

Basic properties of GPDs and modelling of the latter

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We present here a new method based on the Radon transform to model Generalised Parton Distributions (GPDs). It allows to fulfil all theoretical constraints applying on GPDs, especially polynomiality and positivity at the same time. More specifically, we show how polynomiality can be systematically restored within the framework of the overlaps of Lightfront Wave Functions (LFWFs). It provides a systematic way to extend models defined solely in the DGLAP kinematical region to the ERBL one. We then exemplify our approach using LFWFs models.

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1. Introduction

Since they have been introduced two decades ago [1, 2, 3], Generalised Parton Distributions (GPDs) have been under deep theoretical and experimental studies. On the experimental side, we are entering the precision era [4], being sensitive to higher-twist and next-to-leading order effects [5]. On the theory side, progresses have been also performed, allowing us to extract Form Factors of exclusive processes [6, 7, 8] with a better control of uncertainties. Another breakthrough was made in the possibility to take into account higher-twist kinematical effects [9, 10] shown to be sizeable [11]. On top of this, new open source software like PARTONS [12] are now available.

Modelling GPDs remains however a challenge and various approaches have been developed. Phenomenologists mostly rely on Double Distributions [1, 3] and more precisely on the Radyushkin Ansatz [13] (see for instance [14, 15]) and on conformal space approaches [16, 17]. Model based on dynamical assumptions of QCD are still not in sight for the nucleon, although some attempts had been performed in the meson sector [18, 19]. Lattice calculations of GPDs Mellin moments have also been performed [20] and attempts to compute quasi-GPDs start to emerge [21].

Within this general picture, we present here briefly the new approach we developed in [22, 23] to allow the computation of GPD models fulfilling *a priori* all the theoretical constraints required by the symmetries of QCD. Our framework allow notably to fulfil at the same time both positivity and polynomiality, which are known to be hard to conciliate together *a priori*.

2. Basic Properties of GPDs

GPDs are defined as the Fourier transform of a non-perturbative matrix element of a quark operators depending on a light-like distance. For a pseudo-scalar hadron (a pion), one has [24]:

$$H(x, \xi, t) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P + \frac{\Delta}{2} | \bar{\psi} \left(-\frac{z}{2} \right) \gamma^+ \psi \left(\frac{z}{2} \right) | P - \frac{\Delta}{2} \rangle \Big|_{z^+ = z_{\perp} = 0}, \quad (2.1)$$

where P is the average momentum of the hadron, Δ the momentum transfer, x is the average momentum fraction carried by the active quark, the skewness $\xi = \Delta^+ / (2P^+)$ represent half of the fraction of longitudinal momentum exchanged and $t = \Delta^2$ is the usual Mandelstam variable. Through this work, lightfront coordinate will be used : $v^{\pm} = (v^0 \pm v^3) / \sqrt{2}$.

2.1 Positivity and Lightfront Wave Functions

The lightfront quantization allows one to expand hadron states on a Fock basis [25]:

$$|H, p, \lambda\rangle = \sum_{N, \beta} \int dx_1 \dots dx_N \frac{dk_1^{\perp} \dots dk_N^{\perp}}{(16\pi^3)^{N-1}} \delta_x \delta_{k_{\perp}} \Psi_N(x_1, k_1^{\perp} \dots x_N, k_N^{\perp}) |q_1 \dots q_N\rangle, \quad (2.2)$$

with $\delta_x = \delta(1 - \sum_i x_i)$ and $\delta_{k_{\perp}} = \delta^{(2)}(P_{\perp} - \sum_i k_i^{\perp})$ and Ψ_N being the N -body lightfront wavefunctions (LFWFs). This expansion provides one with a systematic way to compute GPDs in terms of overlaps of LFWFs [26]. However, the type of overlap depends on the kinematical region investigated. Indeed for $|x| \geq |\xi|$ (the DGLAP region), the overlap involves LFWFs of the same

number of partons (schematically $\sum_N \bar{\Psi}_N \Psi_N$) yielding a formal scalar product, and therefore the Cauchy-Schwartz inequality associated. This yields the positivity property [27]:

$$|H(x, \xi, t)| \leq \sqrt{q\left(\frac{x-\xi}{1-\xi}\right) q\left(\frac{x+\xi}{1+\xi}\right)}, \quad (2.3)$$

where q is the pion PDF. However, in the $|x| \leq |\xi|$ kinematical region, the initial state has two more partons than the final one, leading to an off-diagonal overlap formula (schematically $\sum_N \bar{\Psi}_{N-2} \Psi_N$). Therefore, a N -body truncation in the Fock basis (2.2) yields an ambiguous description of the ERBL region, and breaks the polynomiality properties of GPDs described below.

2.2 Polynomiality and Double Distributions

The Mellin moments of GPDs need to be polynomials in ξ due to Lorentz covariance:

$$M_n(\xi, t) = \int_{-1}^1 dx x^n H(x, \xi, t) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} c_{i,n}(t) \xi^{2i} + \text{mod}(n, 2) \xi^{n+1} c_{n+1,n}(t). \quad (2.4)$$

Introducing a function $D(x/\xi, t) \Theta(|\xi| - |x|)$ labelled the D -term such that:

$$\int_{-|\xi|}^{|\xi|} dx x^n D\left(\frac{x}{\xi}, t\right) = \text{mod}(n, 2) \text{sign}(\xi) \xi^{n+1} c_{n+1,n}(t), \quad (2.5)$$

one can directly find the Ludwig-Helgason consistency condition:

$$\int_{-1}^1 dx x^n \left(H(x, \xi, t) - \text{sign}(\xi) \Theta(|\xi| - |x|) D\left(\frac{x}{\xi}, t\right) \right) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} c_{i,n}(t) \xi^{2i}, \quad (2.6)$$

and consequently, $H - D$ is the Radon transform of another object, called the Double Distribution¹ (DD). The latter were originally introduced in [1, 3] as the 2D Fourier transform of non-local matrix element. It is clear here that the polynomiality property and the existence of DDs are equivalent, and contrary to what may have been thought previously, DD are not simply a convenient way to fulfil the polynomiality property.

3. Inverse Radon Transform

The Radon transform admit an inverse transform, which can be used to compute DD from GPDs. However, proceeding naively, one would need to know the GPDs not only for $\xi \in [-1, 1]$ but for $\xi \in \mathbb{R}$, i.e. including the Generalised Distribution Amplitude kinematical sector, *a priori* precluding useful applications of the inverse Radon transform. We showed in [22] that the situation is actually much better. Using the Boman and Todd-Quinto theorem [28], one can prove that the DGLAP region is sufficient to entirely constraint the Double Distributions up to D -term like singularities. This generalises previous results obtain within specific functional forms [29, 30]

This result is crucial, as it allows to develop a new modelling strategy, combining the best of the LFWFs and DDs worlds to fulfil *a priori* both the positivity and polynomiality properties. The strategy consists in computing the DGLAP region from LFWFs, and then computing the DDs from

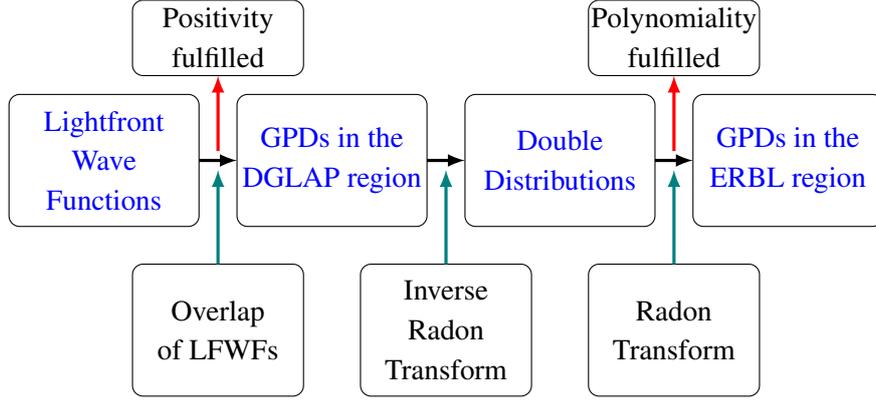


Figure 1: Modelling strategy for GPDs fulfilling all theoretical constraints.

the DGLAP region only. This way LFWFs will ensure positivity while DDs polynomiality. A sketch of the strategy is shown on figure 1.

Our strategy has been tested on pion GPDs. One of the technical difficulties to overcome is the fact that the inverse Radon transform is ill-posed in the sense of Hadamard. That is, numerical noise must be handled with care. We developed an algorithm based on finite-element techniques which has been benchmarked using models whose inversion was known algebraically. An example is given in figure 2 using a Nakanishi-based model [19, 31] for the pion GPD. Possible D -term like singularities can be partially fixed in this case using the so-called soft pion theorem [23].

To conclude, we have presented a new method to model GPDs in such a way that all theoretical constraints are a priori guaranteed. The tests performed for the pion demonstrate the feasibility of our approach. The generalisation to the nucleon case is considered, especially since models of nucleon LFWFs are currently being developed (see e.g. [32]).

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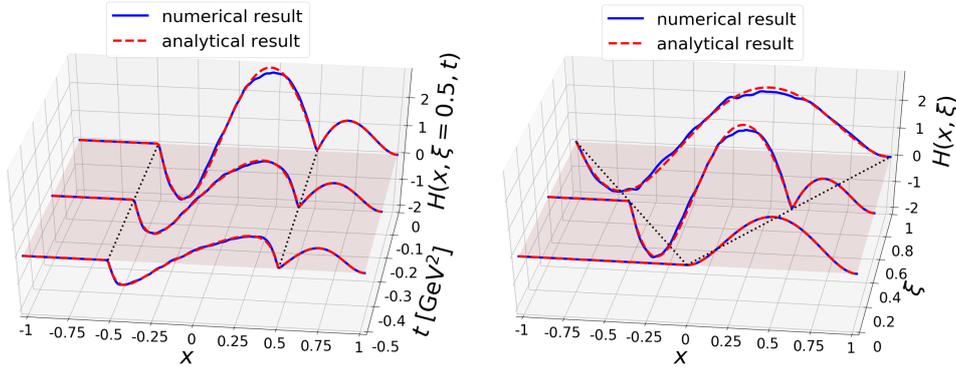


Figure 2: Example of reconstruction algorithm tested on the pion GPD model of [19, 31]. Red curve : algebraic answer to the inversion problem; Blue curve: numerical results.

¹The presentation is done in the Polyakov-Weiss DD scheme, but is valid for all DD scheme [22].

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