PROCEEDINGS OF SCIENCE

The QCD energy-momentum tensor for massive states of arbitrary spin

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We present the parametrisation of the energy-momentum tensor (EMT) for massive hadrons of any spin, writing explicitly the expansion in terms of gravitational form factors (GFFs). Such a complete and general parametrisation allows one to derive universal properties that are valid for all hadrons independently of their spin.

Light Cone 2019 - QCD on the light cone: from hadrons to heavy ions - LC2019
16-20 September 2019
Ecole Polytechnique, Palaiseau, France

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PoS(LC2019)036

1. Introduction

The energy-momentum tensor (EMT) is a fundamental object of study in quantum field theories such as QCD, see e.g. [1, 2] and references therein. The hadronic matrix element of the (local) EMT operator is parametrised in terms of gravitational form factors (GFFs), in analogy to the hadronic matrix elements of the electromagnetic current operator which are parametrised in terms of electromagnetic form factors. The GFFs encode properties that are of great interest for hadrons, such as their mass and angular momentum and their spatial distributions [3, 1, 4, 5]. The study of GFFs can shed light on novel properties of hadrons, such as the way that stress and shear forces are distributed within them [6, 3, 2, 7].

For a long time in the past, the main focus of hadronic physics has been devoted to spin-$\frac{1}{2}$ particles, due to the proton's abundance as a stable particle and its central role in the building of visible matter. However, there is an increasing interest in the study of higher-spin hadrons, as unique tools to study the dynamics of internal constituents beyond the degrees of freedom typical of a single spin-1/2 nucleon (proton and neutron) [8, 9, 10, 11, 12, 13].

Even though measurements of the GFFs for higher-spin particles are hardly feasible experimentally, looking at higher-spin problems is desirable from a broader theoretical point of view. A natural question is whether it is possible to characterize the role of the spin of the state in shaping the structure of the EMT and, consequently, the number of GFFs. Similarly, one might wonder what is the best systematic approach to find the complete EMT parametrisation such that the form factors counting depends only on the total spin j [8, 14]. In this paper, inspired and based on refs. [15, 12, 16, 14], we present the complete parametrisations for the EMT for massive states of arbitrary spin. We single out all the possible "core" structures, or "seeds", i.e. Lorentz structures that contribute to the expansion of the matrix element, and associate to them a "tower" of elements, whose number depends on the spin of the particle. Finally, we point out that one can use a more general approach, recently derived and presented in ref. [14], based on a covariant version of the spin multipole expansion. This is useful to derive some universal properties and relations, that turn out to be valid for all on-shell particles, independently of their spin, structure, and mass [12, 16].

2. Parametrisations of the EMT

We introduce the average four-momentum $P = (p' + p)/2$ and the four-momentum transfer $\Delta = p' - p$ with $\Delta^2 = t$, satisfying the onshell conditions $P^2 + \Delta^2/4 = M^2$ and $P \cdot \Delta = 0$. The polarisation of physical states is described by a generalised polarisation tensor (GPT) $\eta(p,\lambda)$ as in [17, 12, 16]. GPTs are defined such that the covariant density matrix in a given representation of the Lorentz group

$$\rho^A{}_B(p,\lambda,\lambda') = \eta^A(p,\lambda)\,\overline{\eta}_B(p,\lambda') \tag{2.1}$$

has normalisation $\mathrm{Tr}[\rho(p,\lambda,\lambda')] = \delta_{\lambda\lambda'}$.

Let us consider a particle of mass M and spin j. When $j = n$ is integer, we choose to work with the $(\frac{n}{2},\frac{n}{2})$ representation, where the GPT $\eta(p,\lambda) \sim \varepsilon_{\alpha_1\cdots\alpha_n}(p,\lambda)$ is totally symmetric, traceless and satisfies the subsidiary condition

$$p^\alpha \varepsilon_{\alpha\alpha_2\cdots\alpha_n}(p,\lambda) = 0. \tag{2.2}$$

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When $j = n + \frac{1}{2}$ is half-integer, we choose to work with the $(\frac{n+1}{2}, \frac{n}{2}) \oplus (\frac{n}{2}, \frac{n+1}{2})$ representation, where the GPT $\eta(p,\lambda) \sim u_{\alpha_1\cdots\alpha_n}(p,\lambda)$ is totally symmetric, traceless and satisfies the subsidiary conditions[1]

$$\begin{aligned} p^{\alpha} u_{\alpha\alpha_2\cdots\alpha_n}(p,\lambda) &= 0, \\ (\not{p} - M)\, u_{\alpha_1\cdots\alpha_n}(p,\lambda) &= 0, \\ \gamma^{\alpha} u_{\alpha\alpha_2\cdots\alpha_n}(p,\lambda) &= 0. \end{aligned} \tag{2.3}$$

The subsidiary conditions (2.2) and (2.3) simply ensure that the number of degrees of freedom is $2j+1$. For more material on the construction of the GPTs, identities and general relations we refer to ref. [14], especially its appendices.

Thanks to the Lorentz invariance of the theory, a rank-2 tensor $T^{\mu\nu}$, such as the EMT, can be expressed as a sum of Lorentz tensors built out of the Minkowski metric $g^{\mu\nu}$, the totally antisymmetric Levi-Civita pseudo-tensor[2] $\varepsilon^{\mu\nu\rho\sigma}$, and the four-vectors of the problem P^μ and Δ^μ. The structures can be separately conserved, i.e. they vanish when contracted with Δ^μ, or not. Each of these Lorentz structures is multiplied by a Lorentz scalar function of $t = \Delta^2$ and are referred to as the GFFs. Various parametrisations have been proposed in the literature for spin-0, $\frac{1}{2}$, 1 [18, 19, 20, 21, 22, 1, 9, 10, 11, 15, 13], and higher spins [8]. When $j = n$ is integer, we find that the EMT matrix element can be written in terms of the following basis [14]

$$\begin{aligned} T^{\mu\nu,\alpha_1'\cdots\alpha_n'\alpha_1\cdots\alpha_n}(P,\Delta) = {} & 2P^\mu P^\nu \sum_{(k,n)} F_{1,k}(t) \\ & + 2\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right) \sum_{(k,n)} F_{2,k}(t) \\ & + 2g^{\mu\nu}M^2 \sum_{(k,n)} F_{3,k}(t) \\ & - P^{\{\mu} g^{\nu\}[\alpha_n'}\Delta^{\alpha_n]} \sum_{(k,n-1)} F_{4,k}(t) \\ & - \left[\Delta^{\{\mu} g^{\nu\}\{\alpha_n'}\Delta^{\alpha_n\}} - g^{\mu\nu}\Delta^{\alpha_n'}\Delta^{\alpha_n} - g^{\alpha_n'\{\mu} g^{\nu\}\alpha_n}\Delta^2\right] \sum_{(k,n-1)} F_{5,k}(t) \\ & + g^{\alpha_n'\{\mu} g^{\nu\}\alpha_n} M^2 \sum_{(k,n-1)} F_{6,k}(t) \\ & + \Delta^{[\alpha_n'} g^{\alpha_n]\{\mu} g^{\nu\}[\alpha_{n-1}'}\Delta^{\alpha_{n-1}]} \sum_{(k,n-2)} F_{7,k}(t) \\ & - P^{[\mu} g^{\nu][\alpha_n'}\Delta^{\alpha_n]} \sum_{(k,n-1)} F_{8,k}(t) \\ & - \Delta^{[\mu} g^{\nu]\{\alpha_n'}\Delta^{\alpha_n\}} \sum_{(k,n-1)} F_{9,k}(t), \end{aligned} \tag{2.4}$$

where $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$ and $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$. The symbol of sum $\sum_{(k,n)}$ is defined as:

$$\sum_{(k,n)} \equiv \sum_{k=0}^{n} \left[\prod_{i=1}^{k} \left(-\frac{\Delta^{\alpha_i'}\Delta^{\alpha_i}}{2M^2} \right) \prod_{i=k+1}^{n} g^{\alpha_i'\alpha_i} \right]. \tag{2.5}$$

[1] Note that the first condition is superfluous since it can be derived from the other two.

[2] We use the convention $\varepsilon_{0123} = +1$.

When $j = n + \frac{1}{2}$ is half-integer, we find a similar basis

$$
\begin{aligned}
T^{\mu\nu,\alpha'_1\cdots\alpha'_n\alpha_1\cdots\alpha_n}(P,\Delta) = {} & 2P^\mu P^\nu \sum_{(k,n)} F_{1,k}(t) \\
& + 2\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right) \sum_{(k,n)} F_{2,k}(t) \\
& + 2g^{\mu\nu}M^2 \sum_{(k,n)} F_{3,k}(t) \\
& + P^{\{\mu}\tfrac{i}{2}\sigma^{\nu\}\rho}\Delta_\rho \sum_{(k,n)} F_{4,k}(t) \\
& - \left[\Delta^{\{\mu}g^{\nu\}\{\alpha'_n}\Delta^{\alpha_n\}} - g^{\mu\nu}\Delta^{\alpha'_n}\Delta^{\alpha_n} - g^{\alpha'_n\{\mu}g^{\nu\}\alpha_n}\Delta^2\right] \sum_{(k,n-1)} F_{5,k}(t) \\
& + g^{\alpha'_n\{\mu}g^{\nu\}\alpha_n}M^2 \sum_{(k,n-1)} F_{6,k}(t) \\
& + \Delta^{[\alpha'_n}g^{\alpha_n]\{\mu}g^{\nu\}[\alpha'_{n-1}}\Delta^{\alpha_{n-1}]} \sum_{(k,n-2)} F_{7,k}(t) \\
& + P^{[\mu}\tfrac{i}{2}\sigma^{\nu]\rho}\Delta_\rho \sum_{(k,n)} F_{8,k}(t) \\
& - \Delta^{[\mu}g^{\nu]\{\alpha'_n}\Delta^{\alpha_n\}} \sum_{(k,n-1)} F_{9,k}(t).
\end{aligned}
\tag{2.6}
$$

The symmetric conserved part (associated to the GFFs $F_{i,k}$ with $i = 1,2,4,5,7$) is parametrised in terms of $2(j+1) + 3\lfloor j\rfloor - \theta(\lfloor j\rfloor > 1)$ GFFs, and the symmetric non-conserved part ($i = 3,6$) is parametrised in terms of $2\lfloor j\rfloor + 1$ GFFs, where $\lfloor j\rfloor$ is the floor of the spin, i.e. the largest integer smaller or equal to j. The antisymmetric conserved part ($i = 8$) is parametrised in terms of $\lceil j\rceil$ GFFs, where $\lceil j\rceil$ is the ceiling of the spin, i.e. the smallest integer greater or equal to j. The antisymmetric non-conserved part ($i = 9$) is parametrised in terms of $\lfloor j\rfloor$ GFFs. In total we get $4j + 5\lfloor j\rfloor + 3 - \theta(\lfloor j\rfloor > 1)$ GFFs which agrees with former results for spin 0, $\frac{1}{2}$, and 1. We arranged the bases so to maximise the number of conserved terms. This is especially important for the EMT, where non-conserved terms can also be present and they characterise the separate quark and gluon contributions [4, 7].

3. Universal properties

Inspecting the expressions (2.4) and (2.6) for the lower spin cases, one can bring to light the connection between the seeds in the chosen representation and the Lorentz generators $S^{\mu\nu}$ of the same representation, properly arranged. As extensively elaborated in [14], writing the EMT matrix element as an expansion of Lorentz generators, cast in spin multipoles, is indeed a more general and natural way of looking at the symmetry properties of these local operators.

This Section is based on a selection of results from refs. [15, 12, 16, 14]. We focus on a few terms of this multipole expansion, namely those on which the Poincaré generators put constraints. They correspond to terms that contain at most one power of Δ and one Lorentz generator $S^{\mu\nu}$. The (truncated) EMT expansion can then be written, for any spin value, as follows[3] [23, 12, 14]:

$$T^{\mu\nu}(P,\Delta) = P^{\{\mu}P^{\nu\}}F_1(t) + iP^{\{\mu}S^{\nu\}\rho}\Delta_\rho F_2(t) + \cdots . \tag{3.1}$$

The GFFs $F_1(t)$ and $F_2(t)$ are usually called $A(t)$ and $A(t)+B(t)$ respectively in the literature. As shown in [12, 16], the zero momentum transfer value of these objects is solely constrained by Poincaré symmetry and it assumes a constant value, independently of spin:

$$F_1(0) = F_2(0) = 1. \tag{3.2}$$

The (local) EMT matrix element, parametrised in terms of GFFs, is connected to a non-local operator written in terms of generalised parton distributions (GPDs) [21, 24, 25]. Denoting with $\mathcal{O}^\mu$ the GPD operator for both quarks and gluons, as in [12], the relation between $\mathcal{O}^\mu$ and the EMT involves its second Mellin moment, namely :

$$\int_{-1}^{1} dx\, x\, \mathcal{O}^\mu = \frac{T_q^{\mu\nu} n_\nu}{2(P\cdot n)^2}, \tag{3.3}$$

where the non-locality is indicated by the light-like vector n. Note that $\mathcal{O}^\mu$ is the operator which appears in the amplitude of the deeply virtual Compton scattering (DVCS). Restricting ourselves to the twist-2 part of eq. (3.3) (obtained from the contraction with the vector n) and neglecting terms with a higher power of Δ, the GPD correlator reads:

$$\mathcal{O}^\mu n_\mu = H_1(x,\xi,t) + \frac{iS^{\alpha\rho} n_\alpha \Delta_\rho}{P\cdot n} H_2(x,\xi,t) + \cdots , \tag{3.4}$$

where $\xi = -(\Delta\cdot n)/(2P\cdot n)$ is the light-front longitudinal momentum transfer. It is therefore possible to generalise Ji's sum rule [21] such that it holds independently of the spin of the states, and is universal for all hadrons of arbitrary spin. From Eq. (3.2) it follows that:

$$J^z = \sum_{a=q,g} \int_{-1}^{1} dx\, x H_2^a(x,0,0) = F_2(0) = 1, \tag{3.5}$$

which states that for any hadron with polarisation vector pointing along, say, the z-direction, the total angular momentum in the rest frame (summed over quarks and gluons) is constant and spin-universal.

4. Conclusions

In this work we have parametrised the EMT in terms of gravitational form factors. Our approach, which we refer to as the tensor product approach, consists in building all possible structures that appear in the GFFs expansion. We have found that the counting of GFFs depends on the spin value j in a non trivial way. The symmetry properties of the EMT can be exploited to derive universal relations on the GFFs which are independent of the spin. This paper is based on refs. [15, 12, 16, 14], to which we refer for further developments on the topic.

[3] We suppress the GPT indices for simplicity.

5. Acknowledgements

The authors wish to thank the organisation committee of the Light Cone 2019 meeting. This work has been financially supported by the Agence Nationale de la Recherche under the projects No.ANR-18-ERC1-0002 and ANR-16-CE31-0019.

References

[1] E. Leader and C. Lorcé, *The angular momentum controversy: What's it all about and does it matter?*, *Phys. Rept.* **541** (2014) 163 [1309.4235].

[2] M. V. Polyakov and P. Schweitzer, *Forces inside hadrons: pressure, surface tension, mechanical radius, and all that*, *Int. J. Mod. Phys.* **A33** (2018) 1830025 [1805.06596].

[3] M. V. Polyakov, *Generalized parton distributions and strong forces inside nucleons and nuclei*, *Phys. Lett.* **B555** (2003) 57 [hep-ph/0210165].

[4] C. Lorcé, *On the hadron mass decomposition*, *Eur. Phys. J.* **C78** (2018) 120 [1706.05853].

[5] C. Lorcé, L. Mantovani and B. Pasquini, *Spatial distribution of angular momentum inside the nucleon*, *Phys. Lett.* **B776** (2018) 38 [1704.08557].

[6] M. V. Polyakov and A. G. Shuvaev, *On 'dual' parametrizations of generalized parton distributions*, hep-ph/0207153.

[7] C. Lorcé, H. Moutarde and A. P. Trawiński, *Revisiting the mechanical properties of the nucleon*, *Eur. Phys. J.* **C79** (2019) 89 [1810.09837].

[8] A. A. Cheshkov and M. Shirov, Yu, *Invariant parametrization of local operators*, Soviet Physics JETP **17** (1963) .

[9] B. R. Holstein, *Metric modifications for a massive spin 1 particle*, *Phys. Rev.* **D74** (2006) 084030 [gr-qc/0607051].

[10] Z. Abidin and C. E. Carlson, *Gravitational form factors of vector mesons in an AdS/QCD model*, *Phys. Rev.* **D77** (2008) 095007 [0801.3839].

[11] S. K. Taneja, K. Kathuria, S. Liuti and G. R. Goldstein, *Angular momentum sum rule for spin one hadronic systems*, *Phys. Rev.* **D86** (2012) 036008 [1101.0581].

[12] S. Cotogno, C. Lorcé and P. Lowdon, *Poincaré constraints on the gravitational form factors for massive states with arbitrary spin*, *Phys. Rev.* **D100** (2019) 045003 [1905.11969].

[13] M. V. Polyakov and B.-D. Sun, *Gravitational form factors of a spin one particle*, *Phys. Rev.* **D100** (2019) 036003 [1903.02738].

[14] S. Cotogno, C. Lorcé, P. Lowdon and M. Morales, *Covariant multipole expansion of local currents for massive states of any spin*, 1912.08749.

[15] W. Cosyn, S. Cotogno, A. Freese and C. Lorcé, *The energy-momentum tensor of spin-1 hadrons: formalism*, *Eur. Phys. J.* **C79** (2019) 476 [1903.00408].

[16] C. Lorcé and P. Lowdon, *Universality of the Poincaré gravitational form factor constraints*, 1908.02567.

[17] P. Lowdon, K. Y.-J. Chiu and S. J. Brodsky, *Rigorous constraints on the matrix elements of the energy-momentum tensor*, *Phys. Lett.* **B774** (2017) 1 [1707.06313].

[18] H. Pagels, *Energy-Momentum Structure Form Factors of Particles*, *Phys. Rev.* **144** (1966) 1250.

[19] J. F. Donoghue and H. Leutwyler, *Energy and momentum in chiral theories*, *Z. Phys.* **C52** (1991) 343.

[20] I. Yu. Kobzarev and L. B. Okun, *GRAVITATIONAL INTERACTION OF FERMIONS*, *Zh. Eksp. Teor. Fiz.* **43** (1962) 1904.

[21] X.-D. Ji, *Gauge-Invariant Decomposition of Nucleon Spin*, *Phys. Rev. Lett.* **78** (1997) 610 [hep-ph/9603249].

[22] B. L. G. Bakker, E. Leader and T. L. Trueman, *A Critique of the angular momentum sum rules and a new angular momentum sum rule*, *Phys. Rev.* **D70** (2004) 114001 [hep-ph/0406139].

[23] D. G. Boulware and S. Deser, *Classical General Relativity Derived from Quantum Gravity*, *Annals Phys.* **89** (1975) 193.

[24] R. L. Jaffe and A. Manohar, *The G(1) Problem: Fact and Fantasy on the Spin of the Proton*, *Nucl. Phys.* **B337** (1990) 509.

[25] X.-D. Ji, *Off forward parton distributions*, *J. Phys.* **G24** (1998) 1181 [hep-ph/9807358].

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