PROCEEDINGS OF SCIENCE

The homogeneous Bethe-Salpeter equation for fermion-scalar systems in Minkowski space

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The approach for solving the Bethe-Salpeter equation of bound systems with spin degrees of freedom has been further tested by investigating a fermion-scalar bound state. The main outcome of such a study is given by the extraction of genuinely dynamical information on both the valence probabilities and light-front distributions (longitudinal and transverse ones) and it has been achieved by exploiting the well-known Nakanishi integral representation of the Bethe-Salpeter amplitude. The whole set of obtained results improves the perspective of actually constructing a phenomenological framework where the dynamical features of a bound system can be studied directly in Minkowski space.

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1. Introduction

The present contribution is devoted to illustrate the recent investigation of a bound system composed by a fermion and a scalar, interacting through the exchange of scalar or vector quanta, within a truly relativistic quantum-field-theory (QFT) framework [1]. It has to be emphasized that such a study is part of a collective effort aiming at exploring the potentiality of the approach for solving the Bethe-Salpeter equation (BSE)[2], based on the so-called Nakanishi integral representation of the vertex function, describing in QFT a bound state (see, e.g., for a general introduction Ref. [3]). The field of application ranges from bound systems without spin dof to the ones with such a fundamental feature, starting from simple interaction kernel and free propagators (see, e.g., Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]). The main target of the investigation has been the homogeneous BSE, pictorially shown in Fig. 1, that amounts to an integral equation fulfilled by an unknown amplitude, related to the aforementioned vertex function, once we take into account the propagators of the constituents. The dynamical content of the BSE is given by the interaction kernel, composed by 2PI diagrams. Noteworthy, each term contributing to the kernel generates an infinite set of quanta exchanges, given the mathematical structure of any integral equation. This feature qualifies the BSE as a non perturbative tool for analyzing the dynamics inside a bound state, as well as inside a scattering state, once the inhomogeneous version of BSE is considered (see, e.g., Ref. [14]). In

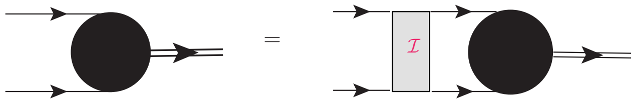

Figure 1: Pictorial representation of the homogeneous Bethe-Salpeter equation for a two-body system. The blob is the unknown Bethe-Salpeter amplitude describing a bound system in QFT, and the interaction kernel is indicated by $\mathscr{I}$, containing, in principle, an infinite set of 2PI diagrams.

view of both forthcoming and in progress experimental efforts for reconstructing the 3D structure of hadronic systems with a very high accuracy, it is quite desirable and useful to develop a fully covariant and non perturbative description of a bound system, with spin dof, in order to study the phenomenology of the spin-$k_\perp$ correlations. It has to be stressed that both longitudinal and transverse momentum distributions do not belong to the spacelike sector of the Minkowski space, and this represents a challenge for the nowadays theoretical approaches.

The overall strategy, we would like to pursue, can be summarized in two main steps: i) first, one has to train and educate his-own physical intuition through *simple* applications of the Bethe-Salpeter equation (BSE) in Minkowski momentum-space, like two-scalar, two-fermion, fermion-scalar, etc. systems, by adopting, e.g., the ladder approximation and disregarding self-energy and vertex corrections; then one should try to extend the framework by taking into account the gap-equations for the needed self-energy contributions, e.g. by including results from lattice [15], or more formally by consistently cutting the tower of Dyson-Schwinger equations.

The Nakanishi Integral Representation (NIR) of both the BS amplitudes (3- and 4-legs transition amplitudes, for two- and three-body systems) and the self-energies (2-legs transition amplitudes) plays a pivotal role in the construction of the non perturbative tool we are discussing. Moreover the light-front (LF) variables, $x^\pm = x^0 \pm x^3$ and $\mathbf{x}_\perp \equiv \{x^1, x^2\}$, are very suitable for managing analytic integrations and spin dof in a very effective way in Minkowski space. Finally, one should mention a methodological advantage of the Bethe-Salpeter framework: one can add dynamical effects in a controlled way, so that the interplay among different dynamical features can be analyzed more strictly.

2. Generalities

The BSE (cf Fig. 1), without both self-energy and vertex corrections, reads for a $J^\pi = [1/2]^+$ state (i.e. the quantum numbers of a nucleon) as follows

$$\Phi^N(k,p,J_z) = G_0(p/2-k)S(p/2+k) \int \frac{d^4k'}{(2\pi)^4}\, i\mathscr{K}^{Ld}(k,k',p)\, \Phi^N(k',p,J_z)\ , \tag{2.1}$$

with

$$G_0(q) \ = i\, \frac{1}{(q^2-m_S^2+i\varepsilon)}\ , \qquad S(q) \ = i\, \frac{\not{q}+m_F}{(q^2-m_F^2+i\varepsilon)} \tag{2.2}$$

where $m_{F(S)}$ is the mass of the constituent fermion (scalar boson).

In our approach, the ladder approximation for scalar and vector exchanges is adopted, i.e.

$$i\mathscr{K}_s^{Ld}(k,k',p) = -i\, \frac{\lambda_S^s\lambda_F^s}{(k-k')^2-\mu^2+i\varepsilon}\ , \quad i\mathscr{K}_v^{Ld}(k,k',p) = -i\, \frac{\lambda_S^v\lambda_F^v\ (\not{p}-\not{k}-\not{k}')}{(k-k')^2-\mu^2+i\varepsilon}\ , \tag{2.3}$$

with μ the mass of the exchanged boson, and $\lambda_S^{s(v)}\ \lambda_F^{s(v)}$ the product of coupling constants at the interaction vertexes for a scalar (vector) exchange.

From general principles, the BS amplitude of the $[1/2]^+$ fermion-scalar state contains two unknown scalar functions ϕ_i, without exchange symmetry, viz

$$\Phi_{BS}(k,p) = \Big[O_1(k)\ \phi_1(k,p) + O_2(k)\ \phi_2(k,p)\Big]\ U(p,s) \tag{2.4}$$

where

$$O_1(k) = \mathbb{I}\ , \qquad O_2(k) = \frac{\not{k}}{M}\ , \qquad (\not{p}-M)\ U(p,s) = 0\ . \tag{2.5}$$

In the above definitions, the mass of the bound system is $M = 2\bar{m} - B$, with $\bar{m} = (m_F + m_S)/2$ and B the binding energy.

By using the proper Dirac traces, one easily obtains the 2-channel system of integral equations for $\phi_i^{s(v)}$ (the superscripts recall the type of interactions), viz

$$\phi_i^{s(v)}(k,p) = \frac{i}{(p/2-k)^2-m_S^2+i\varepsilon}\ \frac{i}{(p/2+k)^2-m_F^2+i\varepsilon} \int \frac{d^4k'}{(2\pi)^4}$$
$$\times\ \frac{(-i\lambda_S^{s(v)}\lambda_F^{s(v)})}{(k-k')^2-\mu^2+i\varepsilon}\ \sum_{j=1,2} \mathscr{C}_{ij}^{s(v)}(k,k',p)\ \phi_j^{s(v)}(k',p)\ , \tag{2.6}$$

where $\mathscr{C}_{ij}^{s(v)}(k,k',p)$ can be analytically evaluated [1]. The final step is given by the introduction of the NIR of $\phi_i^{s(v)}(k,p)$, that reads as follows

$$\phi_i(k,p) = \int_{-\infty}^{\infty} d\gamma' \int_{-1}^{1} dz' \frac{g_i(\gamma',z';\kappa^2)}{[k^2+z'p\cdot k-\kappa^2-\gamma'+i\varepsilon]^3}\ , \tag{2.7}$$

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with $g_i(\gamma, z; \kappa^2)$ the so-called Nakanishi weight functions (NWFs), and $\kappa^2 = \bar{m}^2 - M^2/4$. The numerical task is represented by the calculation of such NWFs, from which the whole BS amplitude can be reconstructed. In order to proceed in the numerical evaluation, one expands the NWFs on a basis given by the Cartesian product: *Laguerre Polynomials* $\otimes$ *Gegenbauer Polynomials*. Hence, the familiar (in the NIR approach!) generalized eigenvalue problem can be obtained, i.e.

$$A\,\vec{v} = \alpha\, B\,\vec{v}$$

where $\vec{v}$ is the eigenvector, whose components are the two sets of coefficients of the aforementioned expansion, and α the corresponding eigenvalue, namely the strength of the interaction, in the adopted ladder approximation. Noteworthy, the validity of the whole approach is checked *a posteriori*, once the generalized eigenvalue problem admits solutions.

3. Results

Interestingly, in the scalar-exchange case, for increasing values of $B/\bar{m}$, the size of the system shrinks, and repulsion starts to sizably oppose the binding. This can be heuristically explained by realizing that the fermion-scalar vertex, for on mass-shell fermions, contains the scalar density $u^\dagger\ \gamma^0\ u$. The Dirac matrix γ^0 generates a minus sign in front of the contribution produced by the small components, that become more and more relevant when the system behaves more and more relativistically. This feature makes understandable the different size in the values of α needed to

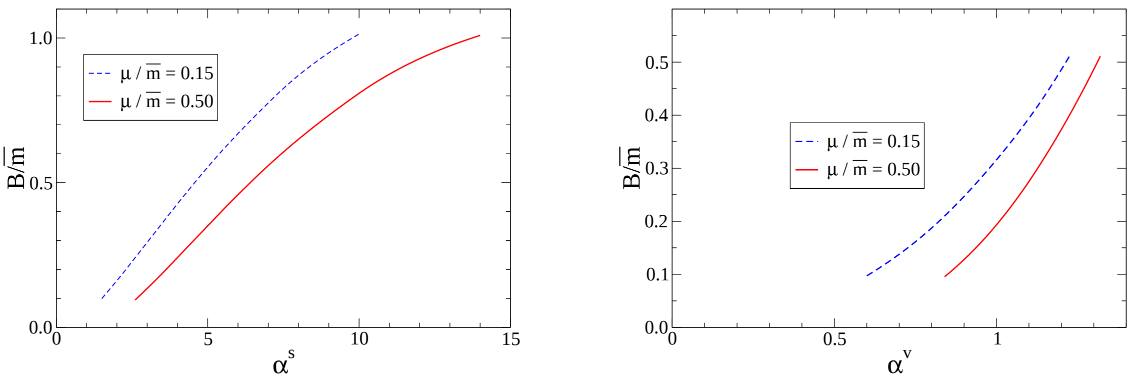

Figure 2: The binding energy, $B/\bar{m}$ (with $\bar{m} = m_F = m_S$), vs the coupling constants for $\mu/\bar{m} = 0.15$ and $\mu/\bar{m} = 0.50$. Left panel: calculations for a scalar exchange. Right panel: calculations for a vector exchange.

bind the system, in the scalar and vector exchanges, as illustrated in Fig. 2. Another important difference between the two plots is the maximal values of the binding energy per unit mass, $B/\bar{m}$. This is related to the very nature of the quanta exchange and the scale invariant properties of the interaction Lagrangian [1]. A typical and distinctive outcome of a dynamical investigation carried out directly in Minkowski space, is given by the possibility of evaluating the valence probability P_{val} i.e. the probability to find the Fock state with the lowest number of particles in the bound state under scrutiny. The results for the scalar (left panel) and vector (right panel) exchanges are illustrated in Fig. 3. Strikingly, one recognizes that the aforementioned repulsion drives the pattern in the scalar case. In the physical space, one can also evaluate both longitudinal and transverse-momentum LF distributions, shown in Fig. 4, for a fermion in the valence component of a *mock nucleon* with $B/\bar{m} = 0.1$, $m_F = 2m_S$ and a vector exchange. The comparison between the full calculation and the contribution from a spin configuration where the constituent spin is parallel to the one of the bound state, confirms the expectation of the leading role of the s-wave in the $[1/2]^+$ state, we are studying. The interested reader can find more details in Ref. [1].

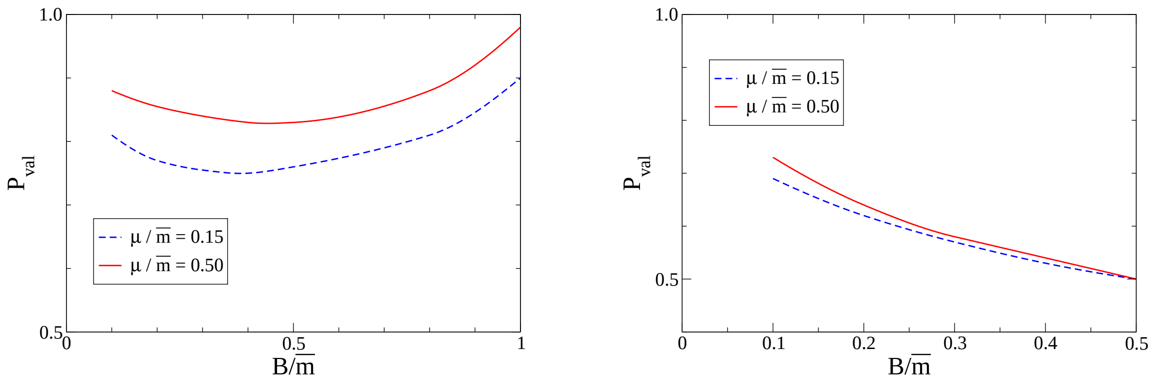

Figure 3: The valence probability vs the binding energy $B/\bar{m}$ $(\bar{m} = m_F = m_S)$, for $\mu/\bar{m} = 0.15$ and $\mu/\bar{m} = 0.50$. Left panel: calculations for a scalar exchange. Right panel: calculations for a vector exchange.

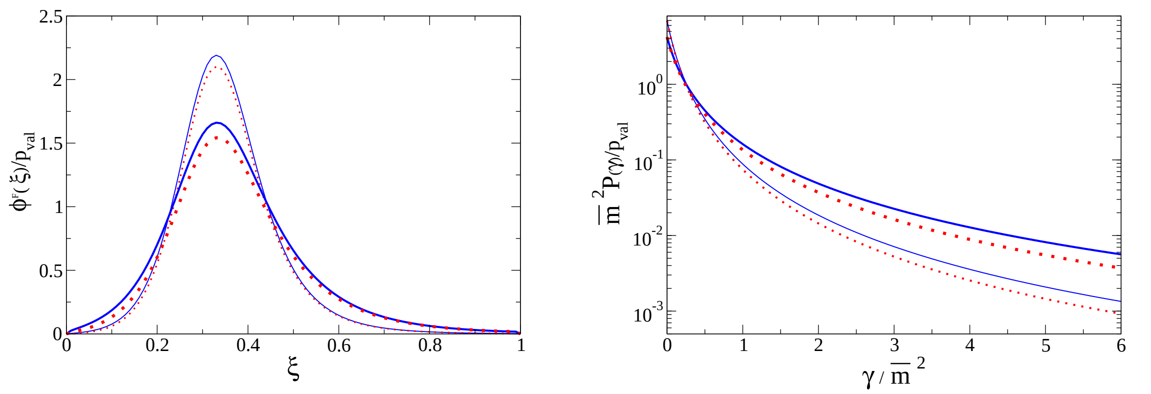

Figure 4: Light-front distributions for a fermion in the valence component of $[1/2]^+$ state, with $B/\bar{m} = 0.1$, $m_F = 2m_S$, and a vector exchange. Thin lines: $\mu/\bar{m} = 0.15$. Thick lines: $\mu/\bar{m} = 0.50$. Solid lines: full calculations. Dotted lines: contribution from the configuration with the constituent spin parallel to the spin of the bound state. Right panel: longitudinal distributions. Left panel: transverse-momentum distributions.

In conclusion, looking at the whole amount of encouraging results obtained by adopting NIR, one can clearly realize that an effort aimed at extending the approach by including the self-energies of the constituents, both exploiting the lattice data and investigating the gap-equation in a more formal way, is definitely worthwhile.

Acknowledgments

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