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Calculating the static gravitational two-body potential to fifth post-Newtonian order with Feynman diagrams

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We discuss the first-time calculation of the static gravitational two-body potential up to fifth post-Newtonian(PN) order. The results are achieved through a manifest factorization property of the odd PN diagrams. The factorization property is illustrated also at first and third PN order.

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1. Introduction

The classic Newton potential of two gravitationally interacting massive bodies receives post-Newtonian(PN) corrections due to effects of general relativity(GR). They can be calculated systematically in perturbation theory in the non-relativistic limit for weak curvature and small velocities. The expansion is performed in virial-related quantities like the relative squared velocity $v^2 \sim G_N \times m/r$ and the compactness $R_S/r \sim G_N m/r$, where G_N is the Newton constant, m is the mass, r the relative distance of the binary components and R_S is the Schwarzschild radius. A given n-th PN order has a manifest power counting in terms of powers ℓ of the Newton constant G_N and powers k of the velocities v squared. The n-th PN order is then given by $n = k + \ell - 1$.

The first PN order is known already since long and was determined by Einstein, Infeld and Hoffmann [1]. It helped in the understanding of phenomena which are observable within our own solar system and which arise due to effects of GR, like for example, it contributes to explain the perihelion precession of mercury.

The direct observation of gravitational waves emitted by a coalescing binary system through the LIGO and Virgo collaborations [2] was a tremendous success and probe of GR. The PN corrections are here important for the construction of wave form templates which are used in the LIGO/Virgo [3, 4] data analysis pipeline [5, 6] for the detection of the gravitational waves. The second and third PN order has been calculated in refs. [7, 8] and in refs. [9, 10, 11], respectively. The fourth PN order was first determined in [12, 13, 14] and confirmed by [15, 16, 17, 18, 19] and [20, 21, 22, 23]. Future observatories, like the Einstein Telescope [24] and LISA [25] are expected to gain at least an order of magnitude in sensitivity with respect to current observatories. As a result of this also an increased precision of the theory description is desirable [26, 27].

In general there are different approaches to solve the gravitational two-body problem, where in the following we will focus solely on the effective field theory (EFT) approach [28, 29, 30, 31, 32, 33]. In the EFT approach the problem of computing PN corrections to the gravitational two-body potential can be traced back to the calculation of Feynman diagrams. In particular the rich methodology commonly used in particle physics for the determination of loop integrals is in this approach directly applicable in order to accomplish such calculations. In the following we focus on the determination of the conservative sector of the gravitational two-body potential in the static limit up to fifth PN order.

2. The effective action

Following the lines of refs. [34, 20], the action S which describes the gravitational interaction can be decomposed into two contributions $S = S_{\text{pp}} + S_{\text{bulk}}$. The world-line point-particle action S_{pp} is representing the two binary components with masses m_1 and m_2. They are considered as spinless point masses. We also neglect tidal effects. The point-particle action reads:

$$S_{\text{pp}} = -\sum_{i=1,2} m_i \int \mathrm{d}\tau_i = -\sum_{i=1,2} m_i \int \sqrt{-g_{\mu\nu}(x_i)\mathrm{d}x_i^{\mu}\mathrm{d}x_i^{\nu}}. \tag{2.1}$$

The bulk action S_{bulk} consists out of the Einstein-Hilbert action plus a gauge fixing term [35, 15]:

$$S_{\text{bulk}} = 2\Lambda^2 \int \mathrm{d}^{d+1}x\sqrt{-g}\left[R(g) - \frac{1}{2}\Gamma^{\mu}\Gamma_{\mu}\right], \tag{2.2}$$

where the harmonic gauge condition has been adopted and Γ^{μ} is given through the Christoffel symbol $\Gamma^{\mu}_{\rho\sigma}$ in the equation $\Gamma^{\mu} = g^{\rho\sigma}\Gamma^{\mu}_{\rho\sigma}$. Furthermore, $\Lambda^{-2} = 32\pi G_N L^{d-3}$, where d is the spatial dimension and L an arbitrary length scale which takes care about the proper mass dimension in dimensional regularization. It vanishes in physical observables in the limit $d \to 3$. For the metric tensor we use the Kaluza-Klein parametrization [36, 37]

$$g_{\mu\nu} = e^{2\phi/\Lambda}\begin{pmatrix} -1 & A_j/\Lambda \\ A_i/\Lambda & e^{-c_d\phi/\Lambda}\gamma_{ij} - A_iA_j/\Lambda^2 \end{pmatrix}, \tag{2.3}$$

where its degrees of freedom are parametrized by three fields: a scalar field ϕ, a vector field A_i and a symmetric tensor field σ_{ij}. The symbols c_d and γ_{ij} are given by $c_d = 2(d-1)/(d-2)$ and $\gamma_{ij} = \delta_{ij} + \sigma_{ij}/\Lambda$, where the indices i, j run over all d spacial dimensions. It turns out that in the static limit the vector fields A_i do not contribute to our calculation. The effective action is obtained by integrating out the remaining gravity fields

$$\exp[\mathrm{i}S_{\mathrm{eff}}] = \int \mathrm{D}\phi\mathrm{D}\sigma_{ij}\exp\left[\mathrm{i}\left(S_{\mathrm{pp}} + S_{\mathrm{bulk}}\right)\right], \tag{2.4}$$

which is perturbatively expanded.

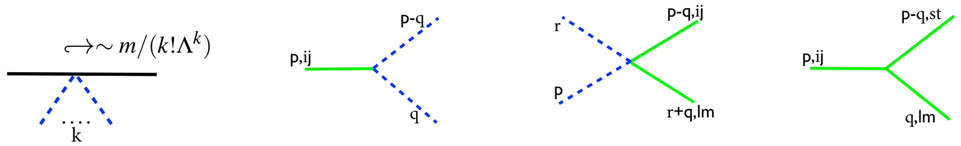


Figure 1: The vertices which contribute to the calculation of the static two-body potential up to fifth PN order are shown. The first diagram shows the matter-ϕ^k vertex, where the solid black lines stand for the non-dynamical external sources. In the three bulk vertices the solid green lines correspond to tensor fields, whereas the blue dashed lines correspond to scalar fields. The external momenta are p, q, r and the indices of the tensor fields are given by i, j, l, m, s, t.

In fig. 1 the four vertices which are required for the calculation of the static contribution to the two-body potential up to 5th PN order are shown. From the point-particle action one obtains the first vertex of fig. 1, while the remaining three originate from the bulk action. The bulk vertices are distinguished by the fact that they contain either zero or two scalar fields. A typical Feynman diagram is then for example given by the tree-level graph which is shown in fig. 2. Its calculation delivers the well known Newton potential.

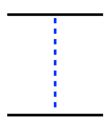


Figure 2: The 0PN diagram is shown.

3. Calculational strategy and factorization property

For the calculation of the static part of the two-body potential up to fifth PN order we consider only the classic contribution of our Feynman diagrams and do not take into account any quantum corrections. In general the original gravity Feynman diagrams depend on the ingoing and outgoing momenta p_1, p_2, p_3 and p_4, like shown in fig. 3, however, it turns out that the corresponding loop integrals are only functions of the momentum transfer $p_3 - p_2 = p = p_1 - p_4$. As a result of this the corresponding loop integrals can be represented by self-energy type diagrams as illustrated in fig. 3, see also ref. [21].

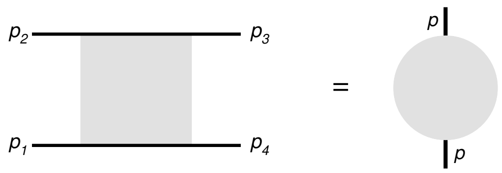
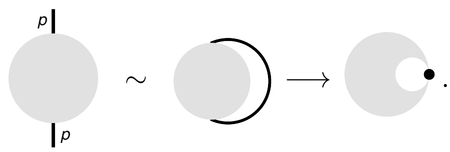


Figure 3: An illustration of the mapping of gravity Feynman diagrams to self-energies as used in ref. [21].

The contribution of a given amplitude to the two-body potential V is obtained by performing the Fourier transform, i.e. going from momentum to coordinate space:

$$V = \mathrm{i} \lim_{d\to 3} \int_p e^{\mathrm{i}p\cdot r} \tag{3.1}$$



In ref. [38] a theorem was shown, which states that

static graphs at odd $(2n+1)$*-PN orders are factorizable*,

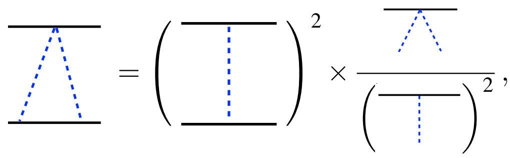
with $n \in \mathbb{N}_0$. A factorizable graph has at least one matter-ϕ^k vertex with $k > 1$ (see. fig. 1), while a prime graph contains only matter-ϕ^k vertices with $k = 1$ [38]. The factorization property allows one to recursively determine a given odd PN order from the known lower PN ones. This strongly simplifies the determination of odd higher PN orders compared to performing a direct calculation of the appearing loop integrals, which becomes increasingly difficult with an increasing number of loops. The factorization becomes apparent diagrammatically when doing the Fourier transform of the amplitude to coordinate space as shown in eq. (3.1). The additional integration over p can be interpreted as an additional loop integration, which can be visualized by joining the external legs of the self-energy into an additional propagator-like line and pinching it to a point [38] as it is illustrated on the r.h.s. of eq. (3.1).

The contribution of a factorizable graph to the potential can then just be obtained by multiplying together the results for the lower PN subgraphs [38]

$$V_n^{\text{factorizable}} = (V_{L,n_1} \times V_{R,n_2}) \times \mathscr{K} \times \mathscr{C}, \tag{3.2}$$

with $n = n_1 + n_2 + 1$, where V_{L,n_1} and V_{R,n_2} is the potential of the left and right subgraph. The factor $\mathscr{K}$ takes into account the new matter-ϕ^k vertex which emerges through glueing the two subgraphs together. The factor $\mathscr{C}$ is a combinatoric factor.

In order to illustrate this method let us consider the well known static odd 1PN and 3PN orders. The 1PN potential can be obtained from a single static diagram. According to the factorization theorem, it can be decomposed in terms of two Newton diagrams



(3.3)

where the individual factors on the *r.h.s.* indeed lead to the known result for the one-loop diagram,

$$\frac{G_N^2 m_1^2 m_2}{2r^2} = \left(-\frac{G_N m_1 m_2}{r}\right)^2 \times \frac{G_N m_2/2}{\left(\sqrt{G_N} m_2\right)^2} . \tag{3.4}$$

In this example, the factor $\mathscr{K}$ of eq. (3.2) is given by the fraction on the *r.h.s.* of eq. (3.3), while $\mathscr{C} = 1$. We have adopted the convention that $m_{1(2)}$ refers to the bottom (top) line in the diagrams. The static 1PN potential is then given by $V_{\text{static}}^{(1\text{PN})} = \frac{G_N^2 m_1^2 m_2}{2r^2} + (m_1 \leftrightarrow m_2)$.

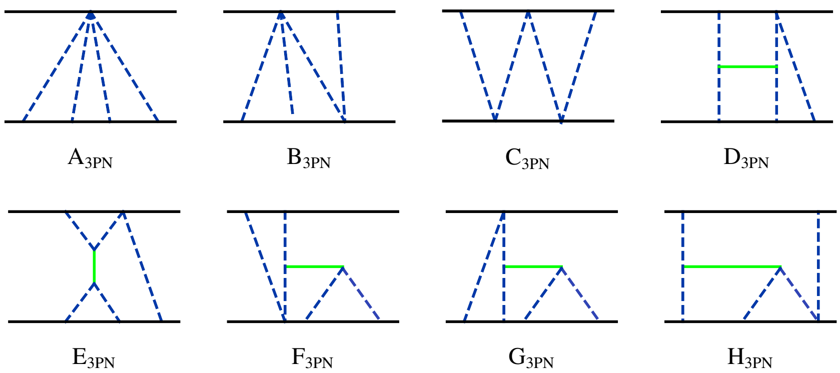
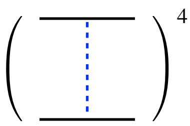


Figure 4: The 3PN graphs are shown. The first graph in the second line (bottom-left) does not contribute to the 3PN potential, because its 2PN subdiagram vanishes.

At the 3PN order there are eight static graphs which are shown in fig. 4. Their contributions to the Lagrangian were computed in [34]. In light of the factorization theorem, we identify two classes of diagrams:

1. In this set, we consider three diagrams composed out of four Newtonian factors, corresponding to the first three diagrams of fig. 4 ($\text{A}_{3\text{PN}}$, $\text{B}_{3\text{PN}}$, $\text{C}_{3\text{PN}}$). We represent them as:

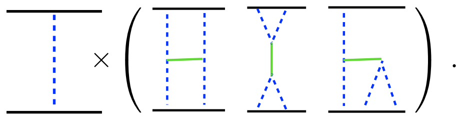


(3.5)

Their contribution to the 3PN potential is:

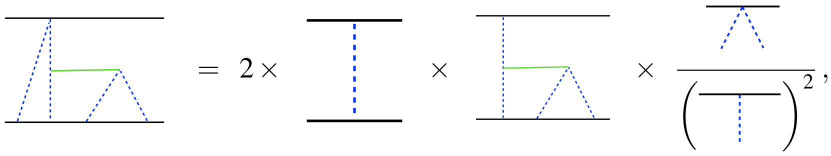
$$V_{N^4} = \frac{1}{24}\frac{G_N^4 m_1^4 m_2}{r^4} + \frac{G_N^4 m_1^3 m_2^2}{r^4} + (m_1 \leftrightarrow m_2) . \tag{3.6}$$

2. The other five diagrams of fig. 4 ($\text{D}_{3\text{PN}}$, $\text{E}_{3\text{PN}}$, $\text{F}_{3\text{PN}}$, $\text{G}_{3\text{PN}}$, $\text{H}_{3\text{PN}}$), are built as products of one Newtonian term and the three static 2PN prime graphs, combined in all possible ways, schematically represented as:



(3.7)

For illustration purposes, the factorization theorem can be verified for one of them:



(3.8)

($\mathscr{C} = 2$ here) amounting to

$$\frac{G_N^4 m_1^4 m_2}{3r^4} = 2 \times \left(-\frac{G_N m_1 m_2}{r}\right) \times \left(-\frac{G_N^3 m_1^3 m_2}{3r^3}\right) \times \frac{G_N m_2/2}{(\sqrt{G_N} m_2)^2}\,, \tag{3.9}$$

in agreement with [34]. The contribution to the potential from the five diagrams in class 2 is:

$$V_{N\times 2\mathrm{PN}} = \frac{1}{3}\frac{G_N^4 m_1^4 m_2}{r^4} + 5\frac{G_N^4 m_1^3 m_2^2}{r^4} + (m_1 \leftrightarrow m_2)\,. \tag{3.10}$$

The total static 3PN contribution of the diagrams belonging to classes 1 and 2 is, in agreement with the literature:

$$V_{\mathrm{static}}^{(3\mathrm{PN})} = V_{N^4} + V_{N\times 2\mathrm{PN}} = \frac{3}{8}\frac{G_N^4 m_1^4 m_2}{r^4} + 6\frac{G_N^4 m_1^3 m_2^2}{r^4} + (m_1 \leftrightarrow m_2)\,. \tag{3.11}$$

4. The results

The static contribution to the two-body potential at fifths PN order can be determined recursively from the lower PN results with the help of the factorization property of ref. [38] as discussed in sec. 3. The 154 five-loop diagrams can be subdivided here into four classes according to their factorization property. The first class consist out of products of six Newtonian diagrams. The second class consists out of products of three Newtonian graphs and the prime 2PN diagrams. The third class are products of a Newtonian diagram and the 4PN prime graphs. Finally the last class are products of the 2PN prime graphs.

The ingredients which are needed in order to determine the static 5PN contribution are only the static 2PN and 4PN prime graphs. All static 5PN contributions can then be obtained through the factorization property. The static 2PN and 4PN prime graphs are known since long from literature, as discussed in sec. 1. In particular the static 4PN contribution has been computed in the EFT approach in ref. [21] by employing techniques for calculating Feynman diagrams which are used in high energy particle physics. The appearing 50 diagrams are subdivided into two sets. The first set contains simpler integrals which can be computed with the kite rule [39, 40]. The second set of four-loop integrals has been reduced systematically to a small set of seven master integrals (MI) with integration-by-parts identities [39, 40] using Laporta's algorithm [41, 42]. They are shown in fig. 5. The reduction has been performed in two ways, with an in-house code based on `Form` [43, 44, 45]

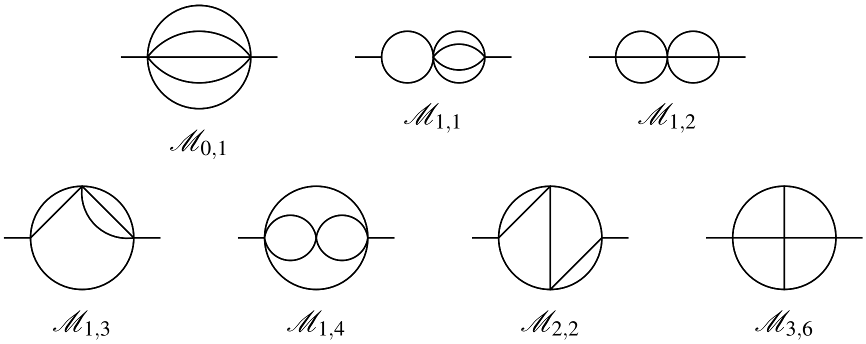


Figure 5: The master integrals appearing in the calculation of the static 4PN contribution are shown.

and with the program `Reduze` [46, 47]. Five of the seven MIs $\{\mathscr{M}_{0,1}, \mathscr{M}_{1,1}, \mathscr{M}_{1,2}, \mathscr{M}_{1,3}, \mathscr{M}_{1,4}\}$ can be calculated straightforwardly in a closed analytical form in d dimensions. They are expressible in terms of Γ-functions. The MI $\mathscr{M}_{2,2}$ always appears multiplied by sufficient high positive powers of $\varepsilon = (d-3)$ in the amplitude, so that it drops out in the limit $\varepsilon \to 0$. The remaining seventh MI $\mathscr{M}_{3,6}$ is known in an expansion in ε in refs. [48, 21, 49].

Having all even PN orders at hand, the static 5PN contribution to the gravitational two-body potential has been obtained for the first time in ref. [38] with the help of the factorization property. It reads:

$$V_{\text{static}}^{(5\text{PN})} = \frac{5}{16}\frac{G_N^6 m_1^6 m_2}{r^6} + \frac{91}{6}\frac{G_N^6 m_1^5 m_2^2}{r^6} + \frac{653}{6}\frac{G_N^6 m_1^4 m_2^3}{r^6} + (m_1 \leftrightarrow m_2). \tag{4.1}$$

The result has been checked in the test-particle limit, in which one considers the body with mass m_2 as a test particle in the gravitational field of the body with mass m_1, corresponding to the Schwarzschild limit which recovers the term linear in m_2 and with the highest power in m_1. At fifth PN order this check has been used for the first-time in ref. [38]. In this limit the static effective Lagrangian reads $\mathscr{L}_{\text{static}}^{m_1 \gg m_2} = -m_2\sqrt{1-G_N m_1/r}/\sqrt{1+G_N m_1/r}$. Its expansion permits to extract this contribution to the potential at each PN order. Hence, the Schwarzschild metric allows to predict the coefficients of the terms of the form $G_N^\ell m_1^\ell m_2/r^\ell$ at any n-th PN order with $\ell = n+1$, for example, at 6th PN order the coefficient of the term $G_N^7 m_1^7 m_2/r^7$ reads $-5/16$. Finally eq. (4.1) has been confirmed in ref. [50] by an independent calculation.

5. Summary and conclusion

We studied the gravitational two-body potential at fourth and fifth PN order in the EFT approach to GR in the static limit. Its calculation can be mapped onto the determination of four- and five-loop self-energies, which can be solved with tools commonly used in high-energy particle physics. We established a factorization property of the static diagrams appearing at odd PN orders, so that these contributions can be determined recursively from the lower PN order results and no loop integrals need to be computed. We verified the validity of our factorization theorem at the lower odd PN orders and applied it to the fifth PN order in order to do a first-time calculation of the static contributions to the gravitational two-body potential. The factorization property is also applicable to a large subset of even-PN diagrams, which simplifies their calculation. As a result of this the factorization property is a powerful tool to simplify higher order PN calculations.

Acknowledgments

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