PROCEEDINGS OF SCIENCE

Sub-eikonal corrections and low-x helicity evolution

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Most of the progress in high-energy Quantum Chromodynamics has been obtained within the eikonal approximation and infinite Wilson-line operators. Evolution equations of Wilson lines with respect to the rapidity parameter encode the dynamics of the hadronic processes at high energy. However, even at high energy many interesting aspects of hadron dynamics are not accessible within the eikonal approximation, the spin physics being an obvious example. The higher precision reached by the experiments and the possibility to probe spin dynamics at future Electron Ion Colliders make the study of deviations from eikonal approximation especially timely.
I will present the high-energy sub-eikonal corrections and the low-x helicity evolution through the high-energy Operator Product Expansion.

Light Cone 2019 - QCD on the light cone: from hadrons to heavy ions - LC2019
16-20 September 2019
Ecole Polytechnique, Palaiseau, France

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PoS(LC2019)034

1. Introduction

The Operator Product Expansion at high-energy [1] applied to a T-product of two electromagnetic currents is the generalization of the usual OPE in light-ray operators in DGLAP regime. In high energy the OPE is given by a convolution of a coefficient function (Impact Factor) with the matrix element of infinite Wilson lines operators. The evolution of the Wilson-line operators with respect to rapidity, the Balitsky-Kovchegov equation, provides the relevant dynamics of partons at high energy and at high density. The solution of the BK equation and its convolution with the impact factor gives the DIS cross-section, provided that we have a model for the initial non-perturbative condition. The high-energy OPE for DIS is now available at NLO: the evolution equation and the impact factor are now known at NLO[2, 3, 4] and the convolution with the solution of the NLO evolution equation in the linear case is also available for $\gamma^*\gamma^*$ scattering cross section since in this case the initial conditions are fully perturbative and have been calculated [5, 6].

The high energy OPE so far known is not applicable if one would like to include spin. To this end one has to extend the OPE to sub eikonal corrections. The first step is to calculate the quark and gluon propagators in the background of quarks and gluons up to sub-eikonal corrections [7]. Using these propagators the high-energy OPE provides new impact factors convoluted with matrix elements of new operators. The evolution of these new operators result in new evolution equations which are relevant for the spin dynamics at high-energy.

It is known that at high-energy (low-x_B) the dynamics is driven by gluons. It was shown [8] that when the quarks are included (BFKL in the t-channel) the resummation is of the $(\alpha_s \ln^2 \frac{1}{x_B})$ type. This is different than the usual low-x resummation $(\alpha_s \ln \frac{1}{x_B})$ provided by the BFKL equation.

Authors of refs. [9, 10] have obtained the resummation for the g_1 structure function and shown that the resummation is again of the double log type $(\alpha_s \ln^2 \frac{1}{x_B})$. The goal is to obtain the evolution equations that describe the non-linear spin dynamics. An attempt to reproduce the double log resummation result within the Saturation formalism can be found in ref. [11]. At the moment the two results disagree thus motivating further and independent analysis.

Here we outline the main steps to be performed in the high-energy spin OPE and obtain the relevant evolution equations which will eventually be used to obtain the double log resummation for the helicity evolution.

2. Operator Product Expansion with quark and gluon sub-eikonal terms

The T-product of two electromagnetic currents evaluated in the target state is

$$\langle P,S|\mathrm{T}\{\hat{\bar\psi}(x)\gamma^\mu\hat\psi(x)\hat{\bar\psi}(y)\gamma^\nu\hat\psi(y)\}|P,S\rangle\,. \tag{2.1}$$

Its high-energy Operator Product Expansion with quark sub-eikonal corrections [7] (see fig. 1) is

$$\begin{aligned}\langle\mathrm{T}\{\hat j^\mu(x)\hat j^\nu(y)\}\rangle_{A,\psi,\bar\psi} \overset{x_*>0>y_*}{\ni} g^2\int d^2z_1 d^2z_2\, I^{\mu\nu}\Big\{&\Big[\langle\mathrm{Tr}\{\frac{1}{2}\mathrm{Tr}\{U^\dagger_{z_2}U_{z_1}\}Q_{5\,z_1}\rangle - \frac{1}{2N_c}\langle\mathrm{Tr}\{U^\dagger_{z_2}\tilde Q_{5\,z_1}\}\rangle + c.c\Big]\\ &+g\int_{x_*}^{y_*} d\omega_*\Big[\langle\mathrm{Tr}\{[x_*,y_*]_{z_1}[y_*,\omega_*]_{z_2}\varepsilon^{\rho\sigma}F^\perp_{\rho\sigma}[\omega_*,x_*]_{z_2}\}\rangle + c.c\Big]\Big\}\,,\end{aligned} \tag{2.2}$$

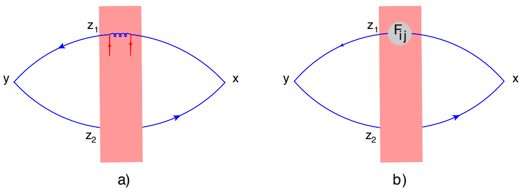

Figure 1: Diagrams of the LO impact factor with quark (left panel) and gluon (right panel) sub-eikonal corrections.

where we defined

$$Q_{5x} = g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* \mathrm{Tr}\{[z_*, z'_*]_x^\dagger \mathrm{tr}\{\psi(z_*, x_\perp)\bar{\psi}(z'_*, x_\perp)\gamma^5 \not{p}_1\}\} \tag{2.3}$$

$$\tilde{Q}_{5ij}(x_\perp) = g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* [\infty p_1, z_*]_x \mathrm{tr}\{\psi(z_*, x_\perp)\bar{\psi}(z'_*, x_\perp)\gamma^5 \not{p}_1\}[z'_*, -p_1\infty]\,. \tag{2.4}$$

The explicit expression of the coefficient functions of the OPE (the impact factors) $I^{\mu\nu}$ will be published in a separate paper. Before we proceed, we need to explain the notation used in equation (2.2). In high-energy it is often convenient to introduce two light-cone vectors $p_1^\mu = \frac{\sqrt{s}}{2}(1,0,0,1)$ and $p_2^\mu = \frac{\sqrt{s}}{2}(1,0,0,-1)$ such that $p_1^\mu p_{2\mu} = \frac{s}{2}$ and $p_1^2 = p_2^2 = 0$, with s the Mandelstam variable for the center-of-mass energy. We also use light-cone coordinates $x_* = \sqrt{s/2}x^+ = p_2^\mu x_\mu$ and $x_\bullet = \sqrt{s/2}x^- = p_1^\mu x_\mu$ with $x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$. Using the two light-cone vectors we decompose the momentum in Sudakov components as $p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$. The gauge link is defined as

$$[x_*, y_*]_x = [x_* p_1 + x_\perp, y_* p_1 + x_\perp] = \mathrm{Pexp}\Big\{ig\frac{2}{s}\int_{y_*}^{x_*} dz_* A_\bullet(z_*, x_\perp)\Big\}\,, \tag{2.5}$$

so, the infinite Wilson line is $U(x_\perp) \equiv U_x = [\infty p_1 + x_\perp, -\infty p_1 + x_\perp]$.

From the OPE given in equation (2.2) we see that the operators for spin dynamics are

$$\mathscr{Q}_5(z_{1\perp}, z_{2\perp}) \equiv \mathrm{Tr}\{\frac{1}{2}\mathrm{Tr}\{U_{z_2}^\dagger U_{z_1}\}Q_{5z_1} - \frac{1}{2N_c}\mathrm{Tr}\{U_{z_2}^\dagger \tilde{Q}_{5z_1}\} + c.c \tag{2.6}$$

$$\mathscr{F}(z_{1\perp}, z_{2\perp}) \equiv ig\frac{s}{2}\int_{x_*}^{y_*} d\omega_* \mathrm{Tr}\{[x_*, y_*]_{z_1}[y_*, \omega_*]_{z_2}\varepsilon^{\rho\sigma}F^\perp_{\rho\sigma}(\omega_*, z_{2\perp})[\omega_*, x_*]_{z_2}\} + c.c\,. \tag{2.7}$$

In the next section we provide the evolution equations for operators $\mathscr{Q}_{5z_1z_2}$ and $\mathscr{F}_{z_1z_2}$.

3. Evolution equations

Let us obtain the evolution equations for the operators $\mathscr{Q}_{5z_1z_2}$ and $\mathscr{F}_{z_1z_2}$. We start with the quark operator. It is convenient to calculate the evolution equation of the operators $\mathrm{Tr}\{U_{z_2}^\dagger U_{z_1}\}Q_{5,z_1}$ and $\mathrm{Tr}\{U_{z_2}^\dagger \tilde{Q}_{5z_1}\}$ separately. Actually, since the evolution of $\mathrm{Tr}\{U_{z_2}^\dagger U_{z_1}\}$ si BK-equation, we really need the evolution of Q_{5,z_1} whose diagram are given in Fig. 2a and b. There are also diagrams that connect the operator $\mathrm{Tr}\{U_{z_2}^\dagger U_{z_2}\}$ with Q_{5,z_1} for which we need the gluon to quark propagator. The contribution for these diagrams will be published in a separate paper.

The result for diagrams given in Fig. 2, omitting the details of the derivation, is

$$
\begin{aligned}
\langle Q_{5z_1}\rangle_{\mathrm{Fig.2}a,b} = &\ \frac{\alpha_s}{4N_c\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\,\frac{1}{(z_1-z)_\perp^2} \\
&\times\Big[N_c\langle \mathrm{Tr}\{U^\dagger_{z_1}U_z\}\,Q_{5z}\rangle-\langle \mathrm{Tr}\{U^\dagger_{z_1}\tilde{Q}_{5z}\}\rangle+2N_c\,\langle \mathrm{Tr}\{U^\dagger_{z_1}\mathscr{F}(z_\perp)\}\rangle\Big]
\end{aligned}
\tag{3.1}
$$

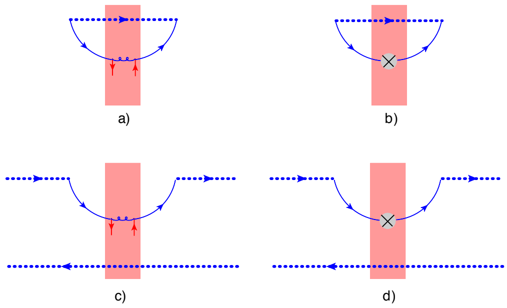

Figure 2: Sample of diagrams for the evolution of operators Q_{5z_1} and $\mathrm{Tr}\{U^\dagger_{z_2}\tilde{Q}_{5z_1}\}$.

Let us consider the diagrams given in Fig. 2c,d for the evolution of the operator $\mathrm{Tr}\{U^\dagger_{z_2}\tilde{Q}_{5z_1}\}$. Also in this case we omit the contribution of diagrams obtained using the gluon to quark propagators (the result will be published in a separate paper). The result is

$$
\begin{aligned}
\langle \mathrm{Tr}\{U^\dagger_{z_2}\tilde{Q}_{5z_1}\}\rangle_{\mathrm{Fig.2}c,d} = &\ \frac{\alpha_s}{4N_c\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\,\frac{1}{(z_1-z)_\perp^2} \\
\times\Bigg[&\Big(N_c\langle \mathrm{Tr}\{U^\dagger_{z_2}U_z\}Q_{5z}\rangle-\langle \mathrm{Tr}\{U^\dagger_{z_2}\tilde{Q}_{5z}\}\rangle+2N_c\langle \mathrm{Tr}\{U^\dagger_{z_2}\mathscr{F}(z)\}\rangle\Big) \\
+&\frac{2N_c(z_1-z_2)_\perp^2}{(z_1-z)^2(z-z_2)^2}\Big(\langle \mathrm{Tr}\{U_zU^\dagger_{z_2}\}\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{5z_1}\}\rangle-N_c\langle \mathrm{Tr}\{U^\dagger_{z_2}\tilde{Q}_{5z_1}\}\rangle\Big)\Bigg]
\end{aligned}
\tag{3.2}
$$

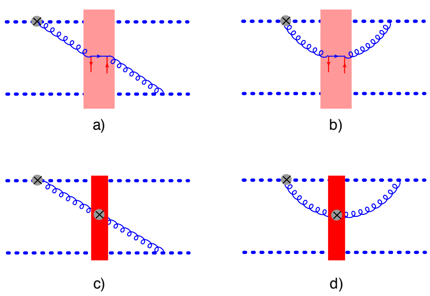

Figure 3: Diagrams for the evolution of the operator F_{ij}. Here we consider F_{ij} quantum field which is depicted as a gray cirlce with a cross inside.

Finally, we consider the gluon operator. A sample of diagrams for its evolution equation are given in figures 3. The result is

$$
\langle \mathrm{Tr}\{\mathscr{F}_{z_1}U^\dagger_{z_2}\}\rangle_{\mathrm{Fig.3}} = \frac{\alpha_s}{4N_c\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\Bigg\{\Big(\frac{(z_1-z,z-z_2)}{(z_1-z)_\perp^2(z_2-z)_\perp^2}+\frac{1}{(z_1-z)_\perp^2}\Big)
$$

$$\times\Big(\langle\mathrm{Tr}\{U_{z_1}U^\dagger_{z_2}U_z\tilde{Q}^\dagger_{5z}\}\rangle+\langle\mathrm{Tr}\{U^\dagger_{z_2}U_{z_1}\tilde{Q}^\dagger_{5z}U_z\}\rangle-N_c\langle\mathrm{Tr}\{U^\dagger_{z_2}\tilde{Q}_{5z}\}\mathrm{Tr}\{U^\dagger_zU_{z_1}\}\rangle-\frac{1}{N_c}\langle\mathrm{Tr}\{U^\dagger_{z_2}U_{z_1}\}Q^\dagger_{5z}\rangle$$
$$\langle\mathrm{Tr}\{U_{z_1}U^\dagger_{z_2}\tilde{Q}_{5z}U^\dagger_z\}\rangle+\langle\mathrm{Tr}\{U^\dagger_{z_2}U_{z_1}U^\dagger_z\tilde{Q}_{5z}\}\rangle-N_c\langle\mathrm{Tr}\{U_{z_1}\tilde{Q}^\dagger_{5z}\}\mathrm{Tr}\{U^\dagger_{z_2}U_z\}\rangle-\frac{1}{N_c}\langle\mathrm{Tr}\{U^\dagger_{z_2}U_{z_1}\}Q_{5z}\rangle\Big)$$
$$+\left[\frac{N_c(z_1-z,z-z_2)}{(z_2-z)^2(z-z_1)^2}+\frac{N_c}{(z_1-z)^2}+N_c\int d^2q_1\frac{e^{i(q_1,z_2-z)}-e^{i(q_1,z_1-z)}}{q^2_{1\perp}}\delta^{(2)}(z-z_1)\right]$$
$$\times\Big(\langle\mathrm{Tr}\{U_{z_1}U^\dagger_z\}\mathrm{Tr}\{U^\dagger_{z_2}\mathscr{F}(z_\perp)\}\rangle-\langle\mathrm{Tr}\{U^\dagger_{z_2}U_z\}\mathrm{Tr}\{U_{z_1}\mathscr{F}^\dagger(z_\perp)\}\rangle\Big)\Big\}\tag{3.3}$$

The evolution equations (3.1), (3.2) and (3.3) present UV and IR divergences. These divergences will be regulated by imposing the double ordering, $\alpha_1\gg\alpha_2\ldots\gg\alpha_n$ and $\beta_1\ll\beta_2\ldots\ll\beta_n$ in order to extract the double log resummation, but there will also be single log contributions that can be resummed trough an evolution equation by introducing appropriate counterterms for the operators.

4. Conclusions

We presented the extension of the high-energy OPE to include spin dynamics. We have used the propagators with sub-eikonal corrections obtained in ref. [7] and performed the OPE to the T-product of two electromagnetic currents. We have also presented the evolution equations of the new operators that describe spin dynamics. These evolution equations contain both single and double log resummation, so in order to compare with the known result [9, 10, 11] we will have to disentangle the two types of resummation, first.

The final goal is not just that of reproducing known results, but that of extending them to include the more rich dynamics provided by the Wilson-line formalism. Like BK equation encodes the non linear dynamics absent in BFKL equation, the high-energy OPE presented here will provide information on the non-linearity of the spin dynamics. This analysis is relevant for experiments that will be performed at the future Electron Ion Collider [12]. The work is in progress.

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