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Renormalization schemes for mixing angles in extended Higgs sectors

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The proper renormalization of mixing angles in quantum field theories is a long-standing problem. It is relevant for the renormalization of the quark mixing matrix in the Standard Model and for various mixing scenarios in theories beyond. In this contribution we specifically consider theories with extended scalar sectors. We describe renormalization schemes for mixing angles based on combinations of observables or symmetry requirements such as rigid or background-field gauge invariance and compare their properties to previous approaches such as $\overline{\mathrm{MS}}$ schemes. We formulate specific renormalization conditions for the mixing angles in the Two-Higgs-Doublet Model and the Higgs-Singlet Extension of the Standard Model and calculate electroweak corrections to Higgs-boson decays via W- or Z-boson pairs within these models for a selection of (new and old) renormalization schemes.

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1. Introduction

Mixing of different fields with identical conserved quantum numbers appears in many theories. The Standard Model contains mixing angles in the fermion sector, the parameters of the quark-mixing matrix. In models with extended Higgs sectors, different scalar fields typically mix, while in models with extended gauge sectors, mixing between different gauge fields takes place. The investigation of the Higgs sector is presently in the focus of particle physics at the LHC. This requires to perform precise calculations in models with extended Higgs sectors and in turn the renormalization of these models including the renormalization of mixing angles. In this contribution we focus on the renormalization of mixing angles in extended Higgs sectors.

Renormalization of mixing angles in extended Higgs sectors has been performed in various models. The renormalization of the mixing angle β in the Minimal Supersymmetric Standard Model (MSSM) was investigated in Refs. [1–5]. Renormalization conditions for the mixing angles α and β of the Two-Higgs-Doublet Model (THDM) were discussed in Refs. [6–13], while the mixing-angle renormalization for the Higgs-Singlet Extension of the Standard Model (HSESM) was investigated in Refs. [12, 14–17]. Finally, the renormalization of mixing angles in the Next-to-Two-Higgs-Doublet Model was performed in Ref. [18].

Previous work on the renormalization of mixing angles has revealed problems in various schemes. $\overline{\mathrm{MS}}$ renormalization of mixing angles can lead to perturbative instabilities. Fixing the renormalized mixing angles from individual physical observables violates symmetries and can give rise to unnaturally large corrections, while renormalization conditions relying on self-energies instead of physical quantities introduce gauge dependences.

Based on the study of the renormalization of $\tan\beta$ in the MSSM, Freitas and Stöckinger [3] formulated desirable properties for the renormalization of mixing angles. Accordingly, mixing-angle renormalization should be (the last condition was added in Ref. [19]):

- *gauge independent*, i.e. renormalized observables should be gauge-independent functions of the renormalized mixing angles;
- *symmetric* with respect to the mixing degrees of freedom, i.e. in particular independent of a specific physical process;
- *perturbatively stable*, i.e. higher-order corrections should not be artificially large;
- *non-singular* for degenerate masses of mixing fields and for extreme mixing angles, i.e. valid in the full parameter space.

The aim of this contribution is to recapitulate the renormalization conditions for mixing angles introduced in Ref. [19] that fulfil these conditions. For more details and further phenomenological applications we refer to Ref. [19].

2. Tadpoles and renormalization

Field theories with spontaneous symmetry breaking involve scalar fields with non-vanishing vacuum expectation (vevs) values $\langle\Phi\rangle = v \neq 0$. Since perturbation theory requires an expansion around the minimum of the potential, shifted fields with vevs are introduced, $\Phi(x) = \bar{v} + H(x)$, with $\langle H\rangle = 0$ at least in leading order (LO). If the expansion is performed about the LO vev v_0, at next-to-leading order (NLO) so-called tadpole contributions T^H from one-particle irreducible diagrams with a single external Higgs field appear. These can be eliminated via an expansion about the corrected vev $v = v_0 + \Delta v$, which leads to tadpole counterterms δt^H that cancel the tadpole contributions of loop diagrams.

In the conventional tadpole scheme, which has been used in the Standard Model for instance in Refs. [20–24], the corrected vev v is used throughout. As a consequence, no explicit tadpole diagrams appear, and the Ward identities are simplified owing to $\langle H \rangle = 0$ in all orders. However, the corrected vev is related to the tadpoles, which are gauge dependent. As a consequence, the bare masses of the gauge bosons, fermions, and the Higgs boson as well as the corresponding counterterms become gauge dependent. This does not matter if all observables are expressed in terms of observables such as physical masses, which is the case in the on-shell schemes used for the Standard Model. If, however, some parameters are renormalized in an $\overline{\mathrm{MS}}$ scheme, these parameters and the S-matrix as a function thereof become gauge dependent. This is, in particular, the case for mixing angles in extended scalar sectors renormalized within the $\overline{\mathrm{MS}}$ scheme [10, 11]. While the conventional tadpole scheme appears in the literature in variants differing in the definition of the Higgs-boson mass, all variants share the properties discussed above. We use in the following the variant defined in Refs. [11, 23] dubbed PRTS in Ref. [19].

An alternative tadpole scheme was introduced in Ref. [25], dubbed FJTS. It uses consistently the bare (LO) vev v_0. As a consequence, all renormalized parameters are gauge independent also in $\overline{\mathrm{MS}}$ schemes. On the down side, $\langle H \rangle \neq 0$ beyond LO, and tadpoles appear in many places, e.g. in the definition of the renormalized masses and in Ward identities. Upon performing the shift $H \to H + \Delta v$ of the Higgs field H by the constant Δv, the explicit tadpoles can be replaced by contributions of Δv and thus transferred to the counterterms of vertex functions with more than one external leg. Even though being fully consistent, the FJTS is prone to large corrections originating from $\overline{\mathrm{MS}}$-renormalized mixing angles [13, 17, 19, 26].

3. Extended Higgs sectors

The mixing between two CP-even scalars appears in many models with extended Higgs sectors. Denoting the fields in the symmetric basis by η_1, η_2 and the fields in the physical mass-eigenstate basis by H_1, H_2, the mixing is described by

$$\boldsymbol{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = R(\alpha)\mathbf{H}, \qquad R(\alpha) = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}, \tag{3.1}$$

with the shorthand notations $c_\alpha = \cos\alpha$ and $s_\alpha = \sin\alpha$ for the mixing angle α.

The renormalization transformations for α and the scalar fields in the physical basis read

$$\alpha_\mathrm{B} = \alpha + \delta\alpha, \qquad \mathbf{H}_\mathrm{B} = (Z^H)^{1/2}\mathbf{H}, \tag{3.2}$$

where the index B denotes bare quantities and the field renormalization constants are parametrized as

$$(Z^H)^{1/2} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z^H_{11} & \frac{1}{2}\delta Z^H_{12} \\ \frac{1}{2}\delta Z^H_{21} & 1 + \frac{1}{2}\delta Z^H_{22} \end{pmatrix}. \tag{3.3}$$

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In the complete on-shell scheme [22, 23] the non-diagonal field renormalization constants are fixed as

$$\delta Z^H_{ij} = \frac{2}{M^2_{\mathrm{H}_i} - M^2_{\mathrm{H}_j}} \Sigma_{ij}(M^2_{\mathrm{H}_j}), \quad i \neq j, \tag{3.4}$$

where M_{H_i} and M_{H_j} are the masses of the scalar bosons H_i and H_j, respectively, and Σ_{ij} their mixing energy.

4. Renormalization schemes for mixing angles based on symmetries

Symmetric renormalization conditions for mixing angles can be obtained upon using rigid symmetry, i.e. the symmetry under global $\mathrm{SU}(2) \times \mathrm{U}(1)$ transformations. Since a spontaneously broken theory can be renormalized in the unbroken phase [20,27,28], the following renormalization transformations are sufficient to absorb the ultraviolet (UV) singularities

$$\alpha_{\mathrm{B}} = \alpha + \delta\alpha, \qquad \boldsymbol{\eta}_{\mathrm{B}} = (Z^\eta)^{1/2} \boldsymbol{\eta}, \tag{4.1}$$

$$(Z^\eta)^{1/2} = \begin{pmatrix} (Z^\eta_1)^{1/2} & 0 \\ 0 & (Z^\eta_2)^{1/2} \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z^\eta_1 & 0 \\ 0 & 1 + \frac{1}{2}\delta Z^\eta_2 \end{pmatrix}. \tag{4.2}$$

Consistency with renormalization in the complete on-shell scheme (3.2), (3.3) requires for the UV-divergent parts

$$(Z^H)^{1/2}\big|_{\mathrm{UV}} = R^{\mathrm{T}}(\alpha + \delta\alpha)(Z^\eta)^{1/2} R(\alpha)\big|_{\mathrm{UV}} \tag{4.3}$$

and as a consequence

$$\begin{aligned} \delta Z^H_{11}\big|_{\mathrm{UV}} &= c^2_\alpha \delta Z^\eta_1\big|_{\mathrm{UV}} + s^2_\alpha \delta Z^\eta_2\big|_{\mathrm{UV}}, \\ \delta Z^H_{22}\big|_{\mathrm{UV}} &= s^2_\alpha \delta Z^\eta_1\big|_{\mathrm{UV}} + c^2_\alpha \delta Z^\eta_2\big|_{\mathrm{UV}}, \\ \delta Z^H_{12}\big|_{\mathrm{UV}} + \delta Z^H_{21}\big|_{\mathrm{UV}} &= 2 c_\alpha s_\alpha (\delta Z^\eta_2 - \delta Z^\eta_1)\big|_{\mathrm{UV}}, \\ \delta Z^H_{12}\big|_{\mathrm{UV}} - \delta Z^H_{21}\big|_{\mathrm{UV}} &= 4\delta\alpha\big|_{\mathrm{UV}}, \end{aligned} \tag{4.4}$$

i.e. UV divergences of mixing-angle counterterms are related to UV divergences of δZ^H_{ij}. This can be used to fix the renormalization of α via [6, 10, 19]

$$\delta\alpha = \frac{1}{4}\left(\delta Z^H_{12} - \delta Z^H_{21}\right) = \frac{\Sigma^H_{12}(M^2_{\mathrm{H}_2}) + \Sigma^H_{12}(M^2_{\mathrm{H}_1})}{2(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})}, \tag{4.5}$$

which is evidently symmetric and process independent.

The counterterm $\delta\alpha$ appears in S-matrix elements only in the combinations

$$\begin{aligned} &-\delta\alpha + \frac{1}{2}\delta Z^H_{12}, \qquad \delta\alpha + \frac{1}{2}\delta Z^H_{21} && \text{from the field rotation } R(\alpha), \\ &(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})\delta\alpha && \text{from relations to parameters of the Higgs potential.} \end{aligned} \tag{4.6}$$

With the condition (4.5), the first two combinations become

$$\frac{1}{2}\delta Z^H_{12} - \delta\alpha = \delta\alpha + \frac{1}{2}\delta Z^H_{21} = \frac{1}{4}\left(\delta Z^H_{12} + \delta Z^H_{21}\right) = \frac{\Sigma^H_{12}(M^2_{\mathrm{H}_2}) - \Sigma^H_{12}(M^2_{\mathrm{H}_1})}{2(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})}. \tag{4.7}$$

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Thus, all three combinations (4.6) are smooth in the limit of degenerate scalar masses $M_{\mathrm{H}_1} \to M_{\mathrm{H}_2}$.

Since the field renormalization constants δZ^{H}_{ij} are gauge dependent, this gauge dependence enters $\delta\alpha$ defined in (4.5). However, as suggested in Refs. [9, 29, 30] for the renormalization of β, one can choose a specific gauge to calculate the counterterms for $\delta\alpha$ and thus fix this renormalization constant once and for all. A convenient choice for this purpose, which does not introduce large artificial parameters, is the 't Hooft–Feynman gauge. Once $\delta\alpha$ is kept fixed, S-matrix elements are gauge-independent functions of the renormalized input parameters as usual.

The method discussed can be applied to the renormalization of β in the THDM by using the mixing between the physical pseudoscalar Higgs boson and the would-be Goldstone boson. Alternatively, one can rely on the definition $\tan\beta = v_2/v_1$ and the framework of the background field method (BFM) [12, 31, 32]. In the BFM, fields are split into background fields $\hat{\Phi}$, which serve as external sources, and quantum fields Φ, which are quantized. By choosing a suitable gauge-fixing term for the quantum fields, rigid gauge invariance is maintained for the background fields [12,32]. This invariance implies Ward identities and restrictions on renormalization constants for the background fields. For instance, the relations (4.4) can be kept exact including finite parts by choosing the field renormalization constants appropriately.

Applying the BFM to the THDM [12] yields the relations for the symmetric field renormalization constants of the scalar fields,

$$
\begin{aligned}
\delta Z^{\hat{\eta}}_1 &= -2\delta Z_e - \frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2}\frac{\delta c_{\mathrm{w}}^2}{c_{\mathrm{w}}^2} + \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} + 2\frac{\Delta v_1}{v_1} + 2\frac{\delta c_\beta}{c_\beta}, \\
\delta Z^{\hat{\eta}}_2 &= -2\delta Z_e - \frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2}\frac{\delta c_{\mathrm{w}}^2}{c_{\mathrm{w}}^2} + \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} + 2\frac{\Delta v_2}{v_2} + 2\frac{\delta s_\beta}{s_\beta},
\end{aligned}
\tag{4.8}
$$

where the shifts Δv_i of the vevs result from tadpole contributions and can be expressed by tadpole counterterms $\delta t_{\hat{H}_i}$ as

$$
\Delta v_1 = -\frac{\delta t_{\hat{H}_1}}{M_{\mathrm{H}_1}^2} c_\alpha + \frac{\delta t_{\hat{H}_2}}{M_{\mathrm{H}_2}^2} s_\alpha, \qquad \Delta v_2 = -\frac{\delta t_{\hat{H}_1}}{M_{\mathrm{H}_1}^2} s_\alpha - \frac{\delta t_{\hat{H}_2}}{M_{\mathrm{H}_2}^2} c_\alpha. \tag{4.9}
$$

Using the relations (4.4) to eliminate $\delta Z^{\hat{\eta}}_1$ in favour of $\delta Z^{\hat{H}}_{ij}$ and solving for $\delta\beta$ yields [19]

$$
\delta\beta = \frac{1}{2} c_\beta s_\beta \left[(s_\alpha^2 - c_\alpha^2)\left(\delta Z^{\hat{H}}_{11} - \delta Z^{\hat{H}}_{22}\right) + 2 c_\alpha s_\alpha \left(\delta Z^{\hat{H}}_{12} + \delta Z^{\hat{H}}_{21}\right)\right] + \frac{e}{2 s_{\mathrm{w}} M_{\mathrm{W}}} \left(s_\beta \Delta v_1 - c_\beta \Delta v_2\right). \tag{4.10}
$$

This renormalization of the mixing angle β is symmetric, process independent, and smooth in the limits of degenerate Higgs masses and extreme mixing angles. A similar renormalization condition was proposed in Ref. [10].

5. Renormalization of mixing angles in the $\overline{\mathrm{MS}}$ scheme

The $\overline{\mathrm{MS}}$ scheme offers a simple framework for the renormalization of mixing angles. In this scheme, the renormalization constants $\delta\alpha$ contain only UV-divergent parts along with some universal finite constants. The $\overline{\mathrm{MS}}$ renormalization of mixing angles is by construction symmetric in the mixing degrees of freedom and independent of a specific observable. The counterterms are gauge independent if the FJTS is used for the treatment of tadpoles. Finally, if perturbatively stable, the residual renormalization-scale dependence of calculated observables offers a diagnostic tool for estimating residual theoretical uncertainties.

The counterterm $\delta\alpha$ can for instance be fixed by taking the UV-singular part of (4.5). However, since the field renormalization constants δZ^H_{ij} enter the S-matrix elements including their complete finite part owing to the LSZ formalism, the cancellation in the terms in the first line of (4.6) becomes incomplete, and these contributions become singular for $M_{\mathrm{H}_i} \to M_{\mathrm{H}_j}$. As a consequence, all $\overline{\mathrm{MS}}$ renormalization schemes for mixing angles give rise to large corrections in the limit of degenerate masses. These effects are enhanced by additional tadpole contributions in the FJTS.

6. Renormalization schemes for mixing angles based on S-matrix elements

Defining the mixing angles from single observables in general leads to unnaturally large corrections [10], since these observables depend on further parameters. A better strategy uses combinations of observables or S-matrix elements that depend exclusively on a mixing angle. Consider as an example the LO matrix elements for the decay of the scalar Higgs bosons H_1 and H_2 into a pair of Z bosons in the HSESM,

$$\mathscr{M}_0^{H_1\to \mathrm{ZZ}} = \frac{e s_\alpha}{s_\mathrm{W} c_\mathrm{W}^2} M_\mathrm{W} (\varepsilon_1^* \cdot \varepsilon_2^*), \qquad \mathscr{M}_0^{H_2\to \mathrm{ZZ}} = \frac{e c_\alpha}{s_\mathrm{W} c_\mathrm{W}^2} M_\mathrm{W} (\varepsilon_1^* \cdot \varepsilon_2^*), \tag{6.1}$$

where $\varepsilon_{1,2}$ denote the polarization vectors of the two Z bosons. As renormalization condition we can require that the ratio of these two matrix elements, which at LO is a function of α only, does not change under renormalization, i.e.

$$\frac{\mathscr{M}^{H_1\to \mathrm{ZZ}}}{\mathscr{M}^{H_2\to \mathrm{ZZ}}} \stackrel{!}{=} \frac{\mathscr{M}_0^{H_1\to \mathrm{ZZ}}}{\mathscr{M}_0^{H_2\to \mathrm{ZZ}}} = \frac{s_\alpha}{c_\alpha} = f(\alpha). \tag{6.2}$$

In the complete on-shell scheme, this yields for the counterterm of the mixing angle α at NLO

$$\delta\alpha = c_\alpha s_\alpha (\delta_{H_2 ZZ} - \delta_{H_1 ZZ}) + \frac{1}{2} c_\alpha s_\alpha (\delta Z^H_{22} - \delta Z^H_{11}) + \frac{1}{2} (\delta Z^H_{12} s_\alpha^2 - \delta Z^H_{21} c_\alpha^2), \tag{6.3}$$

where $\delta_{H_i ZZ} = \delta_{H_i ZZ}(M^2_{\mathrm{H}_i})$ are the unrenormalized relative one-loop corrections to the respective decay matrix elements. This renormalization condition is gauge independent, because it is based on physical S-matrix elements; it is symmetric with respect to the scalar fields H_1 and H_2; it is numerically stable for degenerate masses $M_{\mathrm{H}_1} \sim M_{\mathrm{H}_2}$; and it has smooth limits for extreme mixing angles, i.e. for $c_\alpha \to 0$ or $s_\alpha \to 0$. However, it is restricted to processes with only neutral external particles, since for charged particles the renormalization constant $\delta\alpha$ becomes IR singular, and for the observed Higgs boson with mass 125 GeV the relative correction factors for the corresponding decay have to be evaluated in the unphysical region.

These drawbacks can be lifted by introducing suitable extra neutral fields with a simple coupling structure and considering the limit of vanishing extra couplings [19]. As an example we add an additional fermion singlet ψ to the HSESM with the Lagrangian[1]

$$\mathscr{L}_\psi = \mathrm{i}\bar{\psi}\partial\!\!\!/\,\psi - y_\psi \sigma \bar{\psi}\psi = \mathrm{i}\bar{\psi}\partial\!\!\!/\,\psi - y_\psi (v_1 + H_1 c_\alpha - H_2 s_\alpha)\bar{\psi}\psi. \tag{6.4}$$

[1] We choose a fermion singlet in order to limit the number of independent operators that could mix when renormalizing the extended model.

and consider the limit of vanishing Yukawa coupling y_ψ. Requiring that the ratio of the matrix elements for the decays of the two scalar Higgs bosons into a $\bar{\psi}\psi$ pair does not receive any higher-order corrections in the limit of vanishing y_ψ,

$$\frac{\mathscr{M}^{H_1\to\psi\psi}}{\mathscr{M}^{H_2\to\psi\psi}} \stackrel{!}{=} \frac{\mathscr{M}_0^{H_1\to\psi\psi}}{\mathscr{M}_0^{H_2\to\psi\psi}} \propto -\frac{c_\alpha}{s_\alpha}, \tag{6.5}$$

fixes the mixing-angle counterterm as

$$\delta\alpha = \frac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22})c_\alpha s_\alpha + \frac{1}{2}(\delta Z^H_{12} c^2_\alpha - \delta Z^H_{21} s^2_\alpha). \tag{6.6}$$

Owing to the simple structure of the model the vertex corrections drop out, and the counterterm is fixed by a (gauge-independent) combination of field-renormalization constants only.

This strategy can be applied to the THDM as well. To this end, we add two right-handed fermion singlets to the Lagrangian, $\nu_{1\mathrm{R}}$ transforming under the extra $\mathbb{Z}_2$ symmetry as $\nu_{1\mathrm{R}} \to -\nu_{1\mathrm{R}}$ and $\nu_{2\mathrm{R}}$ transforming as $\nu_{2\mathrm{R}} \to \nu_{2\mathrm{R}}$, so that $\nu_{i\mathrm{R}}$ can only receive a Yukawa coupling to Φ_i. The additional Lagrangian is given by

$$\mathscr{L}_{\nu_\mathrm{R}} = \mathrm{i}\bar{\nu}_{1\mathrm{R}}\partial\!\!\!/\nu_{1\mathrm{R}} + \mathrm{i}\bar{\nu}_{2\mathrm{R}}\partial\!\!\!/\nu_{2\mathrm{R}} - [y_{\nu_1}\bar{L}_{1\mathrm{L}}(\mathrm{i}\sigma_2\Phi_1^*)\nu_{1\mathrm{R}} + y_{\nu_2}\bar{L}_{2\mathrm{L}}(\mathrm{i}\sigma_2\Phi_2^*)\nu_{2\mathrm{R}} + \mathrm{h.c.}], \tag{6.7}$$

and the new Yukawa couplings are considered in the limit $y_{\nu_i} \to 0$. Renormalization conditions can be formulated [19] by requiring that ratios of S-matrix elements or relations of form factors of Higgs decays into fermion singlets do not get perturbative corrections. Thus, one arrives at the following renormalization constants for the mixing angles,

$$\delta\alpha = (\delta_{H_1\nu_1\bar{\nu}_1} - \delta_{H_2\nu_1\bar{\nu}_1})c_\alpha s_\alpha + \frac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22})c_\alpha s_\alpha + \frac{1}{2}(\delta Z^H_{12}c^2_\alpha - \delta Z^H_{21}s^2_\alpha), \tag{6.8}$$

$$\begin{aligned}\delta\beta = {}& \frac{1}{2}c_\beta s_\beta\left[(c^2_\alpha - s^2_\alpha)(\delta Z^H_{11} - \delta Z^H_{22}) - 2c_\alpha s_\alpha(\delta Z^H_{12} + \delta Z^H_{21})\right] + \frac{1}{2}\delta Z_{G_0A_0} \\ & + c_\beta s_\beta\left(\delta_{A_0\nu_2\bar{\nu}_2} + c^2_\alpha\delta_{H_1\nu_1\bar{\nu}_1} + s^2_\alpha\delta_{H_2\nu_1\bar{\nu}_1} - \delta_{A_0\nu_1\bar{\nu}_1} - s^2_\alpha\delta_{H_1\nu_2\bar{\nu}_2} - c^2_\alpha\delta_{H_2\nu_2\bar{\nu}_2}\right),\end{aligned} \tag{6.9}$$

where again $\delta_{H_i\nu_j\bar{\nu}_j} = \delta_{H_i\nu_j\bar{\nu}_j}(M^2_{\mathrm{H}_i})$ are the unrenormalized relative one-loop corrections to the respective decays. These renormalization conditions are gauge independent, symmetric with respect to the scalar fields H_1 and H_2 and behave smoothly in the limits of degenerate masses $M_{\mathrm{H}_1} \sim M_{\mathrm{H}_2}$ and for extreme mixing angles ($s_\alpha \to 0$, $c_\alpha \to 0$, $s_\beta \to 0$, or $c_\beta \to 0$).

7. Numerical results

For illustration, in Fig. 1 we provide numerical results for the decay widths of the light and heavy scalar Higgs bosons into four fermions in a specific scenario of the THDM obtained with an extended version of PROPHECY4F [33–35]. The renormalization schemes as well as the input parameters are defined in Ref. [19]. The input parameters are fixed in the scheme OS12, where the renormalization of the mixing angles is based on (6.8) and (6.9), and converted with NLO precision to the other schemes.

The plots show the scale dependence of the decay widths in the $\overline{\mathrm{MS}}$ renormalization schemes for the mixing angles α and β in the two tadpole schemes PRTS and FJTS, as calculated in

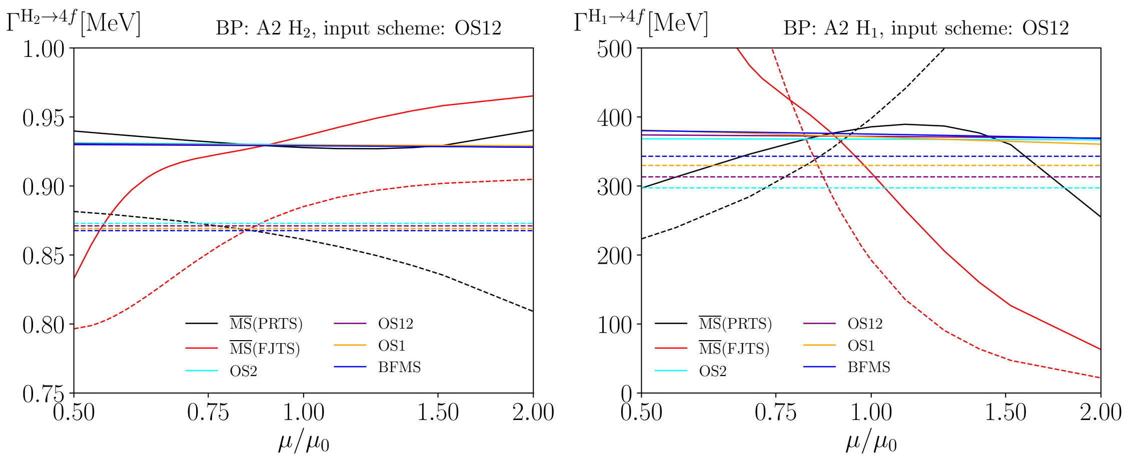

Figure 1: Scale dependence of the decay widths of the light (left) and heavy (right) THDM Higgs bosons for the THDM scenario A2 in different renormalization schemes, with the OS12 scheme as input scheme. LO results are shown as dashed, NLO as full lines; the central scale is set to $\mu_0 = (M_{H_2} + M_{H_1} + M_{A_0} + 2M_{H^+})/5$. (Plots taken from Ref. [19].)

Refs. [13, 26]. While the scale dependence of the decay width of the light Higgs boson is reduced to 2% at NLO in the PRTS, no reduction is found in the FJTS. For the decay of the heavy Higgs boson neither of the $\overline{\text{MS}}$ schemes provides a reduction of scale dependence. In general, the perturbative stability of the results obtained with $\overline{\text{MS}}$-renormalized mixing angles α and β strongly depends on the considered scenario (see Refs. [13, 19, 26] for more examples).

The schemes OS12, OS1, and OS2 (the latter two being variants of OS12) based on sets of physical S-matrix elements and the scheme BFMS defined in (4.5) and (4.10) nicely agree at NLO. In general, uncertainty estimates should be based on schemes that are well behaved and not on the scale dependence of $\overline{\text{MS}}$ schemes if those turn out to be perturbatively unstable.

Further numerical results in different scenarios for the Higgs decays into four fermions and Higgs production in Higgs strahlung and vector-boson fusion in the THDM and the HSESM can be found in Refs. [11–13, 17, 19, 26]. Results for Higgs decays into 2-particle final states in the THDM were presented in Ref. [10].

The renormalization schemes based on rigid symmetry and BFM as well as the renormalization schemes based on combinations of observables have been implemented in 2HDECAY [36], PROPHECY4F3.0 [35], and HAWK 3.0 [37].

8. Conclusions

The mechanism of electroweak symmetry breaking and specifically models with extended Higgs sectors are studied with high accuracy at the LHC. This requires to calculate higher-order corrections in these models which in turn needs renormalization of their parameters including mixing angles. In this contribution we have presented renormalization schemes for mixing angles in the THDM and HSESM that exhibit symmetry with respect to the mixing degrees of freedom, gauge independence, perturbative stability as well as applicability in the full parameter space. The schemes are defined via genuine on-shell renormalization conditions or symmetry principles, so that their generalization to higher orders is well defined and their application to other extensions of the Standard Model feasible. With the implementation of the various types and variants of renormalization schemes in the programs 2HDECAY, PROPHECY4F 3.0 and HAWK 3.0, significant steps towards precision Higgs analysis in the THDM and HSESM are made on the theory side.

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