.1) using a smooth weight falling sufficiently fast at infinity with $|\mathbf{p}|$ in order to avoid infinities due to normalization. A possible choice is to employ a Gaussian wave packets with the same standard deviation parameter for the initial nucleon and the final meson.

Switching to the impact parameter space representation is performed by Fourier transforming the corresponding operator hadronic matrix element with respect to the transverse component $\mathbf{D}$

of the vector $D = \frac{p_\pi}{1-\xi} - \frac{p_N}{1+\xi}$. By construction, the transverse component $\mathbf{D}$ is invariant under the transverse boosts. This ensures that the invariant momentum transfer depends on $\mathbf{p}_N$ and $\mathbf{p}_\pi$ only through $\mathbf{D}$: $\Delta^2 = -2\xi\left(\frac{m_\pi^2}{1-\xi} - \frac{M_N^2}{1+\xi}\right) - (1-\xi^2)\mathbf{D}^2$.

We consider the matrix elements of the diquark-quark operator with the explicit dependence on the transverse position $\mathbf{z}$:

$$\widehat{\mathscr{O}}^{\{uu\}d}_{\rho\tau\chi}(\mathbf{z}) = \int \frac{d\lambda}{4\pi} e^{i(w_3\lambda)(P\cdot n)} u_\rho(0, -\frac{\lambda}{2}, \mathbf{z}) u_\tau(0, -\frac{\lambda}{2}, \mathbf{z}) d_\chi(0, \frac{\lambda}{2} n, \mathbf{z}), \tag{3.2}$$

where we adopt the following convention for the position arguments of quark fields: $q(z) = q(z^+, z^-, \mathbf{z})$.

In order to single out a particular combination of πN TDAs we contract the matrix element of the operator (3.2) over the Dirac indices with a suitable projector. Assuming that for πN TDAs we employ the parametrization of Eq. (10) of Ref. [12] we, as an example, have chosen to contract the matrix element (3.2) with[2] $v^{-1}_{\tau\rho,\chi} = \left(C^{-1}\hat{P}\right)_{\tau\rho} \bar{U}_\chi(p_N, s_N)$. Since we deal with unpolarized nucleon, we sum and average over the nucleon spin s_N. This convolution singles out the following combination of v_3-integrated πN TDAs:

$$\mathscr{H}^{\pi N}(w_3, \xi, \mathbf{D}) = (4M_N^2 - \Delta^2) \int_{-1+|\xi-\xi_3'|}^{1-|\xi-\xi_3'|} dv_3 \frac{1}{2} \Big[(3M_N^2 + m_\pi^2 - \Delta^2) V_1^{\pi N}(w_3, v_3, \xi, \Delta^2)$$
$$+ (-2\Delta^2 - 2M_N^2 + 2m_\pi^2) V_2^{\pi N}(w_3, v_3, \xi, \Delta^2) \Big], \tag{3.3}$$

where Δ^2 is expressed through $\mathbf{D}^2$. The contraction with different projectors[3] will allow to single out other combinations of 8 leading twist-3 proton-to-π^0 uud TDAs. The transition to the impact parameter space then gives

$$\int \frac{d^2\mathbf{D}}{(2\pi)^2} e^{-i\mathbf{D}\cdot\mathbf{b}} \mathscr{H}^{\pi N}(w, \xi, \mathbf{D})$$
$$= \mathscr{N}^{-1} \frac{1+\xi^2}{(1-\xi^2)^2} \sum_{s_N} \left(C^{-1}\hat{P}\right)_{\tau\rho} \bar{U}_\chi(p_N; s_N) \left\langle p_\pi^+, -\frac{\xi\mathbf{b}}{1-\xi} \,\middle|\, \widehat{\mathscr{O}}^{\{uu\}d}_{\rho\tau\chi}(\mathbf{b}) \,\middle|\, p_N^+, \frac{\xi\mathbf{b}}{1+\xi}; s_N \right\rangle, \tag{3.4}$$

where $\mathscr{N}$ is the normalization factor originating from the normalization of the localized states (3.1). Without use of the smooth wave packets it turns to be singular as $\delta^{(2)}(\mathbf{0})$.

Thus we end up with a picture that is completely analogous to the GPD case: the hard probe interacts with a partonic configuration at the transverse position $\mathbf{b}$. The initial state nucleon and the finite state meson are localized around $\mathbf{0}$, but they are shifted one from another by a transverse separation of the order $\xi\mathbf{b}$. This interpretation is presented in Fig. 2. It is also qualitatively consistent with the low Fock component picture proposed in [18].

- In the DGLAP-like I region $w_3 \leq -\xi$ the impact parameter specifies the location where a uu-diquark is pulled out of a proton and then replaced by an antiquark $\bar{d}$ to form the final state meson.

[2] We employ Dirac's "hat" notation $\hat{a} \equiv \gamma_\mu a^\mu$; C is the charge conjugation matrix.

[3] A proper design of the projecting operation establishing connection with corresponding diquark-quark helicity amplitudes still has to be developed.

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- In the DGLAP-like II region $w_3 \geq \xi$ the impact parameter specifies the location where a quark d is pulled out of a proton and then replaced by an antidiquark $\bar{u}\bar{u}$ to form the final state meson.

- In the ERBL-like region $-\xi \leq w_3 \leq \xi$ the impact parameter specifies the location where a three-quark cluster composed of a uu-diquark and a d-quark is pulled out of the initial nucleon to form the final state meson.

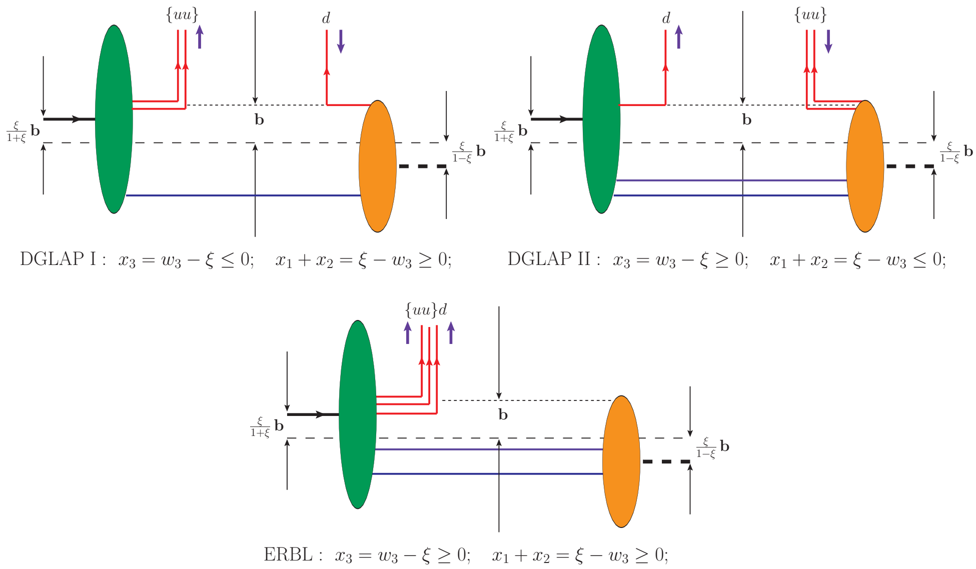

Figure 2: Impact parameter space interpretation for the v_3-integrated uud πN TDA in the DGLAP-like I, DGLAP-like II and in the ERBL-like domains. Solid arrows show the direction of the positive longitudinal momentum flow.

A complementary picture can be obtained from the v_1-integrated πN TDAs. This corresponds to a diquark constructed out of the third and second quarks (du). It makes sense to perform the Fierz transform (see App. B3 of [12]) to the relevant set of the Dirac structures: $h^{\pi N}_{\rho\tau,\chi} \to h^{\pi N}_{\chi\tau,\rho}$. The projection $v^{-1}_{\tau\rho,\chi}$ then involves a different combination of TDAs. The third possible picture resulting from the v_2-integrated πN TDAs should be analogous to the v_1-integrated case since it also corresponds to a $\{ud\}u$-diquark-quark operator.

4. Conclusions

In this paper we propose an interpretation for v-integrated nucleon-to-pion TDAs in the impact parameter space. It offers an intuitive interpretation of the information contained in nucleon-to-meson TDAs in terms of the diquark-quark contents of the corresponding hadrons.

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References

[1] L. L. Frankfurt, P. V. Pobylitsa, M. V. Polyakov and M. Strikman, *Hard exclusive pseudoscalar meson electroproduction and spin structure of a nucleon*, *Phys. Rev. D* **60** (1999) 014010 [hep-ph/9901429].

[2] B. Pire and L. Szymanowski, *Hadron annihilation into two photons and backward VCS in the scaling regime of QCD*, *Phys. Rev. D* **71** (2005) 111501 [hep-ph/0411387].

[3] J. P. Lansberg, B. Pire, K. Semenov-Tian-Shansky and L. Szymanowski, *A consistent model for πN transition distribution amplitudes and backward pion electroproduction*, *Phys. Rev. D* **85** (2012) 054021 [1112.3570 [hep-ph]].

[4] B. Pire, K. Semenov-Tian-Shansky and L. Szymanowski, *QCD description of backward vector meson hard electroproduction*, *Phys. Rev. D* **91** (2015) 094006 [1503.02012 [hep-ph]].

[5] J. P. Lansberg, B. Pire, K. Semenov-Tian-Shansky and L. Szymanowski, *Accessing baryon to meson transition distribution amplitudes in meson production in association with a high invariant mass lepton pair at GSI-FAIR with $\bar{P}$ANDA*, *Phys. Rev. D* **86** (2012) 114033 [1210.0126 [hep-ph]].

[6] B. Pire, K. Semenov-Tian-Shansky and L. Szymanowski, *QCD description of charmonium plus light meson production in $\bar{p}-N$ annihilation*, *Phys. Lett. B* **724** (2013) 99 [1304.6298 [hep-ph]].

[7] V. M. Braun, S. E. Derkachov, G. P. Korchemsky and A. N. Manashov, *Baryon distribution amplitudes in QCD*, *Nucl. Phys. B* **553** (1999) 355 [hep-ph/9902375].

[8] M. Diehl, *Generalized parton distributions*, *Phys. Rept.* **388** (2003) 41 [hep-ph/0307382].

[9] K. Park *et al.* [CLAS Collaboration], *Hard exclusive pion electroproduction at backward angles with CLAS*, *Phys. Lett. B* **780** (2018) 340 [1711.08486 [nucl-ex]].

[10] W. B. Li *et al.* [Jefferson Lab F_π Collaboration], *Unique access to u-channel physics: exclusive backward-angle omega meson electroproduction*, *Phys. Rev. Lett.* **123** (2019) 182501 [1910.00464 [nucl-ex]].

[11] S. Diehl, K. Joo, *Extraction of $A_{LU}^{\sin(\phi)}$ moments from the hard exclusive π^+ channel in a wide range of kinematics with CLAS*, in proceedings of *INPC 2019*, (to be published).

[12] B. Pire, K. Semenov-Tian-Shansky and L. Szymanowski, *πN transition distribution amplitudes: their symmetries and constraints from chiral dynamics*, *Phys. Rev. D* **84** (2011) 074014 [1106.1851 [hep-ph]].

[13] B. Pire, K. Semenov-Tian-Shansky and L. Szymanowski, *A spectral representation for baryon to meson transition distribution amplitudes*, *Phys. Rev. D* **82** (2010) 094030 [1008.0721 [hep-ph]].

[14] M. Burkardt, *Impact parameter dependent parton distributions and off forward parton distributions for $\zeta \to 0$*, *Phys. Rev. D* **62** (2000) 071503 [hep-ph/0005108].

[15] M. Burkardt, *Impact parameter space interpretation for generalized parton distributions*, *Int. J. Mod. Phys. A* **18** (2003) 173 [hep-ph/0207047].

[16] J. P. Ralston and B. Pire, *Femtophotography of protons to nuclei with deeply virtual Compton scattering*, *Phys. Rev. D* **66** (2002) 111501 [hep-ph/0110075].

[17] M. Diehl, *Generalized parton distributions in impact parameter space*, *Eur. Phys. J. C* **25** (2002) 223 [hep-ph/0205208].

[18] B. Pasquini, M. Pincetti and S. Boffi, *Parton content of the nucleon from distribution amplitudes and transition distribution amplitudes*, *Phys. Rev. D* **80** (2009) 014017 [0905.4018 [hep-ph]].