

Impact-parameter dependence of collinearly improved Balitsky-Kovchegov evolution

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The Balitsky-Kovchegov equation has been solved including the impact-parameter dependence. Previous attempts to include this dependence have been spoiled by the presence of the so-called Coulomb tails produced by the evolution. We show, that using the collinearly-improved kernel to the BK equation, the Coulomb tails are heavily suppressed which allows for a correct description of existing data—both of the structure function and exclusive vector meson production—, as well as for the prediction of processes that are feasible for measurement at future facilities such as EICs.

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1. Introduction

Evolution equations are a tool for understanding and predicting QCD driven properties of matter. The evolution in energy (rapidity) for nucleons is given by the Balitsky-Kuraev-Fadin-Lipaton (BFKL) equation [1, 2] that incorporates gluon branching processes. Non-linear gluon behavior such as saturation is taken into account in the Balitsky-Kovchegov (BK) evolution equation [3, 4, 5]. Solutions of this equation for the impact-parameter dependent computation are spoiled by the presence of the so-called Coulomb tails. Coulomb tails are a consequence of the fact, that the BK equation is derived in a purely perturbative way and in the impact-parameter dependent computation, a non-perturbative region is reached. This effect then violates the Martin-Froissart bound and makes data description impossible [6].

In this work we demonstrate that the recently proposed collinearly improved kernel [7] for this equation suppresses large daughter dipole contribution to the evolution which in turn suppresses these non-perturbative regions of Coulomb tails enabling description of data for various processes [8, 9].

2. The Balitsky-Kovchegov equation

The BK equation reads

$$\begin{aligned} \frac{\partial N(r, b; Y)}{\partial Y} = & \int d\vec{r}_1 K(r, r_1, r_2) \left(N(r_1, b_1; Y) \right. \\ & \left. + N(r_2, b_2; Y) - N(r, b; Y) - N(r_1, b_1, Y)N(r_2, b_2; Y) \right), \end{aligned} \quad (2.1)$$

where $\vec{r}_2 = \vec{r} - \vec{r}_1$ and $|\vec{r}_i| \equiv r_i$. The variables b_1 and b_2 denote the magnitudes of the impact parameters of the daughter dipoles.

The collinearly improved kernel [7, 10, 11, 12] is written as

$$K(r, r_1, r_2) = \frac{\bar{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm \bar{\alpha}_s A_1} \frac{J_1(2\sqrt{\bar{\alpha}_s \rho})}{\sqrt{\bar{\alpha}_s \rho}}. \quad (2.2)$$

The value of A_1 is 11/12 and the sign in the third factor is chosen positive when $r^2 < \min(r_1^2, r_2^2)$ and negative otherwise. $\rho \equiv \sqrt{L_{r_1 r} L_{r_2 r}}$, J_1 is the Bessel function and $L_{r_i r} \equiv \ln(r_i^2/r^2)$. The smallest dipole prescription was chosen for the running coupling: $\alpha_s = \alpha_s(r_{\min})$, where $r_{\min} = \min(r_1, r_2, r)$ as in [10].

The region, that contributes the most to the rise of Coulomb tails is the one where large daughter dipoles are emitted [6]. This region is suppressed in the collinearly improved kernel w.r.t. the running coupling kernel [13] (as shown in Fig. 1). In order to compute the evolution, one has to start with an initial condition. We have come up with a prescription

$$N(r, b, Y = 0) = 1 - \exp\left(-\frac{1}{2} \frac{Q_s^2}{4} r^2 T(b_{q_1}, b_{q_2})\right), \quad (2.3)$$

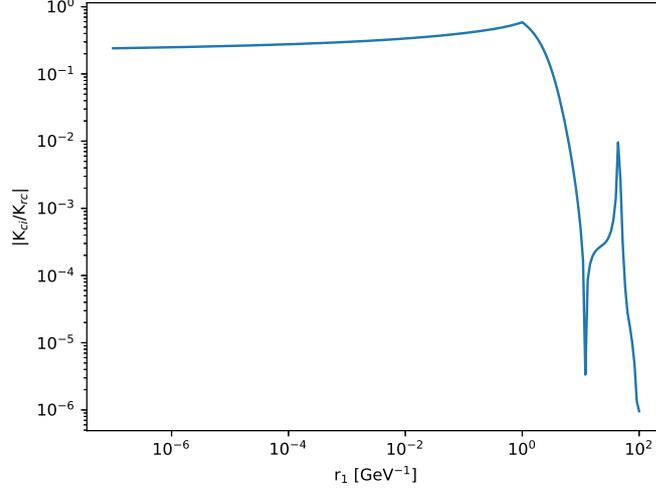


Figure 1: Absolute value of the ratio K_{ci}/K_{rc} at a fixed dipole size $r = 1 \text{ GeV}^{-1}$ and orientation with respect to the daughter dipole $\theta_{rr_1} = \pi/2$ as a function of the daughter dipole size. Figure taken from [8].

where b_{q_i} are the impact parameters of the individual quark and antiquark of the initial bare dipole and

$$T(b_{q_1}, b_{q_2}) = \left[\exp\left(-\frac{b_{q_1}^2}{2B}\right) + \exp\left(-\frac{b_{q_2}^2}{2B}\right) \right]. \quad (2.4)$$

This prescription (with parameters from [8]) combines the approach of the GBW model [14] for the dipole-size dependence and exponential fall-off for the impact parameter space [15, 16, 17, 18, 19]. In order to correctly account for the geometry of the target, we have taken into account the contribution of the two constituent quarks in the initial bare color-dipole separately [8].

3. Results

In Fig. 2 we see the computed scattering amplitude as a function of rapidity, impact parameter and transverse dipole size. Due to the nature of the used kernel, the presence of Coulomb tails in the large- b regions is strongly suppressed [8]. We have taken this scattering amplitude and used it to predict various observables that have been measured in the past to demonstrate that the predictions of this equation are no longer spoiled by the non-perturbative regions and that it can be used for generating predictions for future facilities (see Figs 3 and 4).

4. Summary

We have used the recently proposed collinearly improved kernel to solve the impact-parameter dependent BK equation. Due to the fact, that the time-ordered gluon emissions that are embedded in the collinear resummation [10] suppress the region of large daughter dipoles, we were able to obtain a scattering amplitude that is no longer spoiled by the presence of Coulomb tails and can be

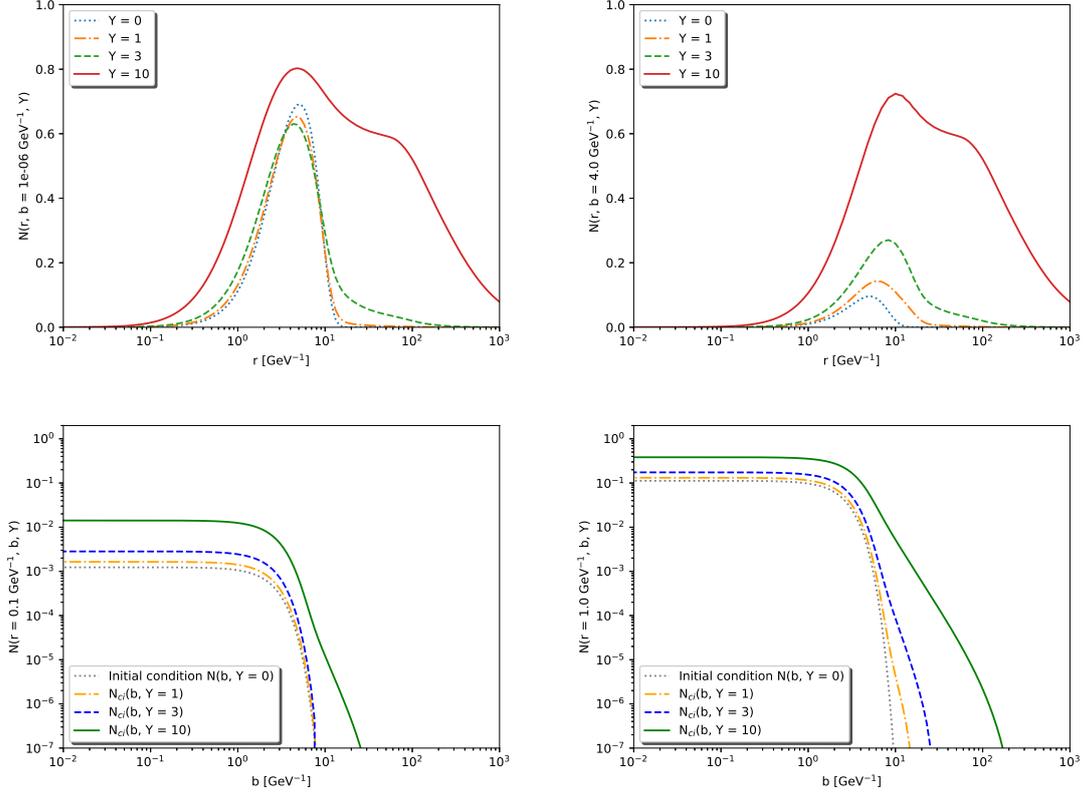


Figure 2: The scattering amplitude as a solution to the BK equation with the collinearly improved kernel as a function of r for $b = 10^{-6} \text{ GeV}^{-1}$ (upper left) and $b = 4 \text{ GeV}^{-1}$ (upper right), and as a function of b at $r = 0.1 \text{ GeV}^{-1}$ (lower left) and at $r = 1 \text{ GeV}^{-1}$ (lower right). Figure taken from [8].

used to compute various phenomena [8]. This is useful for phenomenological applications in QCD namely for the future planned facilities such as LHeC and the EIC [23, 24].

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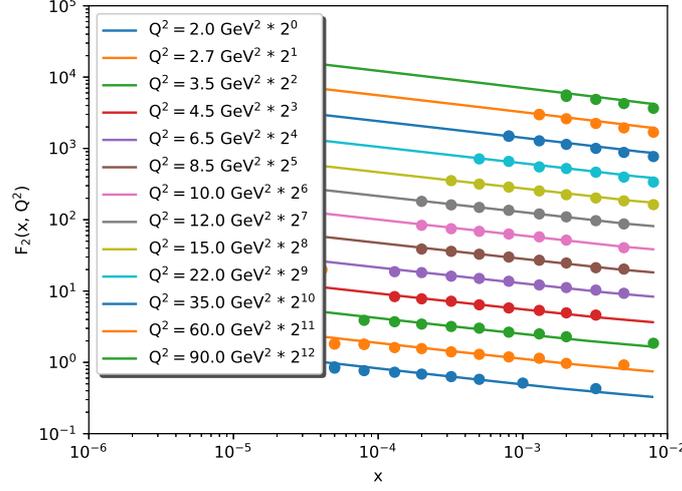


Figure 3: Comparison of the structure function data from HERA [20] (solid circles) to the prediction of the impact-parameter dependent BK equation with the collinearly improved kernel (lines). Figure taken from [8].

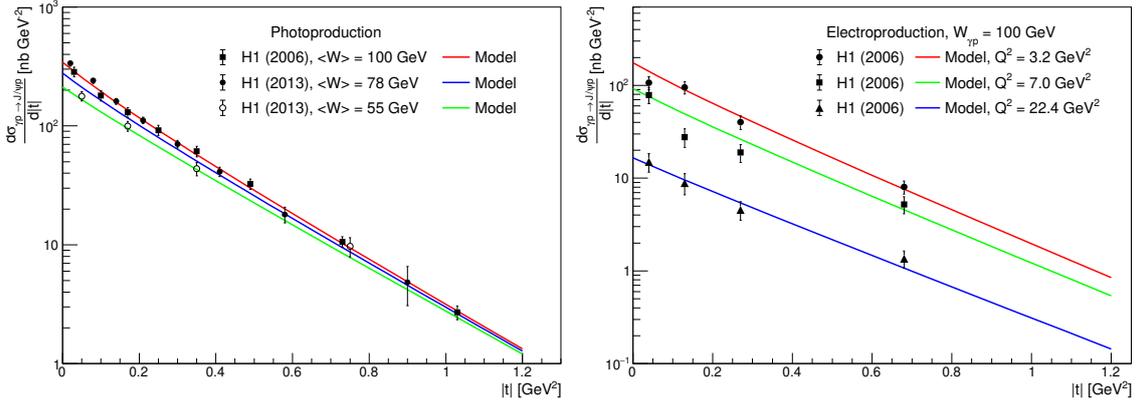


Figure 4: Comparison of the predictions of the model (solid lines) with HERA data from H1 [21, 22] for the $|t|$ dependence of the exclusive photoproduction (left) and electroproduction (right) cross sections of the J/ψ meson. Figure taken from [8].

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