PROCEEDINGS OF SCIENCE

# Gauge boson mass as regulator of dynamics

**Stanisław D. Głazek** [*]

*Faculty of Physics, University of Warsaw*

*E-mail:* stglazek@fuw.edu.pl

Small-x divergences of Abelian gauge theory in the front form of Hamiltonian dynamics are regulated using a mass parameter for gauge bosons, introduced through a mechanism analogous to the spontaneous breaking of global gauge symmetry. A corresponding family of ultraviolet and infrared finite scale-dependent renormalized Hamiltonians, is calculable order-by-order using the renormalization group procedure for effective particles. The second-order terms described here suggest the magnitude of mass corrections that may be involved in resolving the small-x parton and front-form vacuum and zero-mode problems, assuming that the gauge boson mass that counts does not exceed the current upper bound on the photon mass.

[*]Speaker.

## 1. The issue

Front-form (FF) [1] Hamiltonians of quantum gauge theories (QGT) are singular. To become suitable for computation, they require regularization. To remove effects of regularization, they have to be renormalized. The question is how to do it [2]. Among other things, the issue is that in the FF dynamics of QGT the gauge bosons in gauge $A^+ = 0$ have polarization vectors

$$\varepsilon_{p\sigma}^{\mu} = \left( \varepsilon_{p\sigma}^- = \frac{2p^{\perp}\varepsilon_{\sigma}^{\perp}}{p^+}, \ \varepsilon^+ = 0, \ \varepsilon_{\sigma}^{\perp} \right) , \qquad (1.1)$$

where $\varepsilon_{\sigma}^{\perp} = (1 + \sigma, 1 - \sigma)/2$ and $\sigma = \pm 1$. The canonical minimal coupling of a current $j_{\mu}$ with the field $A^{\mu}$ has the form $j_{\mu}A^{\mu}$. This form leads to the Hamiltonian interaction terms $H_I$ that are proportional to $j^+ \varepsilon_{p\sigma}^-$ and thus include the factor $p^{\perp}/p^+$. In the parton model [3], the gauge boson carries a fraction $x$ of momentum $P$ of the system it belongs to and momentum $k$ that is transverse to $P$. In the FF dynamics, one has $p^+ = xP^+$, $p^{\perp} = xP^{\perp} + k^{\perp}$. In perturbative description of the system, one encounters operator products such as $H_I(P^- - H_f)^{-1}H_I$, where $H_f$ denotes the free Hamiltonian. For example, consider a fermion of momentum $P$ that emits and reabsorbs a gauge boson. The sum over the boson polarizations includes summing $|\varepsilon_{p\sigma}^-|^2$, which yields $(k^{\perp}/x)^2$. The sum over intermediate fermion-boson systems involves integrating over the boson $x$ and $k^{\perp}$, for fixed $P$. The issue is that $(k^{\perp}/x)^2$ makes the integral diverge for $x \to 0$ and $k^{\perp} \to \infty$. These divergences reinforce each other.

How to handle the ultraviolet divergence due to large $k^{\perp}$ is understood [4]. A practical method to handle small-$x$ divergences awaits invention. One can limit $p^+$ from below by a cutoff $\delta^+$. This excludes creation of field quanta from the bare vacuum state $|0\rangle$, which can thus be used to build a whole Fock space of states in which the canonical Hamiltonian acts. However, $\delta^+$ breaks boost invariance and one may prefer the cutoff $x > \delta$, where $\delta$ is a pure number that determines the minimal value of $x$ irrespective of the value of $P^+$. We proceed in a different way, focusing on Abelian theory as the simplest one that exhibits $(k^{\perp}/x)^2$ behavior.

We apply the renormalization group procedure for effective particles (RGPEP) [5]. Small-$x$ regularization results from introducing mass for the gauge bosons. The mass is introduced via a mechanism analogous to the spontaneous breaking of global gauge symmetry [6, 7] and we work in the limit that yields Soper's FF version of massive QED [8, 9]. Massive gauge theories have a history of studies using light-front quantization methods with various regularizations, see [10] and references therein. We address the issue of evaluating finite scale-dependent and boost invariant effective Hamiltonian operators. Specifically, we describe the mass corrections and discuss the orders of magnitude of terms that one needs to deal with. The heuristic value of such consideration is that the RGPEP studies can order-by-order (in expansion in powers of a coupling constant) teach us about the FF dynamics of gauge field quanta. This discussion concerns only terms of second order in Abelian Soper's theory.

## 2. RGPEP

The canonical Hamiltonian of a QGT, denoted by $H_{\mathrm{can}}$, is considered an initial condition for solving the differential equation ($\mathscr{H}_f$ is the free part of $H_{\mathrm{can}}$ and $\tilde{\mathscr{H}}_t$ is simply related to $\mathscr{H}_t$, cf. [5])

$$\mathscr{H}_t' = [[\mathscr{H}_f, \tilde{\mathscr{H}}_t], \mathscr{H}_t] , \qquad (2.1)$$

for $\mathscr{H}_t$ so that $\mathscr{H}_{t=0} = H_{\mathrm{can}}$. The equation originates from the similarity renormalization group procedure [11] and results from adapting the double-commutator flow equation for Hamiltonian matrices [12] to the purpose of calculating effective FF Hamiltonian operators $H_t$. Solving Eq. (2.1) for the Hamiltonians $\mathscr{H}_t$ is an intermediate step. They are polynomials in creation and annihilation operators of canonical theory. The polynomial coefficients, say $c_t$, that are found by solving Eq. (2.1), are used to build the Hamiltonians $H_t$ for effective particles. Namely, $H_t$ is obtained by replacing canonical creation and annihilation operators in $\mathscr{H}_t$ by the ones for effective particles that are labeled by $t$. Quantum numbers are the same. The coefficients $c_t$ depend only on these quantum numbers and they are the same in $\mathscr{H}_t$ and $H_t$. The unitary relationship between the canonical and effective particle operators is the source of Eq. (2.1) [5]. One works with operators instead of matrix elements and the generator of the unitary transformation is designed to preserve all seven kinematic Poincaré symmetries of the FF of Hamiltonian dynamics. Divergences of the theory appear in $H_t$ in explicit form. One deals with them by adding counter terms to the canonical Hamiltonian $H_{\mathrm{can}}$. We describe the magnitude of mass counter terms and effective mass corrections one obtains in Soper's theory [8].

## 3. Second-order mass corrections

Applying the RGPEP to Soper's theory, one starts from its FF Hamiltonian density [8, 9],

$$
\begin{aligned}
\mathscr{H} &= \bar{\psi}_f \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{2i\partial^+} \psi_f + \frac{1}{2} A_f^i \left[ (i\partial^\perp)^2 + \kappa^2 \right] A_f^i + \frac{1}{2} B \left[ (i\partial^\perp)^2 + \kappa^2 \right] B \\
&+ g \bar{\psi}_f \slashed{A}_f \psi_f - g \bar{\psi}_f \gamma^+ \psi_f \frac{\kappa}{i\partial^+} iB + \frac{1}{2} g^2 \bar{\psi}_f \slashed{A}_f \frac{\gamma^+}{i\partial^+} \slashed{A}_f \psi_f + \frac{1}{2} \left[ \frac{1}{\partial^+} g \bar{\psi}_f \gamma^+ \psi_f \right]^2 ,
\end{aligned} \quad (3.1)
$$

applies the standard light-front quantization procedure and solves Eq. (2.1). The density includes the fermion field $\psi$, transverse boson field $A$ with two polarizations in gauge $A^+ = 0$ and an additional gauge boson field $B$, associated with the third polarization state of massive vector bosons.

Regularization originates from the RGPEP form factors that result from solving Eq. (2.1) for terms order $g$. They have the form $\mathscr{H}_{t1c.a} = \exp[-t(\mathscr{M}_c^2 - \mathscr{M}_a^2)] \mathscr{H}_{01c.a}$, where $c$ and $a$ refer to operators that create and annihilate quanta and $\mathscr{M}$ denotes their total invariant mass, correspondingly. One multiplies the bare interaction terms by the form factor with $t$ replaced by $t_r$. Singularities due to $(k^\perp/x)^2$ are regulated because the mass $\kappa$ enters $\mathscr{M}^2$ through $(k^{\perp 2} + \kappa^2)/x$. The regularization is lifted when $t_r \to 0$.

In the series expansion $H_t = H_{tf} + g H_{t1} + g^2 H_{t2} + O(g^3)$, the bare coupling constant $g$ differs from the effective coupling constant $g_t$ first in third order. We do not need to distinguish them here, because our discussion only concerns the mass squared terms in $H_t$ that include the free terms from $H_{tf}$ and second-order mass squared corrections from $g^2 H_{t2}$. Namely,

$$
H_{t\psi} = \sum_{\sigma=1}^{2} \int [p] \frac{p^{\perp 2} + m^2 + g^2 \delta m^2(t)}{p^+} \left[ b_{t p \sigma}^\dagger b_{t p \sigma} + d_{t p \sigma}^\dagger d_{t p \sigma} \right] , \quad (3.2)
$$

$$
H_{tA} = \sum_{\sigma=1}^{2} \int [p] \frac{p^{\perp 2} + \kappa^2 + g^2 \delta \kappa_A^2(t)}{p^+} a_{t p \sigma}^\dagger a_{t p \sigma} , \quad (3.3)
$$

$$
H_{tB} = \int [p] \frac{p^{\perp 2} + \kappa^2 + g^2 \delta \kappa_B^2(t)}{p^+} c_{t p}^\dagger c_{t p} . \quad (3.4)
$$

The mass-squared counter terms in the canonical Hamiltonian are adjusted so that the eigenvalue problems for a single physical particle have eigenvalues $m^2$ for fermions and $\kappa^2$ for bosons of types $A$ and $B$. After integration over $k^\perp$,

$$g^2 \delta m^2(t) = \frac{\alpha_g}{4\sqrt{2\pi}} \frac{I_1(t)}{\sqrt{t+t_r}} - \frac{\alpha_g}{4\pi} \left(2m^2 + \kappa^2\right) I_2(t) , \tag{3.5}$$

$$g^2 \delta \kappa_A^2(t) = \frac{\alpha_g}{4\sqrt{2\pi}} \frac{I_3(t)}{\sqrt{t+t_r}} + \frac{\alpha_g}{4\pi} \left(2m^2 + \kappa^2\right) I_4(t) , \quad g^2 \delta \kappa_B^2(t) = \frac{\alpha_g}{4\pi} \kappa^2 I_5(t) , \tag{3.6}$$

where $\alpha_g = g^2/(4\pi)$ and the scale-dependent integrals are

$$I_1(t) = \int_0^1 dx \, \frac{1 + (1-x)^2}{x} \, \text{erfc}\left[\sqrt{2(t+t_r)} \, \delta\mathscr{M}_{fb}^2\right] , \tag{3.7}$$

$$I_2(t) = \int_0^1 dx \, \Gamma\left[0, 2(t+t_r) \, \delta\mathscr{M}_{fb}^4\right] , \tag{3.8}$$

$$I_3(t) = \int_0^1 dx \, \left[x^2 + (1-x)^2\right] \, \text{erfc}\left[\sqrt{2(t+t_r)} \, \delta\mathscr{M}_{f\bar{f}}^2\right] , \tag{3.9}$$

$$I_4(t) = \int_0^1 dx \, \left[1 - \frac{\kappa^2 x(1-x)}{m^2 + \kappa^2/2}\right] \Gamma\left[0, 2(t+t_r) \, \delta\mathscr{M}_{f\bar{f}}^4\right] , \tag{3.10}$$

$$I_5(t) = \int_0^1 dx \, 4x(1-x) \, \Gamma\left[0, 2(t+t_r) \, \delta\mathscr{M}_{f\bar{f}}^4\right] . \tag{3.11}$$

erfc and $\Gamma$ are the complementary error and incomplete gamma functions. Their arguments include $\delta\mathscr{M}_{fb}^2 = \kappa^2/x + m^2/(1-x) - m^2$ and $\delta\mathscr{M}_{f\bar{f}}^2 = m^2/x + m^2/(1-x) - \kappa^2$. In the limit $t \to 0$, Eqs. (3.5) and (3.6) provide the values of the mass-squared counter terms introduced in the initial, canonical Hamiltonian that is regulated using $t_r$.

The effective fermion mass correction behaves like $-\ln(\kappa^2\sqrt{t+t_r})/\sqrt{t+t_r}$ for small $t$. The boson $A$ and $B$ corrections are less singular but they significantly differ from each other, though the physical masses, or eigenvalues of the Hamiltonians $H_t$ for all finite values of $t$, are equal in second order calculation $m^2$ for the fermions and $\kappa^2$ for the both types of bosons, $A$ and $B$.

## 4. Orders of magnitude

We set $\alpha_g = 1/137$ and provide examples of the mass corrections we obtain for $\kappa = m$. This

| $\kappa = m \quad s\,m:$ | 1 | 0.5 | 0.25 | 0.1 | 0.01 | 0.001 |
|---|---|---|---|---|---|---|
| $g^2 \delta m^2/m^2$ | $3.19 \, 10^{-13}$ | $3.03 \, 10^{-4}$ | $1.88 \, 10^{-2}$ | $3.67 \, 10^{-1}$ | $1.04 \, 10^2$ | $1.71 \, 10^4$ |
| $g^2 \delta \kappa_A^2/m^2$ | $2.73 \, 10^{-13}$ | $1.69 \, 10^{-4}$ | $5.17 \, 10^{-3}$ | $4.67 \, 10^{-2}$ | $4.85$ | $4.85 \, 10^2$ |
| $g^2 \delta \kappa_B^2/m^2$ | $5.64 \, 10^{-14}$ | $3.38 \, 10^{-5}$ | $6.21 \, 10^{-4}$ | $1.96 \, 10^{-3}$ | $5.52 \, 10^{-3}$ | $9.09 \, 10^{-3}$ |

case qualitatively illustrates the situation one encounters in commonly considered systems made of two constituents of comparable masses. The parameter $s = t^{1/4}$ can be intuitively understood as the size of effective quanta. The mass corrections are indeed small for the size $s$ comparable or larger than the fermion Compton wavelength. However, they quickly grow when $s$ becomes smaller than the wavelength.

The mass corrections greatly increase when one considers the boson mass $\kappa$ comparable to the experimental upper limit on the photon mass, $m_\gamma < 10^{-18}$ eV [13]. The mass corrections we obtain for the corresponding $\kappa = 10^{-25} m$, assuming $m$ is on the order of the electron mass, are given in the table below. Such large corrections are less harmful to the theory than one might expect

| $\kappa = 10^{-25} m$ , $sm$ : | 1 | $10^{-1}$ | $10^{-3}$ | $10^{-6}$ |
|---|---|---|---|---|
| $g^2 \delta m^2 / m^2$ | $1.62\ 10^{-1}$ | $1.71\ 10^{+1}$ | $1.85\ 10^{+5}$ | $2.05\ 10^{+11}$ |
| $g^2 \delta \kappa_A^2 / m^2$ | $1.39\ 10^{-4}$ | $4.93\ 10^{-2}$ | $4.85\ 10^{+2}$ | $4.85\ 10^{+8}$ |
| $g^2 \delta \kappa_B^2 / m^2$ | $3.17\ 10^{-70}$ | $1.79\ 10^{-53}$ | $8.92\ 10^{-53}$ | $1.96\ 10^{-52}$ |

because the mass corrections are exactly canceled by the self-interactions of effective particles and the eigenvalues continue to not depend on $t$. However, the magnitude of terms that cancel out can be very large unless one keeps the size $s$ of effective particles in the right range. This way the mass correction hierarchy problem is resolved by the finite size of quanta.

## 5. Conclusion

The RGPEP computation of effective mass corrections in Soper's theory suggests a path to take for calculating other Hamiltonian interaction terms in particle theory. Local gauge symmetry with minimal coupling approximates effective interactions of fermions for invariant mass changes much smaller than the inverse of their Compton wavelength. The hierarchy problem for mass squared corrections is similarly resolved. The author hopes to discuss non-Abelian gauge theories with infinitesimally spontaneously broken global gauge symmetry for regularization purposes in a near future elsewhere.

## References

[1] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[2] K. G. Wilson et al., Phys. Rev. D **49**, 6720 (1994).

[3] R. P. Feynman, Phys. Rev. Lett. **23**, 1415 (1969).

[4] K. G. Wilson, Phys. Rev. D **2**, 1438 (1970).

[5] S. D. Głazek, Acta Phys. Polon. B **43**, 1843 (2012).

[6] P. W. Higgs, Phys. Lett. **12**, 132 (1964).

[7] F. Englert, R. Brout, Phys. Rev. Lett. **13**, 321 (1964).

[8] D. E. Soper, Phys. Rev. D **4**, 1620 (1971).

[9] S. D. Głazek, Acta Phys. Polon. B **50**, 5 (2019).

[10] J. R. Hiller, Prog. Part. Nucl. Phys. **90**, 75 (2016).

[11] S. D. Głazek, K. G. Wilson, Phys. Rev. D **48**, 5863 (1993).

[12] F. Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994).

[13] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).