PROCEEDINGS OF SCIENCE

3D imaging of the pion off-shell electromagnetic form factors *

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We study the pion electromagnetic half-off-shell form factors F_1 and F_2 using an exactly solvable manifestly covariant model of a $(3+1)$ dimensional fermion field theory. The model provides a three-dimensional imaging of F_1 and F_2 as a function of (Q^2,t), which are constrained by the Ward-Takahashi identity. The normalization of the charge form factor F_1 is fixed by $F_1(Q^2 = 0, t = m_\pi^2) = 1$ while the other form factor F_2 vanishes, i.e. $F_2(Q^2,t = m_\pi^2) = 0$ for any value of Q^2 due to the time-reversal invariance of the strong interaction. The new form factor defined by $g(Q^2,t) = F_2(Q^2,t)/(t-m_\pi^2)$ is however measurable in the on-mass-shell limit. We note that $g(Q^2 = 0, t = m_\pi^2)$ is related with the pion charge radius.

Light Cone 2019 - QCD on the light cone: from hadrons to heavy ions - LC2019
16-20 September 2019
Ecole Polytechnique, Palaiseau, France

*This work was supported in part by the US Department of Energy under Grant No. DE-FG02-03ER41260 (CJ), by the National Research Foundation of Korea (NRF) under Grant No. NRF-2017R1D1A1B03033129 (HMC), by the project INCT-FNA Proc. No. 464898/2014-5, by CAPES - Finance Code 001, by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under grants 308025/2015-6 (JPBCM), 308486/2015-3 (TF), PVE 401322/2014-9 (CJ), by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) under the thematic projects 2013/26258-4 and 2017/05660-0, and by regular project 2019/02923-5 (JPBCM).

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1. Introduction

The pion is the simplest hadronic system, the valence structure of which is a bound state of a quark and an antiquark. Its structure is known to be parametrized by a single on-mass-shell electromagnetic (EM) form factor, $F_\pi(Q^2)$, which depends on the 4-momentum squared $q^2(=-Q^2)$ of the virtual photon. While $F_\pi(Q^2)$ for the low spacelike momentum transfers Q^2 has been measured directly by elastic scattering of high-energy mesons off atomic electrons, the extraction of $F_\pi(Q^2)$ to higher Q^2 regions through elastic scattering is very difficult experimentally. Thus, $F_\pi(Q^2)$ for the higher Q^2 values has been extracted from the pion electroproduction reaction by exploiting the nucleon's pion cloud as a target, i.e., $F_\pi(Q^2)$ has been extracted from the measurements of the cross sections for the reaction $^1\mathrm{H}(e,e'\pi^+)n$ up to values of $Q^2 = 3.91$ GeV^2 [1, 2, 3]. However, the main problem in using the electroproduction process as a tool for accessing a "pion target" is that the pions in a nucleon's cloud are not real but virtual particles. Accordingly, one cannot access the form factor at the exact pion pole in the actual experiment as the extrapolation to $t \to m_\pi^2$ involves the disallowed kinematic region of the electroproduction ($t<0$). This may raise some questions about the validity of the extrapolation from the off-shell results to the on-shell limit. Furthermore, the EM structure of the off-shell hadron is more complicated than the on-shell hadron and involves more form factors [4]. For instance, the off-shell EM structure of the pseudoscalar meson [4] requires two form factors, which are related by the Ward-Takahashi identity(WTI).

In this work, we discuss the electromagnetic off-shell effects for the pion using an exactly solvable manifestly covariant model of $(3+1)$ dimensional fermion field theory and compare the two off-shell form factors $F_1(Q^2,t)$ and $F_2(Q^2,t)$ with the data extracted from the pion electroproduction reaction [1]. More detailed analysis can be found in Ref. [5].

2. Off-shell Pion Electromagnetic Form Factors

The most general parametrization of the vertex function Γ^μ for the half-on-shell ($p'^2 = m_\pi^2$) and half-off-shell ($p^2 = t < 0$) electromagnetic form factors of the charged pion can be given in terms of the initial and final 4-momenta, p^μ and $p'^\mu (= p^\mu + q^\mu)$, as

$$\Gamma_\mu = (p'+p)_\mu \, F_1(Q^2,t) + q_\mu F_2(Q^2,t). \tag{2.1}$$

where $F_2(Q^2,t) = (t-m_\pi^2)[F_1(0,t) - F_1(Q^2,t)]/Q^2$ is obtained from the WTI [5]. It is known that $F_2(Q^2,t)$ cannot be directly measured in the electroproduction process due to the transversality of the electron current. We note, however, that the new form factor $g(Q^2,t) \equiv F_2(Q^2,t)/(t-m_\pi^2)$ is nonzero in the limit of $t \to m_\pi^2$ although $F_2(Q^2,t)$ itself goes to zero as $t \to m_\pi^2$. The form factor $g(Q^2,m_\pi^2)$ is the new observable in the on-mass-shell limit besides the usual charge form factor $F_1(Q^2,m_\pi^2)$ and should be measurable in the experiment of pion electroproduction. The off-shell form factor $F_2(Q^2,t)$ can be rewritten as the following sum rule,

$$F_1(Q^2,t) - F_1(0,t) + Q^2 g(Q^2,t) = 0. \tag{2.2}$$

Especially, taking the derivative of this sum rule, we find [5] in the on-mass-shell limit $t = m_\pi^2$ and $Q^2 = 0$ that $g(Q^2=0,m_\pi^2) = \langle r_\pi^2\rangle/6$, where r_π is the pion charge radius.

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We explicitly show all those properties of the off-shell pion form factors using the exactly solvable manifestly covariant model. The vertex function for the initial off-shell ($p^2 = t$) and final on-shell ($p'^2 = m_\pi^2$) $q\bar{q}$ bound-state pion coupled to the virtual photon with the 4-momentum q in the fermion field theory can be calculated as

$$\Gamma^\mu = iN_c g_{\pi q\bar{q}}^2 \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[\gamma_5(\not{k}+\not{q}+m_q)\gamma^\mu(\not{k}+m_q)\gamma_5(\not{k}-\not{p}+m_q)]}{[k^2-m_q^2+i\varepsilon][(k+q)^2-m_q^2+i\varepsilon][(p-k)^2-m_q^2+i\varepsilon]}, \tag{2.3}$$

where N_c is the number of colors and $g_{\pi q\bar{q}}$ corresponds to the coupling constant of the $\pi q\bar{q}$ vertex.

Using the Feynman parametrization and the dimensional regularization in $d(=4-2\varepsilon)$-dimensions, we then obtain the two form factors $F_1(Q^2,t)$ and $F_2(Q^2,t)$ as

$$\begin{aligned} F_1(Q^2,t) &= -\frac{N_c g_{\pi q\bar{q}}^2}{8\pi^2}\int_0^1 dx \int_0^x dy \left[(1+3y)\left(\gamma - \frac{1}{\varepsilon} + \frac{1}{2} + \mathrm{Log}C\right) + \frac{\alpha}{C}\right], \\ F_2(Q^2,t) &= -\frac{N_c g_{\pi q\bar{q}}^2}{8\pi^2}\int_0^1 dx \int_0^x dy \left[3(1-2x+y)\mathrm{Log}C + \frac{2\beta-\alpha}{C}\right], \end{aligned} \tag{2.4}$$

where γ is the Euler-Mascheroni constant and $\alpha = (1+y)(E^2-m_q^2) - q\cdot E + 2yp\cdot E - yq\cdot p$, $\beta = (1-x+y)(E^2-m_q^2) + (1-2x+2y)p\cdot E + (x-y)q\cdot p$, and C and E are given in [5]. We should note that the form factor $F_2(Q^2,t)$ is free from the UV divergence since the integration of $(1-2x+y)$ multiplied by the constant factor $(\gamma - 1/\varepsilon + 1/2)$ gives zero in Eq. (2.4).

As the loop correction to the charge form factor $F_1(Q^2, t = m_\pi^2)$ must vanish at $Q^2 = 0$, the charge at $Q^2 = 0$ is given by a subtraction to the contribution by the loop integral. We thus redefine the renormalized charge form factor as

$$F_1^{\mathrm{ren}}(Q^2,t) = 1 + [F_1(Q^2,t) - F_1(0,m_\pi^2)], \tag{2.5}$$

where the loop correction in the square bracket vanishes at $Q^2 = 0$ and $t = m_\pi^2$ and the normalization of the electric charge is fixed by $F_1^{\mathrm{ren.}}(0, t = m_\pi^2) = 1$. The coupling $g_{\pi q\bar{q}}$ is related to the pseudoscalar coupling of the pion *vis-á-vis* partially conserved axial current. From the comparison of the pion decay constant f_π using the same model, we obtain $g_{\pi q\bar{q}} = (2m_q/f_\pi)$ by taking $F_1^{\mathrm{ren}}(0,m_\pi^2) = 1$. In our numerical calculation, however, we take $g_{\pi q\bar{q}}$ as another free parameter in addition to m_q for the best fit of the model calculation compared to the experimental data. From now on, we shall denote $F_1^{\mathrm{ren.}}(Q^2,t)$ as $F_1(Q^2,t)$ for convenience.

3. Results

In our numerical calculation, we tried to find the best fits of the form factor $F_1(Q^2,t)$ compared to the experimental data $F_1^{\mathrm{Exp.}}(Q^2,t)$ for both the off-shell pion ($t \neq m_\pi^2$) and the on-shell pion ($t = m_\pi^2$) by adjusting our model parameters $(m_q, g_{\pi q\bar{q}})$. We found the optimum ranges of quark masses, $0.12 \le m_q \le 0.16$ GeV, and the best-fit for the coupling constants, $g_{\pi q\bar{q}} = (1.32, 1.20, 1.11)$ in unit of $(2m_q/f_\pi^{\mathrm{Exp.}})$ for $m_q = (0.12, 0.14, 0.16)$ GeV, respectively. That is, our phenomenological best fit coupling constants $g_{\pi q\bar{q}}$ are not much different from the values of $2m_q/f_\pi^{\mathrm{Exp.}}$.

The overall landscape of the half-on-shell form factors, $F_1(Q^2,t)$ and $F_2(Q^2,t)$, obtained from $m_q = 0.16$ GeV and $g_{\pi q\bar{q}} = 1.11(2m_q/f_\pi^{\mathrm{Exp.}})$ for spacelike regions are shown in Fig. 1 [5]. The figure represents the 3D plots of $F_1(Q^2,t)$ (top left), $-F_2(Q^2,t)$ (top right), $g(Q^2,t)$ (bottom left) and the sum rule (bottom right) given by Eq. (2.2) for the momentum transfer region $0 \le Q^2 \le 3$ GeV^2 and $m_\pi^2 \ge t \ge -0.4$ GeV^2. While the form factor $F_2(Q^2,t)$ goes to zero as $t \to m_\pi^2$, the form factor $g(Q^2,t)$ is nonzero even in the on-mass-shell limit. Furthermore, $F_1(0,t)$ shows some dependency on t, which is necessary to know in the case of extracting $F_2(Q^2,t)$ from the pion electroproduction data.

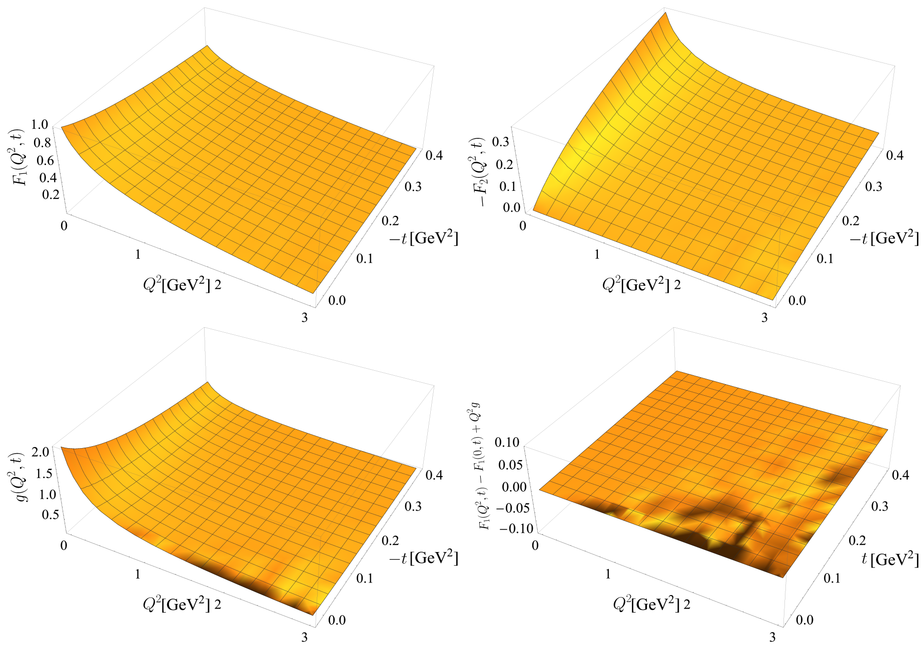

Figure 1: The 3D plots of $F_1(Q^2,t)$ (top left), $F_2(Q^2,t)$ (top right), $g(Q^2,t)$ (bottom left) and the sum rule (bottom right) given by Eq. (2.2) for the spacelike momentum transfer region $0 \le Q^2 \le 3$ GeV^2 and $m_\pi^2 \ge t \ge -0.4$ GeV^2.

The on-shell pion form factors $F_1(Q^2,m_\pi^2)$ (black lines) and $g(Q^2,m_\pi^2)$ (blue lines) from the covariant model for the spacelike region $Q^2 > 0$ are shown in Fig. 2 and compared with the extracted values of $F_1^{\rm Exp.}(Q^2,t=m_\pi^2)$ (black data) and $g^{\rm Exp.}(Q^2,t=m_\pi^2) = [1-F_1^{\rm Exp.}(Q^2,t=m_\pi^2)]/Q^2$ (blue data) [5]. The model parameters in Fig. 2 are $m_q = (0.12, 0.16)$ GeV using the variation of the couplings $g_{\pi q\bar{q}} = (1.32 \pm 0.04, 1.11 \pm 0.04)(2m_q/f_\pi^{\rm Exp.})$, respectively. The solid and dashed lines represent the results obtained from $m_q = 0.12$ and 0.16 GeV using the upper and lower limits of the corresponding $g_{\pi q\bar{q}}$. Unlike the form factor $F_2(Q^2,t)$, the form factor $g(Q^2,t)$ does not vanish in the on-shell limit. We note that the current Particle Data Group [6] average $r_\pi^{\rm Exp} = \sqrt{\langle r_\pi^2 \rangle} = (0.672 \pm 0.008)$ fm for the rms value of the pion charge radius corresponds to $g^{\rm Exp.}(Q^2=0,m_\pi^2) = (1.953 \pm 0.023)$ GeV^{-2}.

4. Conclusions

We investigated the pion electromagnetic half-off-shell form factors $F_1(Q^2,t)$ and $F_2(Q^2,t)$ using the manifestly covariant fermion field theory model. We also note that the ratio of $F_2(Q^2,t)$

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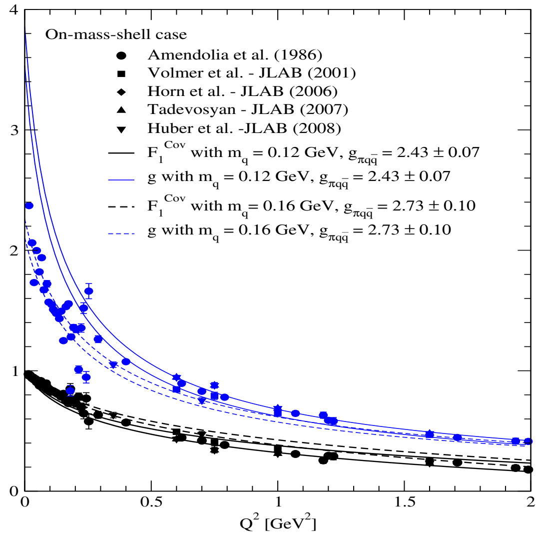

Figure 2: The on-shell pion form factors $F_1(Q^2, m_\pi^2)$ (black lines) and $g(Q^2, m_\pi^2)$ (blue lines) for the spacelike momentum transfer region $0 \le Q^2 \le 2$ GeV^2 compared with the experimental data [1, 2, 3, 7] for $F_1^{\text{Exp.}}$ (black data) and $g^{\text{Exp.}}$ (blue data).

to $t - m_\pi^2$ is nonzero in the limit of $t \to m_\pi^2$ while $F_2(Q^2, t)$ goes to zero as $t \to m_\pi^2$. This led us to define the new form factor $g(Q^2, t) = F_2(Q^2, t)/(t - m_\pi^2)$, which should be measurable even in the on-mass-shell limit on par with the usual charge form factor $F_1(Q^2, m_\pi^2)$. In particular, we obtain the sum rule which relates $g(Q^2, t)$ to $F_1(Q^2, t)$ and note that the value of $g(Q^2 = 0, t = m_\pi^2)$ corresponds to the charge radius of a pion. According to Eq. (2.2), however, one needs the information of $F_1(0, t)$ to determine $g(Q^2, t)$, while no data of $F_1(Q^2, t)$ exist at $Q^2 = 0$ for $t < 0$.

In this work, we used a simple covariant model to provide at least a clear example of demonstration for the simultaneous extraction of both $F_1(Q^2, t)$ and $g(Q^2, t)$ (or $F_2(Q^2, t)$). In our numerical calculations, we show the 3D plots of $F_{1(2)}(Q^2, t)$ and $g(Q^2, t)$ in terms of (Q^2, t) values as shown in Fig. 1. It encourages more in-depth theoretical and experimental efforts to reveal the 3D imaging of the off-shell pion form factors.

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