PROCEEDINGS OF SCIENCE

Tetraquark Properties at Large N_c

Wolfgang Lucha
Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria
E-mail: Wolfgang.Lucha@oeaw.ac.at

Dmitri Melikhov
Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria, and
D. V. Skobeltsyn Institute of Nuclear Physics, M. V. Lomonosov Moscow State University, 119991, Moscow, Russia, and
Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria
E-mail: dmitri_melikhov@gmx.de

Hagop Sazdjian*
Institut de Physique Nucléaire, Université Paris-Sud, CNRS-IN2P3, Université Paris-Saclay, 91405 Orsay Cedex, France
E-mail: sazdjian@ipno.in2p3.fr

Considering quantum chromodynamics in the consistent limit of simultaneous *correlated* increase of the number of colours, N_c, beyond bounds and decrease of the strong coupling to zero allows for solid statements about qualitative features of the class of "truly exotic" tetraquark mesons carrying four mutually distinct quark flavours. Consistency criteria extracted from correlation functions for two-ordinary-meson scattering suggest the existence of more than one such tetraquarks, at least, of two tetraquarks of identical quark-flavour content and large-N_c behaviour of the total decay widths but differing in, and hence discriminable by, their predominant decay modes into two conventional mesons. This pairwise appearance is in conflict with the unique variant of such a four-quark bound state arising from the binding of *diquark* and *antidiquark* to a tetraquark by the strong interactions.

Light Cone 2019 — QCD on the light cone: from hadrons to heavy ions — LC2019
16–20 September 2019
École Polytechnique, Palaiseau, France

*Speaker.

PoS(LC2019)078

1. Approach to Non-Conventional Hadrons: Multiquark Hadrons in Large-N_c QCD

Quantum chromodynamics, the quantum field theory of the strong interactions, does not forbid the formation of hadrons (colour-singlet bound states of quarks and gluons) other than conventional quark–antiquark mesons $(\overline{q}q)$ and three-quark baryons (qqq), *viz.*, of exotic *multiquark* states such as tetraquark $(\overline{q}\overline{q}qq)$, pentaquark $(\overline{q}qqqq)$, and hexaquark $(qqqqqq)$ or $(\overline{q}\overline{q}\overline{q}qqq)$ hadron states. The prospects for such exotics have been studied since long [1]. We focus to the case of tetraquarks.

The framework of our analysis is a generalization of quantum chromodynamics (QCD) dubbed large-N_c QCD [2]: a gauge theory relying on the gauge group $SU(N_c)$, with all quarks transforming, *by assumption*, according to the N_c-dimensional, fundamental representation of $SU(N_c)$, considered in the limit $N_c \to \infty$, with the strong coupling parameter, g_s, behaving like $g_s \propto 1/N_c^{1/2}$. In that limit, QCD catches the main properties of confinement, while getting simplified with respect to secondary complications, *e.g.*, inelasticity or screening effects. $1/N_c$ plays the rôle of a perturbative parameter. The theory's predictions are easily deduced [3]: the spectrum is saturated by an infinite tower of free stable mesons with masses of order $O(N_c^0)$, three-meson interactions behave like $N_c^{-1/2}$, four-meson interactions like N_c^{-1}, and meson decay rates like N_c^{-1}. So, in the limit $N_c \to \infty$ all mesons are stable, which is one of the main features of confinement. Can we derive similar predictions for tetraquarks?

At large N_c, adopting the two-point correlators of colour-singlet tetraquark and ordinary-meson currents, the propagation of a tetraquark becomes equivalent to that of two free ordinary mesons [4]:

$$T(x) \equiv (\overline{q}\overline{q}qq)(x)\ , \quad j(x) \equiv (\overline{q}q)(x) \qquad \Longrightarrow \qquad \langle T(x)\, T^\dagger(0)\rangle \overset{N_c\to\infty}{=} \sum \langle j(x)\, j^\dagger(0)\rangle\, \langle j(x)\, j^\dagger(0)\rangle\ .$$

Tetraquark poles thus cannot appear at leading order but may pop up at subleading orders and might be observable if their widths turn out to be narrow; the latter are expected to fall off like N_c^{-2} [5–10].

2. Tetraquarks of Flavour-Exotic Quark–Antiquark Composition: *Line of Approach*

We are particularly interested in flavour-exotic tetraquarks: bound states of two quarks and two antiquarks involving four different quark flavours, here generically denoted $1,2,3,4 \in \{u,d,s,c,b\}$.

We extract basic features of tetraquarks from their appearance as poles in the amplitudes for the scattering of two ordinary mesons into two ordinary mesons by an investigation [9–11] of four-point Green functions of colour-singlet quark-bilinear currents, serving as meson interpolating operators. Identifying spin and parity as irrelevant for *qualitative* analyses, such a current j_{ab} generically reads

$$j_{ab} \equiv \overline{q}_a\, q_b\ , \qquad a,b = 1,2,3,4\ ,$$

and couples to an ordinary meson $M_{ab} = (\overline{q}_a\, q_b)$ with strength $f_{M_{ab}}$ of known large-N_c behaviour [3]:

$$\langle 0|j_{ab}|M_{ab}\rangle = f_{M_{ab}}\ , \qquad f_{M_{ab}} \propto N_c^{1/2}\ .$$

Consistency requires us to inspect all channels with potential tetraquark poles, by use of the currents

$$j_{12} \equiv \overline{q}_1\, q_2\ , \qquad j_{34} \equiv \overline{q}_3\, q_4\ , \qquad j_{14} \equiv \overline{q}_1\, q_4\ , \qquad j_{32} \equiv \overline{q}_3\, q_2\ .$$

In order to ascertain that a given QCD diagram may contain a tetraquark contribution in form of a pole term, we have to ensure that it receives a four-quark contribution to its s-channel singularities, in addition to gluon singularities that do not modify that diagram's N_c behaviour. If this tetraquark is composed of two quarks and two antiquarks with masses m_j, $j = 1,2,3,4$, each diagram in question should develop, as a function of the Mandelstam variable s, a four-particle branch cut, starting at the branch point $s = (m_1+m_2+m_3+m_4)^2$ [9,10]. The latter's existence can be easily checked by use of the Landau equations [12]. Diagrams that exhibit no s-channel singularity at all or only two-particle (quark–antiquark) singularities cannot contribute to the formation of tetraquarks at N_c-leading order and thus should not be taken into account for the N_c-behaviour analysis of any tetraquark properties. With regard to the allocation of the four unequal quark flavours in initial- and final-state mesons, we encounter two types of scattering reactions, *viz.*, "direct" processes, where the flavour distribution is preserved, and "recombination" processes, where the flavour distribution undergoes rearrangement:

$$\begin{aligned} &M_{12}+M_{34} \longrightarrow M_{12}+M_{34} && \text{(direct channel I)}\ , \\ &M_{14}+M_{32} \longrightarrow M_{14}+M_{32} && \text{(direct channel II)}\ , \\ &M_{12}+M_{34} \longrightarrow M_{14}+M_{32} && \text{(recombination channel)}\ . \end{aligned}$$

For both of the "direct" reactions, we scrutinize [9–11] the two four-current Green functions (Fig. 1)

$$\Gamma_{\rm I}^{\rm (dir)} \equiv \langle T(j_{12}\, j_{34}\, j_{12}^\dagger\, j_{34}^\dagger)\rangle\ , \qquad \Gamma_{\rm II}^{\rm (dir)} \equiv \langle T(j_{14}\, j_{32}\, j_{14}^\dagger\, j_{32}^\dagger)\rangle\ .$$

Only Feynman diagrams of the categories represented by Fig. 1(b) may receive contributions from a tetraquark; the large-N_c behaviour of the two sets *potentially* supporting tetraquarks (T) is the same:

$$\Gamma_{\rm I,T}^{\rm (dir)} = O(N_c^0)\ , \qquad \Gamma_{\rm II,T}^{\rm (dir)} = O(N_c^0)\ .$$

Likewise, for the "recombination" reaction we study [9–11] the four-current Green function (Fig. 2)

$$\Gamma^{\rm (recomb)} \equiv \langle T(j_{12}\, j_{34}\, j_{14}^\dagger\, j_{32}^\dagger)\rangle\ .$$

Only Feynman diagrams of the type of Fig. 2(c) may have tetraquark poles, behaving at large N_c like

$$\Gamma_{\rm T}^{\rm (recomb)} = O(N_c^{-1})\ .$$

Since the large-N_c behaviours of the pole contribution to direct and recombination correlators differ,

$$\Gamma_{\rm I,T}^{\rm (dir)} = O(N_c^0)\ , \qquad \Gamma_{\rm II,T}^{\rm (dir)} = O(N_c^0)\ , \qquad \Gamma_{\rm T}^{\rm (recomb)} = O(N_c^{-1})\ ,$$

reproducing [9,10] these large-N_c findings (Fig. 3) requires (at least) two tetraquarks, say, T_A and T_B, with different couplings to the ordinary-meson pairs and related transition amplitudes behaving like

$$\begin{aligned} &A(T_A \longleftrightarrow M_{12}\,M_{34}) = O(N_c^{-1})\ , \qquad && A(T_A \longleftrightarrow M_{14}\,M_{32}) = O(N_c^{-2})\ , \\ &A(T_B \longleftrightarrow M_{12}\,M_{34}) = O(N_c^{-2})\ , \qquad && A(T_B \longleftrightarrow M_{14}\,M_{32}) = O(N_c^{-1})\ . \end{aligned}$$

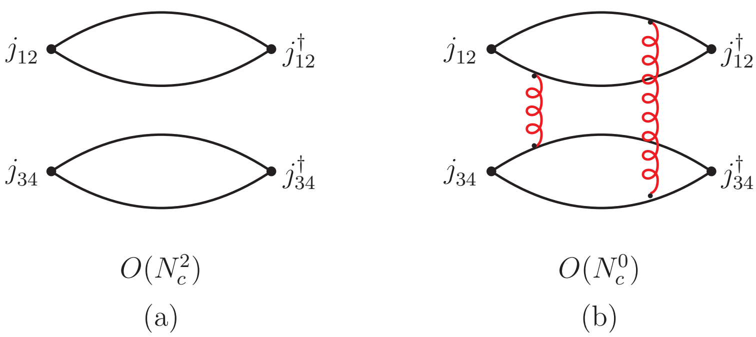

Figure 1: Examples of the (a) leading and (b) subleading Feynman diagrams for *direct* Green functions $\Gamma_{\rm I}^{\rm (dir)}$ [10, Fig. 1]; similar Feynman diagrams exist for *direct* Green functions $\Gamma_{\rm II}^{\rm (dir)}$. Curly red lines indicate gluons.

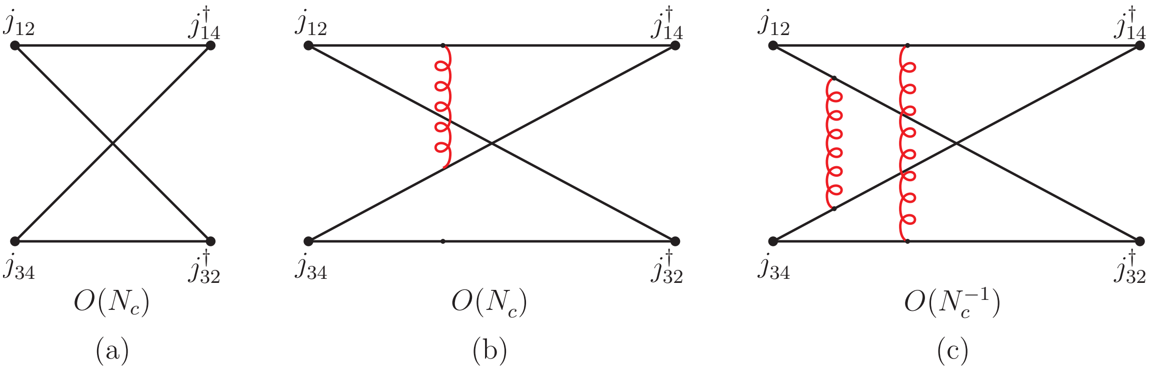

Figure 2: Typical representatives of the (a) leading or (b,c) subleading Feynman diagrams for *recombination* Green functions $\Gamma^{(\text{recomb})}$ [10, Fig. 2(a,b,e)]; as in Fig. 1, curly red lines indicate internal exchanges of gluons.

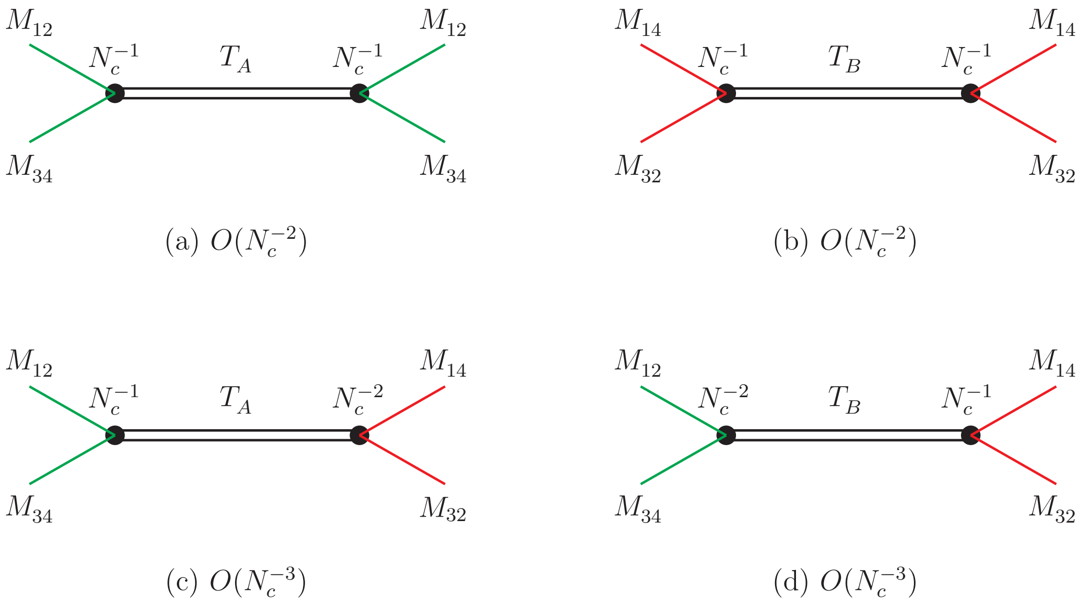

Figure 3: Meson–meson scattering amplitudes: N_c-leading contributions at two tetraquark poles [10, Fig. 7].

From the N_c-leading tetraquark–two-ordinary-meson transition amplitudes, the decay widths of our pair of tetraquarks, T_A and T_B, exhibit the same large-N_c behaviour and are definitely narrow [9–11]:

$$\Gamma(T_A) = O(N_c^{-2}) , \qquad \Gamma(T_B) = O(N_c^{-2}) .$$

The above outcome and the colour structure of the intermediate states in the Feynman diagrams allow us to offer an educated guess on the *flavour structure* of each of the tetraquarks T_A and T_B [10]. Specifically, colour exchange between the quarks characterizes the intermediate states. This hints at

$$T_A \sim (\bar{q}_1\, q_4)\,(\bar{q}_3\, q_2) , \qquad T_B \sim (\bar{q}_1\, q_2)\,(\bar{q}_3\, q_4) .$$

3. Implications and Conclusions: Doubtful Existence of Flavour-Exotic Tetraquarks

These insights tend to favour a singlet–singlet colour structure (Fig. 4) of the two flavour-exotic tetraquark companions, perhaps with mixings of order $O(1/N_c)$ of the two configurations [10]. The

Figure 4: Formation of tetraquarks: two-ordinary-meson (left) vs. diquark–antidiquark (right) configuration.

mutual interactions of the two colour-singlet clusters are no longer confining: the four-quark system is no longer compact but resembles a *molecular* state. Compact multiquarks result from the *diquark* mechanism (Fig. 4) [13–15]: Confining interactions bind colour-nonsinglet diquark and antidiquark clusters to compact states; attractiveness compels diquarks to form in the sole colour-antisymmetric representation. That uniqueness is at odds with the N_c-driven need for two unequal tetraquarks [11]. Hidden dynamical mechanisms [16,17] can avoid quark–antiquark over diquark forces' dominance. Molecular-like tetraquarks are tougher: Typical effective meson–meson couplings are of order N_c^{-1}. Thus, bound-state or resonance formation requires to sum diagrams of different N_c behaviours [18]

Acknowledgements. D. M. is supported by the Austrian Science Fund (FWF), Project No. P29028, H. S. by EU research and innovation programme Horizon 2020, grant agreement No. 824093, and D. M. and H. S. by joint CNRS/RFBR Grant No. PRC Russia/19-52-15022.

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