

# The dipole picture and the non-relativistic expansion

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We study exclusive quarkonium production in the dipole picture at next-to-leading order (NLO) accuracy, using the non-relativistic expansion for the quarkonium wavefunction. This process offers one of the best ways to obtain information about gluon distributions at small  $x$ , in ultraperipheral heavy ion collisions and in deep inelastic scattering. The quarkonium light cone wave functions needed in the dipole picture have typically been available only at tree level, either in phenomenological models or in the nonrelativistic limit. In this paper, we discuss the compatibility of the dipole approach and the non-relativistic expansion and compute NLO relativistic corrections to the quarkonium light-cone wave function in light-cone gauge. Using these corrections we recover results for the NLO decay width of quarkonium to  $e^+e^-$  and we check that the non-relativistic expansion is consistent with ERBL evolution and with B-JIMWLK evolution of the target. The results presented here will allow computing the exclusive quarkonium production rate at NLO once the one loop photon wave function with massive quarks, currently under investigation, is known.

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## 1. Introduction

Exclusive quarkonium production is an ideal observable to study the gluon distribution at low  $x$  in Deep Inelastic Scattering and UltraPeripheral collisions. On one hand, exclusive processes depend on the gluon distribution quadratically (as opposed to inclusive ones, which depend on it linearly). On the other hand, we expect heavy quarkonium to be less sensitive to non-perturbative phenomena since the heavy quark mass  $m$  is much bigger than  $\Lambda_{QCD}$ .

We are going to study this process using light cone perturbation theory (LCPT). The application of LCPT to the study of exclusive process is called the dipole picture (for a review see [1]). In this framework, at leading order, the cross-section for the process of a virtual photon to interact with a nucleus (without destroying it) and transform into a quarkonium state fulfils the following equation

$$\frac{d\sigma_{T,L}^{\gamma^*+N\rightarrow HQ+N}}{dt} = \frac{1}{16\pi} \left| \int d^2r_\perp \int_0^1 \frac{dz}{4\pi} (\Psi_{HQ}^* \Psi_{\gamma^*})_{T,L} \sigma_{q\bar{q}} \right|^2. \quad (1.1)$$

Here  $\Psi_{\gamma^*}$  is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality  $Q$ , which fulfils  $Q \gg \Lambda_{QCD}$ . Therefore,  $\Psi_{\gamma^*}$  is dominated by perturbative physics. The dipole cross-section  $\sigma_{q\bar{q}}$ , is the quantity that contains the information about the gluon density in the nucleus. Finally,  $\Psi_{HQ}$  is the quarkonium wave function. Note that eq. (1.1) is just the LO result, at higher orders gluons have to be included in the wave functions of the photon and quarkonium. As a result, it is also needed to take into account the cross-section of the nucleus with more complex partonic configurations, such as  $\sigma_{q\bar{q}g}$  and so on.

Our aim is to obtain as much information as possible about the properties of the nucleus from the measurements of quarkonium exclusive production. However, the precision at which this can be done depends on our knowledge of the wave function of quarkonium. Several models have been used in the literature to describe this wave function (see [1] for a discussion of some of them). In this work we are going to study the results that can be obtained by treating quarkonium as a non-relativistic system. There are many reasons to consider this an interesting approach. First, it is generally considered that this is a good approximation. In fact, it is a very common approach in the literature when studying other problems related with quarkonium. Examples of this are the study of inclusive production, spectroscopy and quarkonium decays. Moreover, the non-relativistic limit is a well-defined limit of QCD that is worth studying on its own right. Finally, the non-relativistic limit has been used previously in the dipole picture literature at leading order [2, 3] and we aim to develop a framework that allows to extend these results to higher orders in perturbation theory.

This manuscript is organized as follows, in section 2 the quarkonium wave function in the non-relativistic limit is discussed. In section 3 we describe some cross-checks that this wave function is able to fulfil. In section 4 we discuss the application of these results to the computation of exclusive quarkonium production in the limit  $Q \gg m$ . Finally, in section 5 we give the conclusions. A more extended discussion of these results can be found in [4].

## 2. Quarkonium light cone wave function in the non-relativistic limit

At leading order the light cone wave function can be computed only taking into account non-relativistic quarks and their interactions. In LCPT they are defined by having a transverse momen-

tum  $p_\perp$  much smaller than  $m$  and a momentum fraction close to  $\frac{1}{2}$ . We allow relativistic degrees of freedom to be part of the wave function of quarkonium. However, they appear as perturbations, that is to say, they appear during small time periods. The mathematical structure of the light cone wave function of quarkonium in our approach is the following. The quarkonium wave function for short distances  $\mathbf{x}_\perp \sim \frac{1}{m}$  fulfils

$$\int dz f(z) \Psi_{HQ}^n(z, \mathbf{x}_\perp) = \sum_{m,k} \int dz f(z) C_{n \leftarrow m}^k(z, \mathbf{x}_\perp) \left( \frac{\nabla}{m} \right)^k \int \frac{d\lambda}{4\pi} \phi^m(\lambda, \mathbf{0}). \quad (2.1)$$

In this formula the index  $n$  and  $m$  represent the particle content; for higher Fock states the single two-particle relative coordinates  $z, \mathbf{x}_\perp$  are replaced by the appropriate variables. The notation  $\Psi_{HQ}$  refers to the full light-cone wave function of quarkonium, while the  $\phi$ 's are the wavefunctions restricted to the case in which all particles are non-relativistic (therefore  $\lambda \equiv z - \frac{1}{2} \ll 1$ ). Here  $f(z)$  is just a test function written to represent the fact that the equality is only valid when integrating over  $z$ . Note that each power of  $\left( \frac{\nabla}{M} \right)$  acting on  $\phi$  gives a suppression of order  $v$ . At the moment we are interested in the leading order corrections in  $\alpha_s$ . In this approximation we only need to take into account terms with  $k = 0$ . More precisely, we only need to compute the leading order corrections of  $C_{q\bar{q} \leftarrow q\bar{q}}^0$  and the leading order contribution to  $C_{q\bar{q}g \leftarrow q\bar{q}}^0$ .

### 3. Cross-checks

The first cross-check we discuss is the computation of the light-cone distribution amplitude. This is a quantity that encodes the properties of hadrons in computations in which the energy of the collision is much bigger than the energy scales ruling the behaviour of the hadron. In the light-cone gauge, it can be obtained from the light cone wave function in the limit  $r_\perp = 0^1$ . Moreover, this quantity must fulfil the Efremov-Radyushkin-Brodsky-Lepage (ERBL) equation [5, 6].

$$\frac{\partial D^i(z)}{\partial \log \mu^2} = \frac{\alpha_s C_F}{2\pi} \int_0^1 dz' K_{L,T}(z, z') D^i(z'), \quad (3.1)$$

where  $K_{L,T}(z, z')$  is the ERBL kernel. We have checked explicitly in [4], both in longitudinal and transverse polarization, that the distribution amplitudes obtained by our wave functions fulfil this requirement.

Now we move to the discussion of our second cross-check. The cross-section of the process of quarkonium decaying into a pair of leptons can be computed knowing the light cone wave function of quarkonium. At the same time, in the context of NRQCD and related approaches the leading order corrections to this cross-section are well-known [7]. Knowing this, we can deduce that

$$\int_0^1 dz \sum_n \Psi_{HQ}^n(z, \mathbf{0}_\perp) = \left( 1 - \frac{2\alpha_s C_F}{\pi} \right) \int d\lambda \phi(\lambda, \mathbf{0}). \quad (3.2)$$

Note that the previous equation is valid when all divergences are regulated using dimensional regularization. This is not our case since, as it is usually done in LCPT literature, use a cut-off  $x_0$  to regulate in the  $+$  component. This will have consequences in the rest of this subsection.

<sup>1</sup>In practice this is not so straightforward, to take the limit  $r_\perp \rightarrow 0$  introduces ultraviolet divergences which we regulate using dimensional regularization

We focus on the case of longitudinal polarization. Performing this computation we obtain

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, \mathbf{0}_\perp) = \left( 1 + \frac{\alpha_s C_F}{\pi} \left( \frac{1}{x_0} - 2 \right) \right) \int d\lambda \phi(\lambda, \mathbf{0}). \quad (3.3)$$

The origin of the  $\frac{1}{x_0}$  divergence is the Coulomb singularity due to the exchange of gluons with  $p_\perp \sim mx_0 \ll m$  and  $\lambda \sim x_0 \ll 1$ . The computation is consistent if the following equation is fulfilled

$$\frac{d}{dx_0} \int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, \mathbf{0}_\perp) = -\frac{\alpha_s C_F}{\pi x_0^2} \int d\lambda \phi(\lambda, \mathbf{0}) + \frac{d}{dx_0} \int d\lambda \phi(\lambda, \mathbf{0}) = 0. \quad (3.4)$$

We have checked that it is the case.

#### 4. Exclusive quarkonium production in the $Q \gg m$ limit

Our final goal is to compute the exclusive quarkonium production in the general case. However, in order to do so we would need the one loop corrections to the photon wave function with massive quarks, which are the moment are unknown. We note, however, that there is an outgoing project to perform precisely this computation<sup>2</sup>. In this section we focus on the structure of the divergences in the case  $Q \gg m$  and for longitudinal polarization. We get that all divergences are correctly cancelled. For this it is crucial to take into account the B-JIMWLK evolution of the target.

The tree level result for the exclusive production of quarkonium has two terms which are sensitive to  $x_0$ . One is  $\sigma_{q\bar{q}}$ , which must fulfil the B-JIMWLK evolution equation. The other is the non-relativistic wave function, which fulfils eq. (3.4).

Now we discuss the corrections which only involve the photon wave function. In the limit  $Q \gg m$  and in order to study ultraviolet and collinear singularities, we can use the results with massless quarks of [8].

$$\Psi_\gamma(z, r_\perp)|_{NLO} = \Psi_\gamma(z, r_\perp)|_{LO} (1 + \delta Z_\gamma(z, r_\perp)). \quad (4.1)$$

In our case we need  $Z_\gamma(\frac{1}{2}, r_\perp)$ . We note that this quantity has a dependence on both  $x_0$  and  $\mu$ .

Now we discuss the corrections which only involve the quarkonium wave function. Regarding the dependence on  $\mu$ , this is only given by the heavy quark wave function renormalization coefficient  $Z$ , which fulfils  $\frac{d\delta Z}{d\mu} = \frac{d\delta Z_\gamma(\frac{1}{2}, r_\perp)}{d\mu}$ . The dependence on  $x_0$  can be divided into two terms. One of these cancels the one given by eq. (3.4), and the other is equal to the dependence on  $x_0$  given by the photon wave function.

Finally, we discuss the divergences given by diagrams in which a gluon is emitted before the interaction with the target and absorbed after it, in such a way that the amplitude of the process is proportional to  $\sigma_{q\bar{q}g}$ . The dependence on  $\mu$  is such that it cancels those given by the photon and quarkonium wave functions. The dependence with  $x_0$  can be divided in two pieces. One is proportional to  $\sigma_{q\bar{q}g} - \sigma_{q\bar{q}}$  which cancels the B-JIMWLK evolution of the target. The other is proportional to  $\sigma_{q\bar{q}}$  and cancels the divergences given by the photon and quarkonium wave functions, except for the piece which is cancelled by eq. (3.4). With this we have finished the discussion of the cancellation of all divergences which appear in the calculation.

<sup>2</sup>See contributions of G. Beuf and R. Paatelainen on these proceedings.

## 5. Conclusions

In this work we have discussed the next-to-leading order corrections to the heavy quarkonium light-cone wave function in the non-relativistic limit. As cross-checks, we have seen that the light-cone distribution amplitude that we can obtain from it fulfils the ERBL equation and that using our results we can obtain the one loop corrections to the decay of quarkonium into leptons. To our knowledge, this is the first computation of this process in the light-cone gauge. Regarding the application of our computation to the study of exclusive quarkonium production, which is our final goal, we have checked, by studying the  $Q \gg m$  limit, that all divergences are correctly cancelled. Once the photon wave function with massive quarks is known we are ready for phenomenological applications.

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