Quark mass function from a OGE-type interaction in Minkowski space

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The quark self-energy is calculated in both the spacelike and timelike regions of Minkowski space from an effective one-gluon-exchange and a constant interaction. The gauge dependence of the quark mass function is analysed and the model parameters are fixed by comparison of the results to the available lattice QCD data in the spacelike region.

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1. Introduction

The theoretical description of hadrons as bound states of QCD requires an essentially non-perturbative treatment. In particular, for the light mesons and the pion, the implementation of the low-energy phenomenon of dynamical chiral symmetry breaking (DχSB) is indispensable. In this work we report on recent results on the quark mass function in the Covariant Spectator Theory (CST) [1]. The CST [2, 3] is a covariant non-perturbative approach based on quantum field theory in Minkowski space and it has already been applied successfully to quarks and mesons in a number of previous papers [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. It uses a covariant quark-quark interaction kernel that consists of Lorentz vector one-gluon-exchange (OGE) interaction, a generalized constant interaction, and a scalar-pseudoscalar generalization of a linear confining potential that is consistent with DχSB. In the non-relativistic limit, the covariant kernel reduces to the well-known "Cornell" potential and is therefore suitable for all mesons.

2. CST quark self-energy

In CST, the quark self-energy Σ (times a renormalization constant Z_2) is given by the 3-dimensional covariant integral [1]

$$Z_2\Sigma(\not{p}) \equiv \frac{M(p^2)}{Z(p^2)} - m_0 + \left[1 - \frac{1}{Z(p^2)}\right]\not{p} = \frac{Z_2^2}{2}\sum_{\sigma=\pm}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{m}{E_k}\mathscr{V}(p,\hat{k}_\sigma)\left(\frac{m+\hat{\not{k}}_\sigma}{2m}\right), \quad (2.1)$$

where M is the quark mass function, Z the wave function normalization, m_0 denotes the bare quark mass, $\sigma = \pm$ labels the positive- and negative-energy on-shell momenta (corresponding to the positions of the quark propagator poles at $k^2 = m^2$), and $\hat{k}_\sigma = (\sigma E_k, \mathbf{k})$ with $E_k = \sqrt{m^2 + \mathbf{k}^2}$. The invariant functions M and Z defined in Eq. (2.1) satisfy the real mass-pole conditions at $p^2 = m^2$:

$$M(m^2) = m \quad \text{and} \quad Z(m^2) = Z_2\left[1 - 2m\frac{\mathrm{d}M(p^2)}{\mathrm{d}p^2}\bigg|_{p^2=m^2}\right]. \quad (2.2)$$

The interaction kernel is given by

$$\mathscr{V}(p,\hat{k}_\sigma) = \frac{1}{4}\sum_{a=1}^{8}\lambda_a\otimes\lambda_a\left\{\left(\mathbf{1}\otimes\mathbf{1}+\gamma^5\otimes\gamma^5\right)V_\ell(p,\hat{k}_\sigma)\right.$$
$$\left. -\gamma_\mu\otimes\gamma_\nu\left[g^{\mu\nu}-(1-\xi)\frac{q_\sigma^\mu q_\sigma^\nu}{q_\sigma^2}\right]\left[V_g(p,\hat{k}_\sigma)+V_c(p,\hat{k}_\sigma)\right]\right\}. \quad (2.3)$$

Here V_ℓ is a covariant generalization of the linear confining potential [13], ξ is a parameter specifying the linear covariant gauge, $q_\sigma = p - \hat{k}_\sigma$,

$$V_g(p,\hat{k}_\sigma) = \frac{4\pi\eta\alpha_s g(y)}{M_g^2 + |q_\sigma^2|} \quad (2.4)$$

is an OGE-type interaction with η a mixing parameter (discussed below), α_s the *unrenormalized* strong coupling constant, $g(y)$ a regularization form factor depending on the covariant variable $y^2 = \frac{(p\cdot\hat{k}_\sigma)^2}{p^2\hat{k}_\sigma^2}$, and

$$V_c(p,\hat{k}_\sigma) = \frac{(1-\eta)CE_k}{2m}(2\pi)^3\delta^3\left(\mathbf{k} - \frac{m}{\sqrt{p^2}}\mathbf{p}\right)h(p^2) \quad (2.5)$$

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is a covariant form of a constant potential with the unrenormalized strength C and $h(p^2)$ is a strong quark form factor normalized as $h(m^2) = 1$. The OGE and constant parts are both assumed to be vector interactions with the same gauge dependence. This is because the constant kernel can be regarded as part of—or a correction to—the OGE part, with their respective effective strengths related by the mixing parameter η and chosen such that always the same constituent quark mass m is generated through Eq. (2.2) [1]. Furthermore, in the OGE kernel, Eq. (2.4), a finite gluon mass $M_g = 0.6$ GeV has been introduced, and the prescription $q_\sigma^2 \to -|q_\sigma^2|$ has been applied to remove the unwanted singularity in the gluon propagator [1]. In addition, both the large-$\mathbf{k}^2$ and small-p^2 singular behaviors of the OGE contribution are regularized by an appropriately-chosen form factor $g(y)$, which introduces a cutoff parameter Λ_g. Finally, recall that V_ℓ with a Lorentz scalar-pseudoscalar structure does not contribute to the self-energy [11], which simplifies the calculation considerably.

3. Results and Conclusion

The main result of this work is the quark mass function calculated in the chiral limit ($m_0 = 0$) and for 3 different linear covariant gauges ($\xi = 0, 1, 3$), as summarized in Fig. 1.
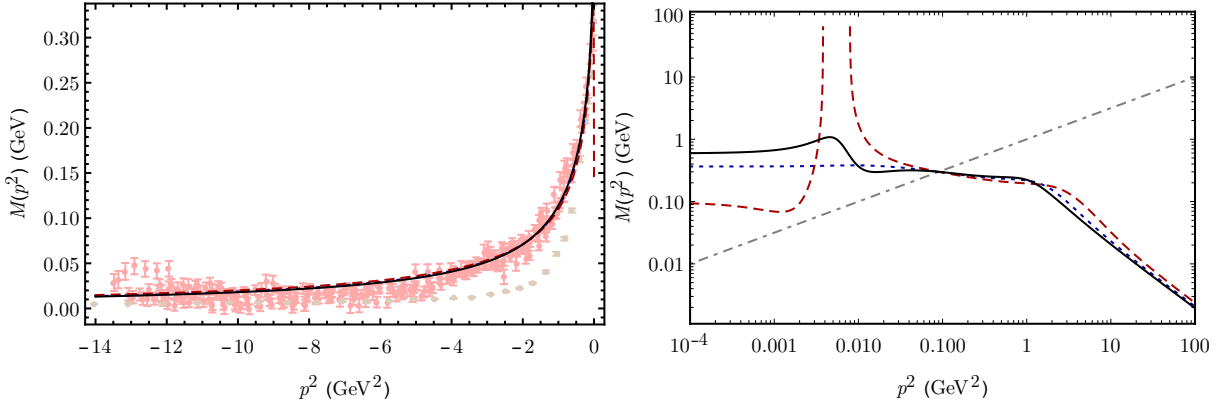

Figure 1: The mass function (in GeV) vs. p^2 (in GeV^2) for $\xi = 0$ and $\Lambda_g = 0.9$ GeV (red dashed line), $\xi = 1$ and $\Lambda_g = 0.6$ GeV (blue dotted line), and $\xi = 3$ and $\Lambda_g = 0.45$ GeV (black solid line). The left panel shows the spacelike region with lattice QCD data taken from [17] (red data points) and [18] (brown data points). The right panel shows the timelike region on logarithmic scales where the intersection of the curves with $\sqrt{p^2}$ (gray dotdashed line) marks the ξ-independent on-shell point $p^2 = m^2$ defining the chiral constituent quark mass.

We have chosen a chiral constituent quark mass of $m = 0.3$ GeV and an effective renormalized OGE strength $\eta Z_2^2 \alpha_s = 0.5$. The cutoff parameter Λ_g was then roughly adjusted, separately for each gauge, to agree with the lattice QCD data [17]. The Landau gauge ($\xi = 0$) result has a pole at some small timelike p^2. Such poles are not present in the Feynman ($\xi = 1$) and Yennie ($\xi = 3$) gauges. Further, it should be mentioned that, as a consequence of the prescription to make the dressed gluon propagator non-singular, the mass function develops a discontinuity at $p^2 = 0$ (not displayed in the figure). The discontinuity depends on the gauge, and we find that the quark mass function is continuous at $p^2 = 0$ only for the Yennie gauge, which therefore is the preferred gauge

in this CST calculation. For timelike p we find a strong dependence of the quark mass function on the gauge. However, on-mass-shell quantities, such as the constituent quark mass are gauge-parameter independent – a nice feature of our CST approach that has been proven in Ref. [1]. For spacelike p the gauge dependence is very weak, which allowed us to use the existing lattice QCD data to adjust the model parameters. After extending the present calculation to finite bare quark masses, the mass functions can then be used in meson calculations, thereby achieving complete consistency between the quark self-interaction and the quark-antiquark interaction.

Acknowledgments

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