Top quark mass effects in HJ and HH production

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We present predictions for $\mathrm{H}+jet$ and Higgs boson pair production in gluon fusion at NLO QCD. The virtual amplitudes are obtained via numerical integration of the two-loop diagrams, retaining the full dependence on the top-quark mass. After a short discussion of the methods used to obtain the results, we present various phenomenological applications.

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1. Introduction

Despite the tremendous progress in the calculation of multi-loop integrals in the recent years, obtaining analytical expressions for the two-loop amplitudes of two-to-two processes remains challenging if massive particles appear in the loop. A viable alternative to the fully analytical approach is the numerical evaluation of the required loop integrals. In this contribution, we present results based on Refs. [1, 2] and Ref. [3], where the next-to-leading order (NLO) QCD predictions for HH and $\mathrm{H}+jet$ production in gluon fusion have been calculated, respectively. For these processes, the commonly used heavy-top limit (HTL) breaks down for large invariant masses of the final-state system and, therefore, keeping the virtual top quarks in the loops is required to obtain accurate predictions.

Our predictions have been obtained using the program SECDEC [4, 5] to bring the loop integrals into a form which is well suited for numerical integration. An independent calculation of HH production at NLO has been presented in Ref. [6], employing a different strategy for the numerical integration of the two-loop integrals. Furthermore, for both processes, various approximated NLO results are available, which are based on expansions in either m_t, $1/m_t$ or p_T, or on Padé approximants combined with the known threshold logarithms. Furthermore NNLO corrections in the HTL as well as with approximated top-mass effects are available. We refer to the references in Refs. [1–3] and the presentations in Refs. [7, 8] for more details. Very recently, also analytical results for the two-loop integrals appearing in HJ production have been obtained [9–11].

In the following, we first summarize the methods used in Refs. [1–3] to obtain the NLO corrections for $\mathrm{H}+jet$ and HH production. We then present phenomenological results for both processes and we discuss further studies, which are based on these publications.

2. Numerical computation of the NLO corrections

The most difficult part of the calculations are the virtual corrections, since they consist of two-loop four-point diagrams, which depend on four mass scales: the two masses m_t and m_h, as well as the Mandelstam invariants s and t. We write the amplitudes in terms of a form-factor decomposition and we use the program REDUZE [12] to express the contributing loop integrals as linear combinations of master integrals, choosing finite integrals as preferred masters [13]. The reduction step is quite challenging for the processes at hand and we therefore modified the program REDUZE such that it tries to identify equations, which are relevant for the reduction of the integrals appearing in the amplitude, and solve these equations first. With these modifications, we obtained the full reduction for HJ production, first with the mass ratio $m_h^2/m_t^2 = 12/23$ fixed to simplify the reduction, and later with full dependence on all four mass parameters. While we obtained the reduction for all integrals appearing in HJ production, we did not manage to obtain it for the non-planar double-box integrals appearing in HH production. All integrals, including the unreduced non-planar integrals in HH production of up to rank 4, are calculated with the program SECDEC. For the numerical integration of the loop integrals, we employ a Quasi Monte Carlo algorithm [14–16], with which the integration error scales as $1/n$ or better with the number of sampling points n, since the integrand functions generated by SECDEC are sufficiently smooth.

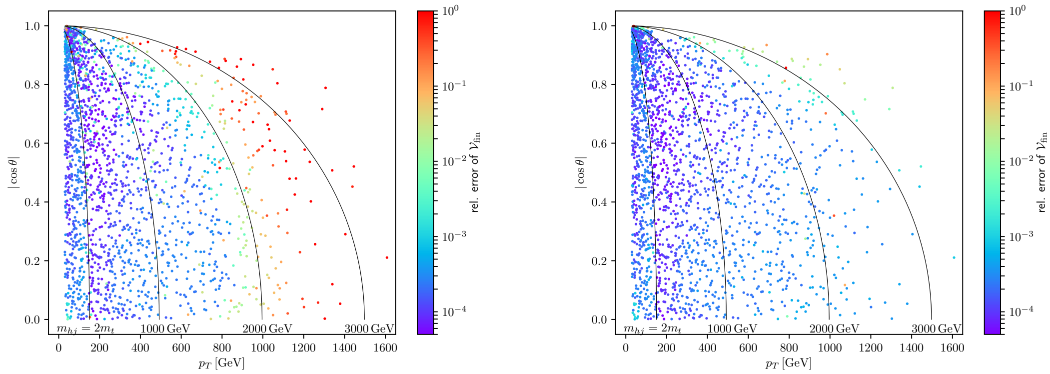

Figure 1: Numerical accuracy of the virtual amplitude results for HJ production. The left plot shows the results used in Ref. [3], whereas the plot on the right hand side shows results after changing the master-integral basis as described in the text. The finite part of the amplitude, $\mathcal{V}_{\rm fin}$, is defined as in Ref. [17] and parton luminosities are included, summing over all subprocesses.

The phase-space integration of the virtual contributions is obtained via reweighing of pre-generated unweighed Born events, such that reliable predictions of phenomenological relevant results can be obtained with a few thousand phase-space points. The individual phase-space points are evaluated on a cluster with Nvidia Tesla K20X GPUs and the accuracy goal for the two-loop form factors has been set to 0.5% (4%) for all (the leading) form factors in HJ (HH) production. The required integration time per phase-space point ranges from $1h$ to $48h$, the run-time constraint of the cluster. For phase-space points with high partonic center-of-mass energy, the numerical integration of the loop integrals is particularly challenging and the desired accuracy of the amplitude can't be reached within the given time constraint. This can be improved by trying to find a master-integral basis with improved convergence of the integrals. Fig. 1 shows the accuracy of the two-loop HJ amplitude for two different sets of master integrals. In the plot on the left-hand side, results obtained in the basis, which was used in Ref. [3], are shown. The plot shows that for most phase-space points a precision at the per-mille level, or better, is reached. However, for large invariant masses $m_{hj} \gtrsim 2\,\mathrm{TeV}$, severe stability problems can be seen. In the plot on the right-hand side, the basis has been changed such that in the Feynman representation of the master integrals, the 2nd Symanzik preferably appears with exponent -1. We also try to avoid poles in $\varepsilon = (4-d)/2$ in the coefficients of the most complicated integrals and we prefer basis choices, where the denominators of the coefficients factor into simple factors. With these improvements, also for most phase-space points in the high invariant-mass region, stable amplitude results can be obtained. Furthermore, the median GPU-time required for evaluating the virtual amplitude reduced from approximately $15h$ to less than $2h$ and the combined file size of the coefficient functions reduced from 360 MB to 100 MB, leading to a significant improvement in the compile time.

The results for the virtual amplitude are stored on disk and we follow two different strategies to combine them with the remaining NLO contributions, where real-radiation matrix elements are generated with the program GoSam [18, 19]: 1) Differential results of the virtual contribution can be directly obtained from the precalculated phase-space points, to which the remaining NLO

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THEORY	LO [pb]	NLO [pb]
HEFT:	$\sigma_{\rm LO} = 8.22^{+3.17}_{-2.15}$	$\sigma_{\rm NLO} = 14.63^{+3.30}_{-2.54}$
$\mathrm{FT}_{\rm approx}$:	$\sigma_{\rm LO} = 8.57^{+3.31}_{-2.24}$	$\sigma_{\rm NLO} = 15.07^{+2.89}_{-2.54}$
Full:	$\sigma_{\rm LO} = 8.57^{+3.31}_{-2.24}$	$\sigma_{\rm NLO} = 16.01^{+1.59}_{-3.73}$

Table 1: Total cross section of $\mathrm{H}+jet$ production at the LHC. Results with full top-mass dependence, as well as the two approximated results HEFT and $\mathrm{FT}_{\rm approx}$ are shown.

contributions can be added at the histogram level. 2) A grid-interpolation framework has been constructed in Ref. [20], which can be used to predict the amplitudes at arbitrary phase-space points. While latter method allows to interface the amplitudes to general Monte Carlo event generators, it also adds an uncertainty stemming from the interpolation, which is small in the majority of the phase-space regions relevant for the LHC, but can be pronounced at very large energies, as will be discussed at the end of section 3.2. For the predictions obtained in Refs. [1–3] we used the more reliable first method and the grid interpolation was only used in subsequent studies.

3. Results

3.1 $\mathrm{H}+jet$ production

We present NLO predictions for $\mathrm{H}+jet$ production at the LHC with a center-of-mass energy of 13 TeV. Jets are defined using the anti-kt algorithm with $R = 0.4$ and $p_{T,j} > 30\,\mathrm{GeV}$. The masses of the Higgs boson and top quark are set to $m_h = 125\,\mathrm{GeV}$ and $m_{\rm t} = m_{\rm h}\sqrt{23/12} \approx 173.055\,\mathrm{GeV}$, respectively, and the PDF4LHC15_nlo [21] parton distributions are used. The top-quark mass is renormalized in the on-shell scheme and we use $\overline{\rm MS}$ renormalization with 5 light quark flavors for the renormalization of the strong coupling, choosing $\mu_0 = (\sqrt{m_{\rm h}^2 + p_{T,\rm h}^2} + \sum_i |p_{t,i}|)/2$ as central value for the renormalization and factorization scale. Scale uncertainties are estimated by the usual 7-point variation.

Predictions for the total cross section of $\mathrm{H}+jet$ production at the LHC are given in table 1. In addition to the result with full top-mass dependence, two approximated results are shown. The HEFT results are obtained in the $m_t \to \infty$ limit. In the $\mathrm{FT}_{\rm approx}$ results, the full top-mass dependence of the LO and real-radiation contributions is included, while the virtual corrections are evaluated in the HEFT and rescaled by the ratio $B_{\rm full}/B_{\rm HEFT}$ of the squared LO matrix elements in the full theory and HEFT. We find that the top-mass effects increase the NLO cross section by 9% (6%) relative to the NLO HEFT ($\mathrm{FT}_{\rm approx}$) result.

The dependence of the cross section on the Higgs boson transverse momentum, $p_{T,\rm h}$, is shown in Figure 2. A comparison of the full theory and HEFT result, given in the left plot, shows significant deviations of the two predictions, which can be attributed to a different scaling behavior of the amplitudes at large $p_{T,\rm h}$ [22, 23]. Despite these large differences, both results lead to K-factors of similar size. However, while the K-factor is nearly constant in the full theory, it slightly decreases in the HEFT al large $p_{T,\rm h}$. The plot on the right-hand side shows a comparison with the $\mathrm{FT}_{\rm approx}$ result, which leads to results similar to the full theory. Taking the full top-mass dependence of

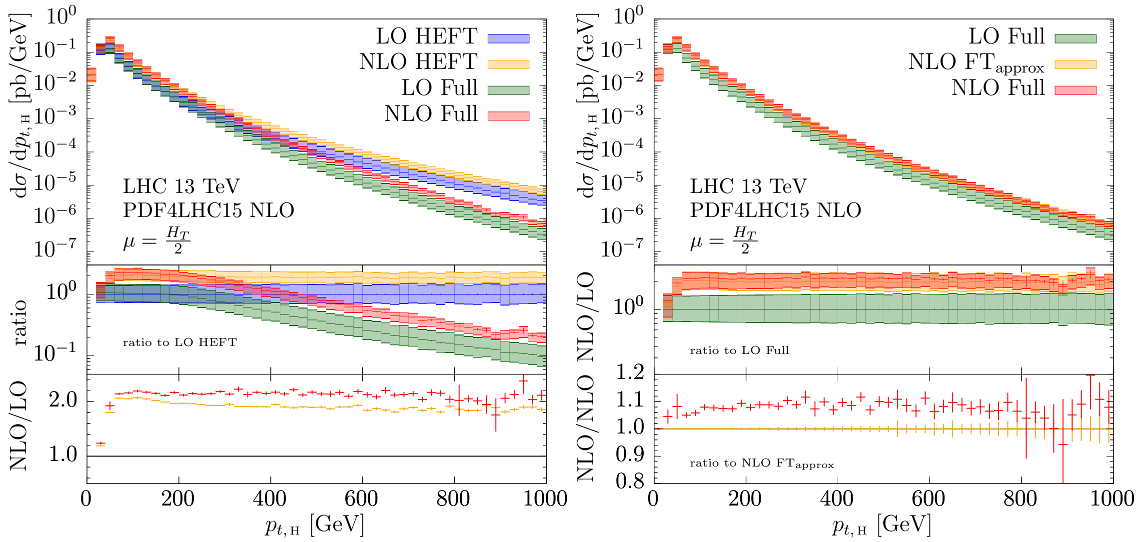

Figure 2: Higgs boson transverse momentum distribution at LO and NLO in QCD. The results with full top-mass dependence are compared to the HEFT (left) and FT_{approx} (right). The small panels show ratios of these results.

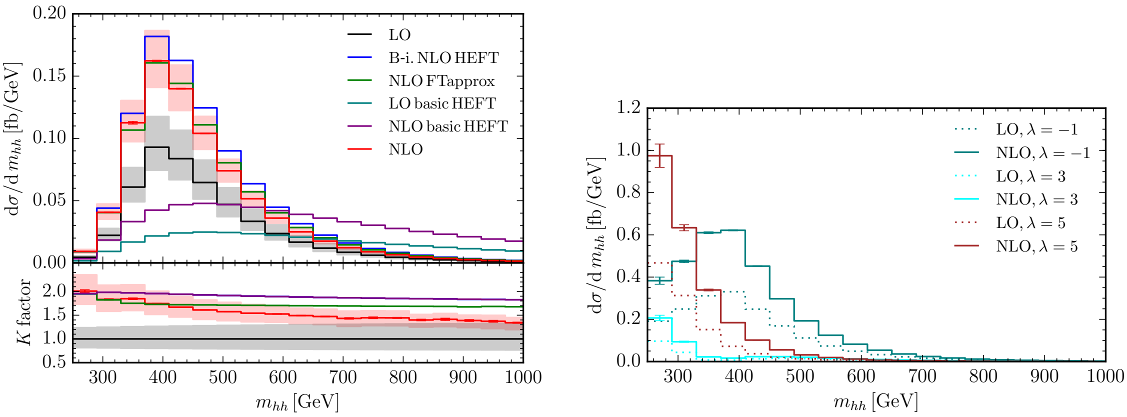

Figure 3: Differential distributions of the Higgs pair invariant mass in the Standard Model (left) and with modified Higgs boson self coupling λ (right).

the virtual contributions into account leads to an increase of about 8% over a large range of the transverse momentum, with slightly smaller corrections at low $p_{T,\mathrm{h}}$.

3.2 HH production

The results for HH production in this section are obtained for the LHC with $\sqrt{s} = 14\,\mathrm{TeV}$. The masses are set to $m_h = 125\,\mathrm{GeV}$, $m_t = 173\,\mathrm{GeV}$ and $\mu_0 = m_{hh}/2$ is used as central value of the renormalization and factorization scale. The remaining parameters are identical to the ones in the previous section.

The NLO QCD predictions for HH production have first been presented in Refs. [1, 2]. The distribution of the invariant mass of the Higgs boson pair, m_{hh}, is shows in Fig. 3. While the top-

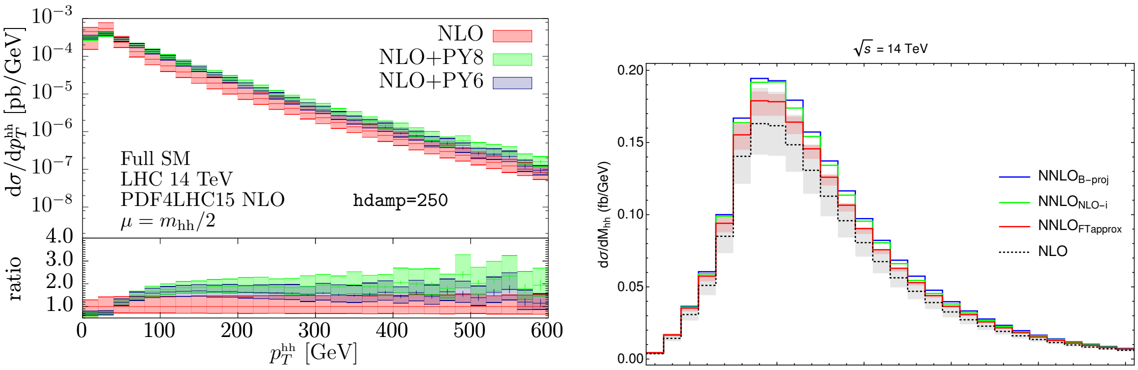

Figure 4: *left:* Effect of a parton shower on the transverse momentum of the di-Higgs system. *right:* Invariant mass distribution of the Higgs boson pair, using various approximations of the top-mass effects at NNLO.

mass effects are small for low invariant masses of the di-Higgs system, they become relevant for large invariant masses, reaching about 30% at $m_{hh} = 1$ TeV, as can be seen in the left plot, where results in the Standard Model are shown. Results with a modified self coupling λ of the Higgs boson are shown in the plot on the right-hand side. It can be seen that varying the self interaction has a large impact on the shape and normalization of the distribution. This sensitivity is caused by interferences of the contributing box- and triangle-type contributions, where only latter involve the Higgs self-interactions.

The NLO corrections to HH production have been included in various phenomenological studies. In Refs. [20, 24] they have been matched to parton showers. It has been observed that the transverse momentum distribution of the di-Higgs system shows a large dependence on the parton shower and matching procedure used, as can be seen in the left plot of Fig. 4. In Ref. [24] these parton-shower uncertainties have been studied in detail and it was argued that they are caused by the parton-shower splittings overestimating the true real radiation matrix elements. The transverse-momentum resummation for Higgs boson pair production has been calculated in Ref. [25].

In Ref. [26] the NNLO HEFT predictions of Ref. [27] have been combined with the full NLO corrections to approximate the top-quark mass effects at NNLO. Three different approaches to combine the calculations have been studied and the resulting invariant mass distributions are shown in the right plot of Fig. 4. The $\mathrm{NNLO}_{\mathrm{FTapprox}}$ prediction is obtained by evaluating all NNLO contributions in the HEFT and, for each parton multiplicity, rescaling them by the ratio of the corresponding LO amplitudes in the full theory and HEFT. Since this approximation, in particular, contains the exact double-real contribution with full top-mass dependence, we consider this result our best prediction. In this approximation, the NNLO corrections increase the total cross section by 12%. The remaining top-mass uncertainties at NNLO, not considering possible mass-scheme dependences, are expected to be at the 3% level. The NNLO predictions have been further improved by resumming the threshold enhanced contributions in Ref. [28].

The implementation of HH production within the PowhegBox [17, 29, 30] framework has been extended in Ref. [31] to allow for the variation of the Higgs boson self interaction λ and the top-quark Yukawa coupling y_t. Effects of anomalous couplings within the Electroweak Chiral

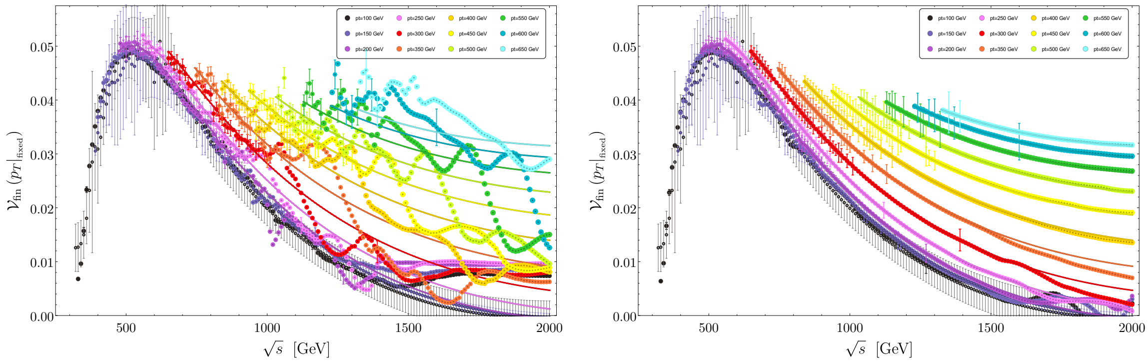

Figure 5: Dependence of the finite part of the two-loop $gg \to HH$ amplitude on $\sqrt{s}$ and p_T. The dots are predictions of the grid-interpolation framework, whereas solid lies show the result of the high-energy expansion. The plot on the left shows results using the original version of the grid, whereas the updated version is shown on the right.

Lagrangian framework have been studied in in Refs. [32, 33].

The grid-interpolation framework [34] for the virtual amplitude of HH production was first constructed in Ref. [20], based on the amplitude results calculated in Refs. [1, 2]. Since the phase-space points in these publications were based on unweighted LO events, they are sparse in the high invariant-mass region, where the cross section is small. Therefore, the grid interpolation has insufficient support in this region and can't predict reliable results. To improve the grid in this phase-space region, in Ref. [35] additional amplitude results based on the high-energy expansion of the amplitudes [36] have been added to the input data of the interpolation. Fig. 5 shows the predictions of the grid interpolation before and after including the additional points from the high energy expansion. The left plot shows that the original version of the grid produced unphysical behaviour in the phase-space region $\sqrt{s} > 1\,\mathrm{TeV}$, but agrees well with the high-energy expansion in the region $600\,\mathrm{GeV} < \sqrt{s} < 1\,\mathrm{TeV}$, where the grid has enough support and the high-energy expansion is still valid. After adding the points of the high-energy expansion to the grid interpolation, the grid produces reliable results for almost all phase-space regions shown in the plot. The updated version of the grid allows to produce reliable results for HH production even in the multi-TeV region, which will become relevant in particular at future hadron colliders.

4. Conclusion

We have presented NLO QCD predictions for H+jet and HH production in gluon fusion, retaining the full dependence on the top-quark mass. Since many of the two-loop integrals appearing in the virtual corrections are not known analytically, we have evaluated all integrals numerically with the program SecDec. For H+jet production we discussed the p_T distribution of the Higgs boson and we showed that the HEFT approximation is not reliable for $p_T \gtrsim m_t$. A similar calculation has been performed for HH production and a grid-interpolation framework for the virtual two-loop corrections has been constructed. This allowed us to include the NLO corrections to HH production in parton-shower programs. A recent improvement of the grid-interpolation framework is the possibility to modify the Higgs boson self interaction. Furthermore, results obtained via a high-energy expansion of the amplitude have been added to the grid to improve its reliability in the high invariant-mass region. The NLO corrections to HH production with full top-mass dependence have been combined with the corresponding NNLO in the HTL to estimate the top-mass effects at NNLO.

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