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The massive gluon and the massless pion

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Lattice simulations of Yang-Mills theories and QCD in the Landau gauge demonstrate that the gluon propagator reaches a finite nonzero value at vanishing momentum. This can be modelled by a simple deformation of the corresponding Faddeev-Popov Lagrangian known as the Curci-Ferrari model. The latter does not modify the known ultraviolet regime of the theory and provides a successful perturbative description of essential aspects of the non-Abelian dynamics in the infrared regime, where, in particular, the coupling remains finite, as also seen in lattice simulations. This opens the possibility of a controlled (semi)perturbative description of various aspects of the infrared QCD dynamics, including correlation functions and the deconfinement phase transition at finite temperature and density. I present recent progress concerning the description of chiral symmetry breaking in this context.

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1. Introduction

Understanding the low-energy dynamics of strong interactions, in particular, confinement and spontaneous chiral symmetry breaking from the QCD Lagrangian remains a challenging task. If the main necessary ingredients to describe the latter have long been identified [1], its description in terms of quarks and gluons degrees of freedom mainly rely on *ad-hoc* modelling of the relevant propagators and vertices, and still lacks an analytical understanding based on a controlled approximation scheme. In a recent work [2], we have proposed such a scheme, exploiting the existence of effective small parameters in (infrared) QCD in the Landau gauge, namely, the inverse number of colors, $1/N_c$, and the pure gauge coupling in the Taylor scheme, g_g. It is well-known that the large-N_c limit correctly capture essential features of the QCD dynamics. Less known—although one of the very important discoveries in the field in recent years—is the fact that the pure gauge coupling in the Landau gauge and in the Taylor scheme remains moderate at all energy scales, as shown by explicit lattice calculations [3]. This suggests that the pure gauge sector of the theory might be under perturbative control. In contrast, lattice results show that the quark-gluon coupling g_q typically becomes large in the infrared and cannot be treated perturbatively. This motivates our proposal in [2] of a double expansion in $1/N_c$ and g_g, keeping all powers of g_q. In particular, this leads, at leading order, to the resummation of the infinite subclasses of rainbow-ladder diagrams for the quark propagator and the quark-antiquark four-point vertex. This provides a first principle justification for these classes of diagrams, which have long been known to correctly capture the main qualitative and quantitative aspects of the phenomenon of spontaneous chiral symmetry breaking [1]. For instance, they predict the dynamical generation of a nonzero mass in the chiral limit and the associated formation of massless quark-antiquark bound states corresponding to the Goldstone modes.

Turning these general statements into controllable calculations, however, requires a modification of the standard perturbation theory based on the Fadeev-Popov (FP) Lagrangian. Indeed, the latter predicts a diverging gauge coupling at a given energy scale (Landau pole), which forbids any calculation in the infrared regime relevant here. It is worth emphasizing that the need to modify the FP gauged-fixed Lagrangian in the infrared is no surprise as the latter is known to be plagued by the Gribov ambiguity problem beyond the ultraviolet, perturbative regime. A guide toward the possible relevant modification of the FP Lagrangian comes from another very important result of gauge-fixed lattice simulations, namely, the fact that the Landau gauge gluon propagator reaches a finite, nonzero value at vanishing momentum, exhibiting the dynamical generation of a screening mass. This suggests that an efficient starting point for a modified perturbation theory in the infrared is to supplement the (Landau gauge) FP Lagrangian with a bare gluon mass term.[1] Such a program has been initiated by Tissier and Wschebor in Ref. [5] and has been shown, indeed, to lead to a well-defined perturbative expansion down to deep infrared momenta. There exist particular (infrared safe) renormalization schemes, where the gluon mass term screens the Landau pole and the running gauge coupling remains finite and, depending on the parameters, moderate, as observed in lattice simulations. The striking result is that one-loop perturbative calculations of the gluon and ghost propagators in the quenched (Yang-Mills) limit compare well with lattice simulations. The

[1]This, in fact, is a particular case of the Curci-Ferrari Lagrangians [4]. Note that adding a mass term to the gauge fixed Lagrangian does not spoil the perturbative renormalizability of the theory.

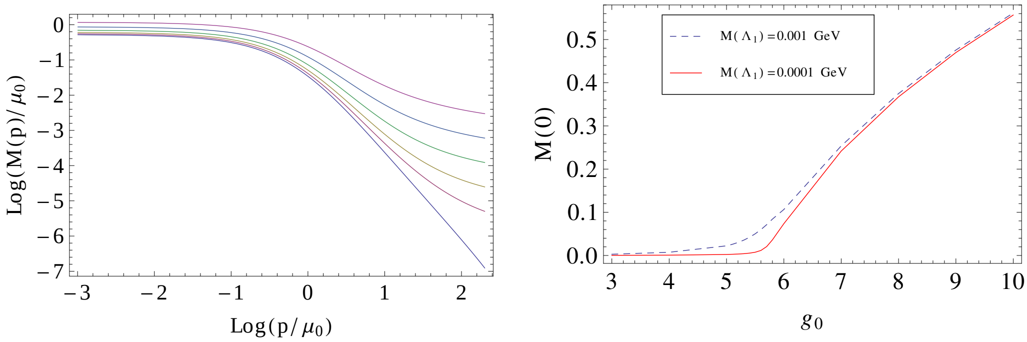

Figure 1: Left: the quark mass function as a function of momentum for decreasing (from top to bottom) values of the tree-level quark mass. At the chiral limit, the quark mass functions decays as a power law (lower curve). Right: the quark mass function at vanishing momentum at the chiral point (red line) and slightly away (blue dashed line) as a function of the typical value of the coupling in the infrared. Chiral symmetry breaking occurs above a critical coupling.

calculations have been recently pushed to two-loop order [6], and the good agreement becomes excellent, a highly nontrivial result! This modified perturbative approach has also been successfully applied to the calculation of propagators and three-point vertices in the vacuum, both in Yang- Mills theories and in QCD with heavy quarks [7], and generalized at nonzero temperature and chemical potential [8], where, for instance, it correctly describes—both qualitatively and quantitatively—the confinement-deconfinement transition for sufficiently heavy quarks.

What about realistic QCD with light quarks? Remarkably, the one-loop perturbative calculations mentioned above show in a self-consistent way that, unlike the pure gauge coupling g_g, the typical quark-gluon coupling g_q that one can extract from the quark-gluon vertex does not remain small enough for a controllable perturbative expansion at infrared momenta, in accordance with lattice results. It is thus consistent, within the massive model itself, to perform an expansion in g_g but not in g_q. As described above, supplementing this with a $1/N_c$ expansion leads, at leading order, to the rainbow-ladder resummation, now with definite expressions for the relevant gluon propagator and quark-gluon vertex, with no *ad-hoc* modelling. Moreover, this allows for a systematic treatment of ultraviolet divergences and of large logarithms through standard renormalization group (RG) techniques. The RG running turns out to be essential for a proper description of the ultraviolet tails of the nonperturbative propagators entering the rainbow-ladder integral equations and, in turn, for a proper description of dynamical chiral symmetry breaking [1, 2].

In the article [2], we have developed the above-mentioned ideas and the corresponding formalism, and we have implemented a toy model for the running parameters, in particular, for the quark-gluon coupling g_q, which is the major player here. The model simply consists in a RG running which reproduces the known ultraviolet behavior at one-loop order and freezes at infrared momenta (the details of the freezing turned out to be of little importance). We refer the reader to the article for details and simply quote the main results here. The left plot of Fig. 1 shows the typical behavior of the quark mass function for various tree-level quark masses. One observes the passage from a so-called massive solution, where the quark mass is described by the standard logarithmic running at large momenta with the appropriate anomalous dimension, and the chiral

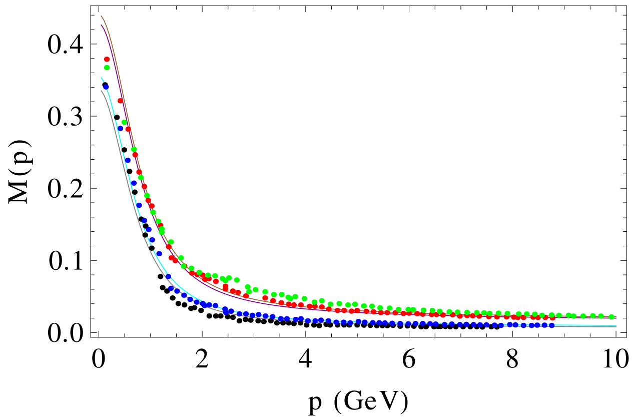

Figure 2: Comparision with lattice data from [9] for $M(p)$ for $M(\mu) = 0.008, 0.01, 0.02, 0.022$ GeV, with $\mu = 10$ GeV. See [2] for details.

symmetry breaking solution, where the quark mass function decays as a power law, up to logarithmic corrections [1]. The right plot of Fig. 1 shows the well-known fact that chiral symmetry breaking—measured here by the nonzero value $M(0)$ of the quark mass function at vanishing momentum in the chiral limit—requires a nonzero value of the quark-gluon coupling—defined here as the value g_0 at saturation in our toy model. Finally, we compare the results of our calculation at leading-order in our expansion scheme with lattice data in Fig. 2. The agreement is remarkable!

In a more refined version of the calculation mentioned here, we avoid the toy model for the RG runnings and compute the latter directly from the model, at the relevant order of approximation in our expansion scheme. As usual, the calculation of the appropriate beta functions requires one to evaluate various relevant couplings at next-to-leading order. This can be done in a systematic way and will be presented in a forthcoming publication. The results are very similar to those mentioned here. The main point of such a calculation is that there are no other parameters than those in the original Lagrangian, that is, in the chiral limit, the coupling[2] and the gluon mass parameter.

With such a systematic entry to the dynamics of chiral symmetry breaking, the obvious thing that comes to mind is to compute measurable physical quantities. The simplest ones are the masses and decay constant of light mesons. In a recent work, we have computed the on-shell quark-antiquark-pion vertex in the chiral limit, from which one can extract the pion decay constant. The value of the latter at vanishing pion mass can be inferred from the physical value by means of chiral perturbation theory at two-loop order: $f_\pi \approx 86$ MeV. Taking the chiral limit for this first calculation allowed us to make drastic simplifications of the relevant rainbow-ladder equations, which reduce to a linear system of one-dimensional integral equations for the four form factors of the vertex under consideration. Such simplifications make very transparent the implementation of the RG running. Again, one can perform a systematic calculation of f_π with no other parameters but those of the Curci-Ferrari Lagrangian. Figure 3 shows our preliminary results for a contour plot of f_π in this two-parameter space.

[2] Although the couplings in the pure gauge and in the quark sectors differ in the infrared, they are related by Slavnov-Taylor identities in the ultraviolet and are thus not independent of one another.

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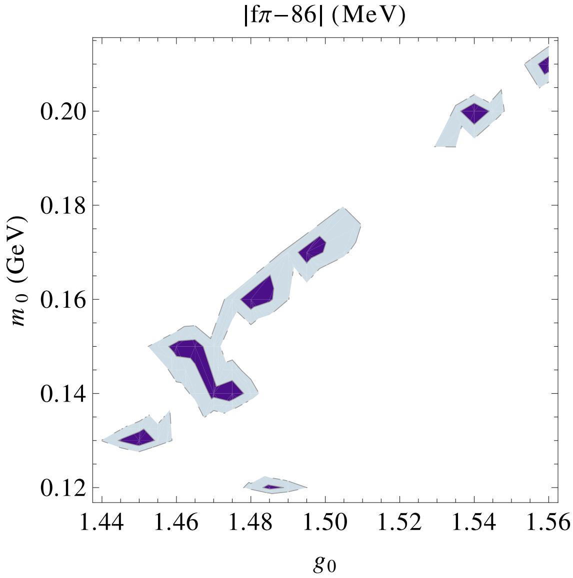

Figure 3: Relative distance of f_π computed at leading order in the present expansion scheme to its value in the chiral limit as a function of the parameters $m_0 = m(\mu)$ and $g_0 = g(\mu)$, with $\mu = 10$ GeV. Dark blue regions are 5% contours and grey ones are 10% contours. Preliminary results.

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