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Planar master integrals for the two-loop light-fermion electroweak corrections to Higgs plus jet production

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We present the analytic computation of the planar master integrals which occurs in the calculation of the two-loop light fermion electroweak corrections to the production of a Higgs boson in association with a jet in gluon-gluon fusion channel. The complete dependence on the electroweak-boson mass has been taken into account. The evaluation of the master integrals is done using the differential equation method with the canonical basis approach. The solution is given as a series expansion in the dimensional regularization parameter up to weight four. The coefficients in the expansion are given in terms of multiple-polylogarithms.

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1. Introduction

The study of the Higgs boson properties represents a crucial test for the Standard Model (SM) and the comprehension of the electroweak symmetry breaking and it can also represent a probe for the searching of New Physics (NP).

The major production channel of the Higgs boson at LHC is given by the gluon-gluon fusion channel. The coupling with gluons is mediated by a heavy-quark loop. In the full theory, the one in which the complete dependence on the heavy-quark mass is considered, Higgs boson production is known at NLO [1, 2].

The computations are more tractable in the Higgs Effective Field Theory (HEFT) in which the mass of top-quark is considered to be infinite. In HEFT the Higgs-gluon coupling is replaced by an effective coupling. In this approximation the Higgs production in the gluon-gluon fusion is known at N^3LO [3, 4, 5, 6, 7]. The transverse momentum distribution of the Higgs boson in gluon-gluon fusion represents a prominent observable for the NP contributions to the Higgs-gluon coupling [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. The p_T distribution is known in HEFT at NNLO [21, 22, 23, 24, 25].

In the region where $p_T \gg m_T$ the HEFT approximation is no more valid hence the transverse momentum distribution gives predictions which can differ considerably from those one computed in the full theory in which the top-mass is treated exactly [26]. The computation of the Higgs plus one jet production is known in the full theory only at LO [27, 28] while the NLO contributions would require the computation of the two-loop four point amplitudes for Higgs plus three partons whose calculation involves elliptic iterated integrals. Though, several approximations exist to Higgs plus jet production including for istance a numerical computation [29], computations in the high-p_T region [30, 31] and finally, in the intermediate p_T region [32, 33].

Due to the increasingly precision of theoretical predictions also the electroweak corrections have started having a role in the phenomenology. In particular, light-fermions represent the bulk of the electroweak corrections to Higgs plus jet production in the gluon-gluon fusion channel [34, 35, 36] increasing by about 5% the LO of gluon-gluon fusion cross section and 2% the N^3LO cross section [37]. The coupling with light-quarks is mediated by an electroweak boson loop, moreover, the coupling with gluons is made by two loops, a light-fermions loop and an electroweak boson loop. In this context, we report the analytic computation of the master integrals (MIs) of two-loop four point amplitudes which are needed in order to evaluate the planar light-fermion electroweak corrections to Higgs plus jet production.

The computation of the master integrals has been perfomed in dimensional regularization using the differential equations method [38, 39, 40, 41, 42, 43]. With the Integration-By-Parts identities (IBPs) [44, 45, 46] and Lorenz Invariant identities [38] we reduced the large number of scalar Feynman integrals to a smaller number of independent master integrals. The reduction to the MIs has been performed using the public softwares `FIRE5` [47] and `LiteRed` [48] which gave a number of 48 MIs. The analytic expression of the MIs in terms of Goncharov polylogarithms [49, 50, 51] up to weight four has been found using the canonical basis approach [42]. The numerical evaluation of the solution is done using the public softaware `GiNaC` [52, 53] and it has been tested successfully against the software `FIESTA` [54] both in the Euclidean and Minkowski regions.

This contribution to the proceedings of RADCOR2019 is based on the work [55].

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2. Notations

In this section we present the definition of the topology (Fig. 1) which enters in the calculation of the light-fermion electroweak corrections to Higgs plus jet production. The external momenta p_i correspond to on-shell gluon with $p_i^2 = 0$ while the external Higgs momentum p_4 is considered as a dynamic variable.

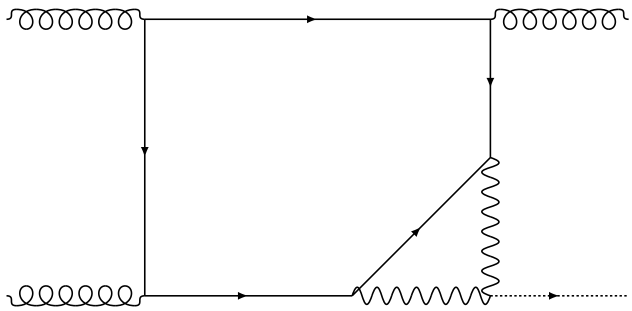

Figure 1: Planar seven-denominator topology for Higgs plus jet production.

The planar integrals are the defined by the seven-denominator,

$$\int \mathscr{D}^d k_1 \mathscr{D}^d k_2 \frac{D_8^{a_8} D_9^{a_9}}{D_1^{a_1} D_2^{a_2} D_3^{a_3} D_4^{a_4} D_5^{a_5} D_6^{a_6} D_7^{a_7}}, \tag{2.1}$$

where $d = 4 - 2\varepsilon$ is the dimensional regularization parameter and a_i for $i = 1, \ldots, 9$ are positive integer numbers. The propagators $D_1, \ldots, D_7$ represent the denominators while D_8, D_9 represent the irreducible scalar products. The planar topology is defined by the following set of propagtors:

$$\begin{aligned} \big\{ &-k_1^2, -(k_1-k_2)^2, -(p_1+k_1)^2, -(p_1+p_2+k_1)^2, -(p_3-k_1)^2, \\ &-(p_1+p_2+k_2)^2 + m_B^2, -(p_3-k_2)^2 + m_B^2, -(p_3+k_1-k_2)^2, -(k_2+p_2)^2 \big\}. \end{aligned} \tag{2.2}$$

The normalization of the integration measure is defined as:

$$\mathscr{D}^d k_i = \frac{d^d k_i}{i\pi^{\frac{d}{2}}} e^{\varepsilon \gamma_E} \left(\frac{m_B^2}{\mu^2} \right)^{\varepsilon}. \tag{2.3}$$

The kinematic invariants are defined by the Mandelstam variables s, t

$$s = (p_1 + p_2)^2, \quad t = (p_1 + p_3)^2. \tag{2.4}$$

We also define the dimensionless variables x, y, z

$$x = -\frac{s}{4m_B^2}, \quad y = -\frac{t}{4m_B^2}, \quad z = -\frac{p_4^2}{4m_B^2}, \tag{2.5}$$

where m_B is the mass of the electroweak bosons. The physical region for the dimensionless kinematic inviarant is given by the following inequalities

$$x < z < 0, \ \ y > 0, \ \ z > x + y. \tag{2.6}$$

3. Master Integrals computations

In this section we present the list of MIs in their precanonical form (Fig. 2) and we discuss briefly the problem of the roots linearization and the determination of the boundary conditions.

3.1 Differential equations

The computation of MIs is performed using the differential equations method [56, 41, 39, 40]. The representation in terms of Goncharov polylogarithms (GPLs) up to weight four in the dimensional regularization parameter ε has been reached with the canonical basis approach [42, 43] which consists in the choice of a peculiar set of MIs for those the system of differential equations can be written in the following form

$$d\vec{f}(\vec{x},\varepsilon) = \varepsilon\, d\tilde{A}(\vec{x})\vec{f}(\vec{x},\varepsilon)\,, \tag{3.1}$$

where $\vec{f}(\vec{x})$ is the master integrals vector and $d\tilde{A}(\vec{x})$ is a 1-form which can be written in a d-log form, namely the element $\tilde{A}_{ij}(\vec{x})$ is a linear combination of logarithms

$$\tilde{A}_{ij}(\vec{x}) = \sum_k c_k \log\left[\alpha_{ij}^{(k)}(\vec{x})\right] \tag{3.2}$$

where the c_k's are rational numbers and $\alpha_{ij}^{(k)}(\vec{x})$ is called a *letter* and is an algebraic function of the kinematic invariants. The set of the all independent letters makes the *alphabet* of the process. The system of differential equations in eq. (3.1) can be solved formally in terms of the Chen iterated integral [57],

$$\vec{f}(\vec{x},\varepsilon) = \mathbb{P}exp\left(\varepsilon\int_\gamma d\tilde{A}(\vec{x})\right)\vec{f}(\vec{x}_0,\varepsilon)\,, \tag{3.3}$$

where $\mathbb{P}$ stands for the path-ordering operator, γ is a path in the space of kinematic invariants and $\vec{f}(\vec{x}_0,\varepsilon)$ is a vector of boundary conditions.

3.2 GPLs representation, roots linearizations and the Alphabet

The Chen iterated integral admits a suitable power series expansion in the ε parameter. Order-by-order in ε the coefficients of the expansion can be written as Goncharov Polylogarithms[49].

GPLs are defined in an iterative way

$$G(w_1,\ldots,w_n;x) = \int_0^x \frac{dt}{t-w_1} G(w_2,\ldots,w_n;t) \tag{3.4}$$

where for $n=0$, we define $G(;x)=1$. The w_i's are called *weights*.

The ε-expansion of the formal solution expressed by eq. (3.3) is given by

$$\vec{f}(x) = \vec{f}^{(0)}(\vec{x}_0) + \sum_{k=1}^{\infty}\varepsilon^k \sum_{j=1}^{k}\int_0^1 dt_1 \frac{\partial\tilde{A}(t_1)}{\partial t_1}\int_0^{t_1} dt_2\frac{\partial\tilde{A}(t_2)}{\partial t_2}\ldots\int_0^{t_{j-1}} dt_j\frac{\partial\tilde{A}(t_j)}{\partial t_j}\vec{f}^{(k-j)}(\vec{x}_0)\,. \tag{3.5}$$

The representation in terms of GPL can be reached only if the letters of the alphabet are rational functions of the kinematic invariants which is not necessarily true since the letters $\alpha(\vec{x})$ are, in general, algebraic functions of the x_i's. This leads to the problem of the roots linearization which can be solved with a variable change. Several approaches to the root linearization has been proposed [58, 59, 60]. The system of differential equations of Higgs plus jet depends on rational functions in the x, y variables while depends on the following root for the variable z:

$$\sqrt{z(1+z)}, \tag{3.6}$$

which can be linearized by the change of variables

$$z(w) = \frac{w^2}{1+2w}. \tag{3.7}$$

With the change of variable given by (3.7) the letters of the alphabet are rational functions in the linearized variables x, y, w.

In the following we report the letters of the alphabet:

$$x_k \in \left\{0, -\frac{1}{4}, \frac{1}{4}\right\}, \tag{3.8}$$

$$y_k \in \left\{0, -\frac{1}{4}, \frac{1}{4}, -x, -\frac{x}{4x+1}\right\}, \tag{3.9}$$

$$\begin{aligned} w_k \in \Big\{ & 0, -\frac{1}{2}, -1, \frac{1}{4}\left(-1-i\sqrt{3}\right), \frac{1}{4}\left(-1+i\sqrt{3}\right), 2x, -\frac{2x}{4x+1}, x-\sqrt{x^2+x}, \\ & x+\sqrt{x^2+x}, \frac{1}{4}\left(-1+4x-\sqrt{16x^2+8x-3}\right), \frac{1}{4}\left(-1+4x+\sqrt{16x^2+8x-3}\right), \\ & 2y, -\frac{2y}{4y+1}, y-\sqrt{y^2+y}, y+\sqrt{y^2+y}, \frac{1}{4}\left(-1+4y-\sqrt{16y^2+8y-3}\right), \\ & \frac{1}{4}\left(-1+4y+\sqrt{16y^2+8y-3}\right), x+y-\sqrt{x^2+2xy+x+y^2+y}, \\ & x+y+\sqrt{x^2+2xy+x+y^2+y}, \frac{-x-y}{2y}, \frac{-x-y}{2x}\Big\}, \end{aligned} \tag{3.10}$$

that correspond to the arguments of the GPLs of the solution.

3.3 Boundary conditions and Numerical checks

Canonical MIs can be written as a linear combination of pre-canonical integrals $\mathscr{T}_i$, $i \in \{1,\ldots,48\}$ MIs which are shown in figure 2.

The determination of the boundary conditions follows on the direct analysis of the behavior of the pre-canonical MIs near singular points.

Let the canonical master integrals $f(x)$ be written as $f(x) = xT(x)$ we are interested in the behavior of T around the point $x = 0$.

- $T(x)$ is finite in $x = 0$:

  Since $T(x)$ is finite and is multiplied by x, in the limit $x \to 0$ the canonical $f(x)$ will vanish in this limit.

- $T(x)$ is log-divergent in $x = 0$:

  Here, $T(x)$ behaves like $\log(x)^a$ with some positive a. But still, since,

  $$\lim_{x\to 0} x \log(x)^a = 0, \;\; \forall a > 0.$$

  Also in this case $f(x)$ is null in $x = 0$.

- $T(x)$ diverges like $1/x$:

  Here, $f(x)$ will be only finite in $x = 0$, but in general we do not know $f(0)$. However we can prove that if the canonical master is finite then we have:

$$\lim_{x\to 0} x \frac{\partial f(x)}{\partial x} = 0. \tag{3.11}$$

  The latter equation can be proved just considering a Laurent expansion of T around $x = 0$.

$$T(x) = \frac{1}{x} + \beta + \mathcal{O}(x), \tag{3.12}$$

  then,

$$\lim_{x\to 0} x \frac{\partial f(x)}{\partial x} = \lim_{x\to 0} x \frac{\partial}{\partial x}(xT(x)) = \lim_{x\to 0} \left[x\left(\frac{1}{x} + \beta\right) + x^2\left(-\frac{1}{x^2}\right)\right]$$
$$= \lim_{x\to 0} \beta x = 0. \tag{3.13}$$

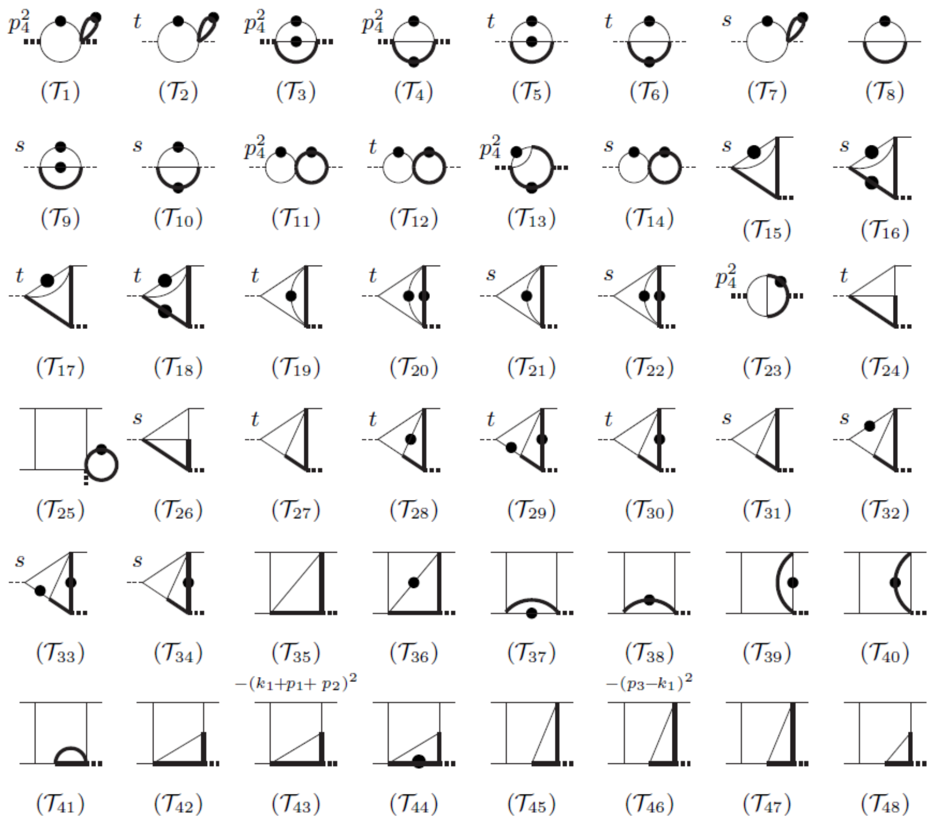

Figure 2: Pre-canonical MIs for the Higgs plus jet production. Internal thick lines represent electroweak vector-boson propagators while internal thin lines represent light-fermion propagators. External thin lines represent massless particles on their mass-shell while the external dotted thick line represents the Higgs boson.

For canonical MIs $f_3,\dots,f_6,f_8,\dots,f_{24},f_{26},\dots,f_{36},f_{38},f_{40},\dots,f_{48}$ the boundary conditions are fixed in $s=t=p_4^2=0$ which is a regular point, while f_1,f_2,f_7,f_{25} are divergent in $s=t=p_4^2=0$, but they are product of known one-loop master integrals. Finally, for f_{37},f_{39} the boundary conditions has been fixed, respectively, in the regular point $(s=0,t=m_B^2,p_4^2=0)$ and $(s=m_B^2,t=0,p_4^2=0)$.

We integrated the system of differential equations in the Euclidean region $x\geq 0$, $y\geq 0$, $w\geq 0$. We evaluated numerically the MIs using the software `GiNaC` [52, 53]. The analytic continuation to the physical region is performed numerically by adding a small imaginary part to s, namely, $s\to s+i0^+$. The physical region is given by:

$$x<\frac{w^2}{1+2w}<0,\ \ y>0,\ \ \frac{w^2}{1+2w}>x+y. \tag{3.14}$$

where no branch cuts are present. We checked different points of the phase space against `FIESTA` [54], finding complete agreement.

4. Conclusions

In this contribution to the proceeding of RADCOR2019 we have computed the analytic expression of the master integrals relevant for the planar two-loop light-fermion electroweak corrections to Higgs plus jet production. The full dependence on the vector boson mass has been taken into account.

The master integrals are evaluated with the differential equations method using the canonical basis approach. The letters of the alphabet depend only on a single square root which has been linearized with an appropriate change of variable. In the linearized variables the matrix associated to the system of differential equations only depends on rational functions. This forms allows us to proceed with a direct integration of the differential equations in such a way the canonical master integrals can be written order-by-order in ε up to weight four in terms of generalized polylogarithms making their numerical evaluation very fast and precise with the aim of dedicated softwares.

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