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New Results in Theories with Reduced Couplings

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The renormalization group invariant, to all orders in perturbation theory, relations among parameters consist the basis of the reduction of couplings concept. N=1 supersymmetric Grand Unified Theories can use the above concept, and even be finite at all loops. We review this idea as well as the resulting theories. We construct: (i) an $N = 1$ $SU(5)$ minimal version, (ii) an all-loop finite $N = 1$ $SU(5)$, (iii) a two-loop finite $N = 1$ $SU(3)^3$ and finally (vi) a Minimal Supersymmetric Standard Model reduced version. The SUSY particle spectra predicted by these models are consistent with the LHC non-observation results.

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1. Introduction

The *reduction of couplings* method [1–4] (see also [5–7]) seems a most promising method which relates independent, in a first view, parameters to a single, “primary” coupling. The method requires the original theory where is applied to be a renormalizable one, and the resulting relation among the parameters to be valid in all energy scales, i.e. are Renormalization Group Invariant (RGI).

The next (natural) step, after the introduction of a novel symmetry through a Grand Unified Theory (GUT) [8–13]), in order to achieve reduction of free parameters of the SM is the relation of the gauge to Yukawa sector (Gauge Yukawa Unification, GUY). This is the central characteristic of the *reduction of couplings* approach. According to that approach, being in a GUT environment, RGI relations are traced between the unification scale and the Planck one. One-loop uniqueness can guarantee all-loop validity of those relations. Moreover, RGI relations can be found which guarantee all order finiteness of a theory. The method has predicted the top quark mass in the finite $N = 1$ $SU(5)$ model [15,16] as well as in the minimal $N = 1$ $SU(5)$ one [14] before its experimental verification [17].

Since SuperSymmetry (SUSY) seems an essential ingredient for the *reduction of couplings* method, the extension to dimension-1 and -2 couplings, or in other words to the supersymmetry breaking sector (SSB), is unavoidable. The supergraph method and the spurion superfield technique played an important role for the progress in that sector, leading to all-loop finite models predicting the SUSY mass spectrum. The all-loop finite $N = 1$ $SU(5)$ model [18] has given a prediction for the Higgs mass compatible with the experimental results [19–22] and a consistent (with the experimental non observation) heavy SUSY mass spectrum. Application of the method to the MMSM, gives consistent results for the Higgs, bottom, top masses and a heavy SUSY spectrum. The (new) `FeynHiggs` code [23–26] has been used for the calculation of the lightest Higgs. Special reference is made to the sum rule for the soft scalar masses (an RGI all-loop relation), which overcomes several phenomenological complications.

2. Theoretical Basis

Let us present the core idea of the *reduction of couplings* method. The target is to single out a basic parameter (the one we shall call the primary one), where all other parameters can be expressed in terms of this one through RGI relations. Such a relation has, in general, the form $\Phi(g_1,\cdots,g_A) =$ const. which should satisfy the following partial differential equation (PDE)

$$\mu\,\frac{d\Phi}{d\mu} = \vec{\nabla}\Phi\cdot\vec{\beta} \;=\; \sum_{a=1}^{A}\beta_a\,\frac{\partial\Phi}{\partial g_a} \;=\; 0\,, \tag{2.1}$$

where β_a is the β-functions of g_a. The above PDE is equivalent to the following set of ordinary differential equations (ODEs), which are called Reduction Equations (REs) [2–4],

$$\beta_g\,\frac{dg_a}{dg} = \beta_a\;,\; a = 1,\cdots,A-1\;, \tag{2.2}$$

where now g is the primary coupling with its corresponding β-function. There are obviously $A-1$ relations in the form of $\Phi(g_1,\cdots,g_A)$ in order to express all other couplings in term of the primary one.

The crucial demand is that the above REs admit power series solutions

$$g_a = \sum_n \rho_a^{(n)} g^{2n+1} , \tag{2.3}$$

which preserve perturbative renormalizability. Without this requirement, we just trade each "dependent" coupling with an integration constant. The power series, which are a set of special solutions, fix that constant. It is very important to point out that the uniqueness of such a solution can be already decided at the one-loop level [2–4]. In supersymmetric theories, where the asymptotic behaviour of several parameters are similar, the use of power series as solutions of the REs are justified. But, usually, the reduction is not "complete", which means that not all of the couplings can be reduced in favor of the primary one, leading to the so called "partial reduction" [27, 28].

We proceed to the reduction scheme for massive parameters, which is far of being straightforward. A number of conditions is required (see for example [29]). Nevertheless, progress has been achieved, starting from [30], and finally we can introduce mass parameters and couplings carrying mass dimension [31, 32] in the same way as dimensionless couplings.

Concider the superpotential

$$W = \frac{1}{2}\mu^{ij}\Phi_i\Phi_j + \frac{1}{6}C^{ijk}\Phi_i\Phi_j\Phi_k , \tag{2.4}$$

and the SSB sector Lagrangian

$$-\mathscr{L}_{\rm SSB} = \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}(m^2)^j_i\phi^{*i}\phi_j + \frac{1}{2}M\lambda_i\lambda_i + \text{h.c.}, \tag{2.5}$$

where ϕ_i's are the scalar fields of the corresponding superfields Φ_i's and M is the gaugino mass which is described by λ.

Let us write down some well known relations:
(i) The β-function of the gauge coupling at one-loop level is given by [33–37]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i T(R_i) - 3C_2(G)\right] . \tag{2.6}$$

(ii) The anomalous dimension $\gamma^{(1)}{}^i_j$, at a one-loop level, of a chiral superfield is

$$\gamma^{(1)}{}^i_j = \frac{1}{32\pi^2}\left[C^{ikl}C_{jkl} - 2g^2C_2(R_i)\delta^i_j\right] . \tag{2.7}$$

(iii) The β-functions of C_{ijk}'s, at one-loop level, following the $N = 1$ non-renormalization theorem [38–40], are expressed in terms of the anomalous dimensions of the fields involved

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma^l_k + C_{ikl}\gamma^l_j + C_{jkl}\gamma^l_i . \tag{2.8}$$

We proceed by assuming that the REs admit power series solutions:

$$C^{ijk} = g\sum_{n=0}\rho^{ijk}_{(n)}g^{2n} . \tag{2.9}$$

Trying to obtain all-loop results we turn to relations among β-functions. The spurion technique [40–44] gives all-loop relations among SSB β-functions [45–51]. Then, assuming that the reduction of C^{ijk} is possible to all orders

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} , \tag{2.10}$$

as well as for h^{ijk}

$$h^{ijk} = -M\frac{dC(g)^{ijk}}{d\ln g} , \tag{2.11}$$

then it can be proven [52, 53] that the following relations are all-loop RGI

$$M = M_0\, \frac{\beta_g}{g}, \tag{2.12}$$

$$h^{ijk} = -M_0\, \beta_C^{ijk}, \tag{2.13}$$

$$b^{ij} = -M_0\, \beta_\mu^{ij}, \tag{2.14}$$

$$(m^2)^i_j = \frac{1}{2}\, |M_0|^2\, \mu\frac{d\gamma^i{}_j}{d\mu}, \tag{2.15}$$

where M_0 is an arbitrary reference mass scale to be specified (note that in both assumptions we do not rely on specific solutions of these equations).
As a next step we substitute the last equation, Eq.(2.15), by a more general RGI sum rule that holds to all orders [54]

$$\begin{aligned} m_i^2+m_j^2+m_k^2 = |M|^2 &\left\{ \frac{1}{1-g^2C_2(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2} \right\} \\ &+\sum_l \frac{m_l^2 T(R_l)}{C_2(G)-8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} , \end{aligned} \tag{2.16}$$

which leads to the following one-loop relation

$$m_i^2+m_j^2+m_k^2 = |M|^2 \,. \tag{2.17}$$

Finally, note that in the case of product gauge groups, Eq.(2.12) takes the form

$$M_i = \frac{\beta_{g_i}}{g_i} M_0 \,, \tag{2.18}$$

where i denotes the group of the product. This will be used in the Reduced MSSM case.

3. Finiteness in N = 1 Supersymmetric Gauge Theories

Consider an $N = 1$ globally supersymmetric gauge theory, which is chiral and anomaly free, where G is the gauge group and g the associated gauge coupling. The theory has the superpotential of Eq.(2.4), while the one-loop gauge and C_{ijk}s β-function are given by Eq.(2.6) and Eq.(2.8) respectively and the one-loop anomalous dimensions of the chiral superfields by Eq.(2.7).
Demanding the vanishing of all one-loop β-functions, Eqs.(2.6,2.7) lead to the relations

$$\sum_i T(R_i) = 3C_2(G)\,, \tag{3.1}$$

$$C^{ikl}C_{jkl} = 2\delta^i_j g^2 C_2(R_i)\,. \tag{3.2}$$

The finiteness conditions for an $N=1$ supersymmetric theory with $SU(N)$ associated group is found in [55] while discussion of the no-charge renormalization and anomaly free requiremnts can be found in [56]. It should be noted that conditions (3.1) and (3.2) are necessary and sufficient to ensure finiteness at the two-loop level [33–37].

The requirement of finiteness, at the one-loop level, in softly broken SUSY theories demands additional constraints among the soft terms of the SSB sector [57], while, once more, these one-loop requirements assure two-loop finiteness, too [58]. These conditions impose restrictions on the irreducible representations R_i of the gauge group G as well as on the Yukawa couplings. For example, since $U(1)$s are not compatible with condition (3.1), the MSSM is excluded. Therefore, a GUT is initially required with the MSSM being its low energy theory. Also, since condition (3.2) forbids the appearance of gauge singlets ($C_2(1)=0$), F-type spontaneous symmetry breaking [59] are not compatible with finiteness. Finally, D-type spontaneous breaking [60] is also incompatible since it requires a $U(1)$ group.

The non trivial point is that the relations among couplings (gauge and Yukawa) which are imposed by the conditions (3.1) and (3.2) should hold at any energy scale. The necessary and sufficient condition is to require that such relations are solutions to the REs (see Eq. (2.10))

$$\beta_g \frac{dC_{ijk}}{dg} = \beta_{ijk} \tag{3.3}$$

holding at all orders. We note, once more, that the existence of one-loop level power series solution guarantees the all-order series.

There exist the following theorem [61,62] which points down which are the the necessary and sufficient conditions in order for an $N=1$ SUSY theory to be all-loop finite. In refs [61–67] it was shown that for an $N=1$ SUSY Yang-Mills theory, based on a simple gauge group, if the following four conditions are fulfilled:

(i) No gauge anomaly is present.

(ii) The β-function of the gauge coupling is zero at one-loop level

$$\beta_g^{(1)} = 0 = \sum_i T(R_i) - 3\,C_2(G). \tag{3.4}$$

(iii) The condition of vanishing for the one-loop anomalous dimensions of matter fields,

$$\gamma^{(1)\,i}{}_j \;=\; 0 = \frac{1}{32\pi^2}\,[\,C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R)\delta^i_j], \tag{3.5}$$

admits solution of the form

$$C_{ijk} = \rho_{ijk} g, \qquad \rho_{ijk} \in \mathbb{C}\ . \tag{3.6}$$

(iv) When considered as solutions of vanishing Yukawa β-functions (at one-loop order), i.e. $\beta_{ijk} = 0$, the above solutions are isolated and non-degenerate.

Then, each of the solutions in Eq.(3.6) can be extended uniquely to a formal power series in g, and the associated super Yang-Mills models depend on the single coupling constant g with a vanishing, at all orders, β-function.

While the validity of the above cannot be extended to non-SUSY theories, it should be noted that reduction of couplings and finiteness are intimately related.

4. The Reduction of Coupling Method in Phenomenological Models

In this section we apply the method to four interesting phenomenological models, namely (i) the Minimal $N=1$ Supersymmetric $SU(5)$, (ii) the Finite $N=1$ Supersymmetric $SU(5)$, (iii) the Finite $SU(N)^3$ and (iv) the MSSM. Discussion on the predictions for quark masses, the light Higgs boson mass, the SUSY breaking scale (defined as the geometric mean of stops), and the full supersymmetric spectra are discussed in Sect. 6.

4.1 The Minimal $N=1$ Supersymmetric $SU(5)$ Model

We start with the partial reduction of the $N=1$ SUSY $SU(5)$ model [14, 30]. Our notation is as follows: $\Psi^I(\mathbf{10})$ and $\Phi^I(\bar{\mathbf{5}})$ refer to the three generations of lepton and quarks ($I=1,2,3$), the adjoint $\Sigma(\mathbf{24})$ breaks $SU(5)$ to $SU(3)_{\rm C}\times SU(2)_{\rm L}\times U(1)_{\rm Y}$ and $\overline{H}(\bar{\mathbf{5}})$ represent the two Higgs superfields for the electroweak symmetry breaking (ESB) [68, 69]. The choice of using only one set of $(\mathbf{5}+\bar{\mathbf{5}})$ for the ESB renders the model asymptoticaly free (i.e. $\beta_g<0$). The superpotential of the model is described by

$$W=\frac{g_t}{4}\,\varepsilon^{\alpha\beta\gamma\delta\tau}\Psi^{(3)}_{\alpha\beta}\Psi^{(3)}_{\gamma\delta}H_\tau+\sqrt{2}g_b\,\Phi^{(3)\alpha}\Psi^{(3)}_{\alpha\beta}\overline{H}^\beta+\frac{g_\lambda}{3}\,\Sigma^\beta_\alpha\Sigma^\gamma_\beta\Sigma^\alpha_\gamma+g_f\,\overline{H}^\alpha\Sigma^\beta_\alpha H_\beta$$
$$+\frac{\mu_\Sigma}{2}\,\Sigma^\gamma_\alpha\Sigma^\alpha_\gamma+\mu_H\,\overline{H}^\alpha H_\alpha\ . \tag{4.1}$$

where only the third generation Yukawa couplings are taken into account. The indices $\alpha,\beta,\gamma,\delta,\tau$ are $SU(5)$ ones. A detailed presentation of the model can be found in [70] and in [71, 72].

Our primary coupling is the gauge one g. In this model the gauge-Yukawa unification can be achieved through two sets of solutions which are asympotically free [70]:

$$a: g_t=\sqrt{\frac{2533}{2605}}g+\mathscr{O}(g^3)\ ,\ g_b=\sqrt{\frac{1491}{2605}}g+\mathscr{O}(g^3)\ ,\ g_\lambda=0\ ,\ g_f=\sqrt{\frac{560}{521}}g+\mathscr{O}(g^3)\ ,$$
$$b: g_t=\sqrt{\frac{89}{65}}g+\mathscr{O}(g^3)\ ,\ g_b=\sqrt{\frac{63}{65}}g+\mathscr{O}(g^3)\ ,\ g_\lambda=0\ ,\ g_f=0\ . \tag{4.2}$$

where the higher order terms denote uniquely computable power series in g. Let us note that the reduction of the dimensionless sector is independent of the dimensionful one. These solutions describe the boundaries of a RGI surface in the parameter space which is AF and where g_f and g_λ could be different from zero. Therefore, a partial reduction is possible where g_λ and g_f are independent (non-vanishing) parameters without endangering AF. The proton decay constraints favour solution a, therefore we choose this one for our discussion. [1]

The SSB Lagrangian is

$$-\mathscr{L}_{\rm soft}=m^2_{H_u}\hat{H}^{*\alpha}\hat{H}_\alpha+m^2_{H_d}\hat{\overline{H}}^*_\alpha\hat{\overline{H}}^\alpha+m^2_\Sigma\hat{\Sigma}^{\dagger\ \alpha}_\beta\hat{\Sigma}^\beta_\alpha+\sum_{I=1,2,3}[m^2_{\Phi^I}\hat{\Phi}^{*\ (I)}_\alpha\hat{\Phi}^{(I)\alpha}$$
$$+m^2_{\Psi^I}\hat{\Psi}^{\dagger\ (I)\alpha\beta}\hat{\Psi}^{(I)}_{\beta\alpha}]+\{\frac{1}{2}M\lambda\lambda+B_H\hat{\overline{H}}^\alpha\hat{H}_\alpha+B_\Sigma\hat{\Sigma}^\alpha_\beta\hat{\Sigma}^\beta_\alpha+h_f\hat{\overline{H}}^\alpha\hat{\Sigma}^\beta_\alpha\hat{H}_\beta$$
$$+\frac{h_\lambda}{3}\hat{\Sigma}^\beta_\alpha\hat{\Sigma}^\gamma_\beta\hat{\Sigma}^\alpha_\gamma+\frac{h_t}{4}\,\varepsilon^{\alpha\beta\gamma\delta\tau}\hat{\Psi}^{(3)}_{\alpha\beta}\hat{\Psi}^{(3)}_{\gamma\delta}\hat{H}_\tau+\sqrt{2}h_b\,\hat{\Phi}^{(3)\alpha}\hat{\Psi}^{(3)}_{\alpha\beta}\hat{\overline{H}}^\beta+\text{h.c.}\}\ , \tag{4.3}$$

[1] $g_\lambda=0$ is inconsistent, but $g_\lambda <\sim 0.005$ is necessary in order for the proton decay constraint [73] to be satisfied. A small g_λ is expected not to affect the prediction of unification of SSB parameters.

where the hat denotes the scalar components of the chiral superfields. The parameters M, μ_Σ and μ_H are treated as independent ones, since they cannot be reduced in a suitable form. The lowest-order reduction for the parameters of the SSB Lagrangian are given by:

$$B_H = \frac{1029}{521}\,\mu_H M\ ,\ B_\Sigma = -\frac{3100}{521}\,\mu_\Sigma M\ , \tag{4.4}$$

$$\begin{aligned} &h_t = -g_t M\ ,\ h_b = -g_b M\ ,\ h_f = -g_f M\ ,\ h_\lambda = 0\ ,\\ &m^2_{H_u} = -\frac{569}{521}M^2\ ,\ m^2_{H_d} = -\frac{460}{521}M^2\ ,\ m^2_\Sigma = \frac{1550}{521}M^2\ ,\\ &m^2_{\Phi^3} = \frac{436}{521}M^2\ ,\ m^2_{\Phi^{1,2}} = \frac{8}{5}M^2\ ,\ m^2_{\Psi^3} = \frac{545}{521}M^2\ ,\ m^2_{\Psi^{1,2}} = \frac{12}{5}M^2\ . \end{aligned} \tag{4.5}$$

We choose the gaugino mass M for characterizing the SUSY breaking scale. Finally, we note that (i) B_Σ and B_H are treated as independent parameters without spoiling the one-loop reduction solution of Eq.(4.5) and (ii) the sum rule still holds despite the specific relations among the gaugino mass and the soft scalar masses.

4.2 The Finite $N = 1$ Supersymmetric $SU(5)$ Model

We proceed now to the finite to all-orders $SU(5)$ gauge theory, where the reduction of couplings is restricted to the third generation. This specific Finite Unified Theory (FUT) was in agreement with the experimental constraints at the time [18] and has predicted, almost five years before its discovery, the light Higgs mass in the range of 121–126 GeV. [2] The particle content of the model has three ($\overline{\mathbf{5}}+\mathbf{10}$) supermultiplets for the three generations of leptons and quarks, while the Higgs sector consists of four supermultiplets ($\overline{\mathbf{5}}+\mathbf{5}$) and one **24**. The finite $SU(5)$ group is broken to MSSM, which of course in no longer a finite theory [14–16, 74–76].

In order for this finite to all-orders $SU(5)$ model to achieve Gauge Yukawa Unification (GYU) should have the following characteristics:
(i) The one-loop anomalous dimensions are diagonal i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.
(ii) The fermions of the $\overline{\mathbf{5}}_i$ and $\mathbf{10}_i$ $(i = 1,2,3)$ are not coupled to the **24**.
(iii) The pair of the MSSM Higgs doublets are mostly consisted from the 5 and $\bar{5}$ Higgs and couple to the third generation

The superpotential of the model, with an enhanced symmetry due to the reduction of couplings, is given by [77, 78]:

$$\begin{aligned} W = &\sum_{i=1}^{3}\Big[\,\frac{1}{2}g_i^u\,\mathbf{10}_i\mathbf{10}_i H_i + g_i^d\,\mathbf{10}_i\overline{\mathbf{5}}_i\overline{H}_i\,\Big] + g_{23}^u\,\mathbf{10}_2\mathbf{10}_3 H_4 \\ &+ g_{23}^d\,\mathbf{10}_2\overline{\mathbf{5}}_3\overline{H}_4 + g_{32}^d\,\mathbf{10}_3\overline{\mathbf{5}}_2\overline{H}_4 + g_2^f H_2\,\mathbf{24}\,\overline{H}_2 + g_3^f H_3\,\mathbf{24}\,\overline{H}_3 + \frac{g^\lambda}{3}(\mathbf{24})^3\ . \end{aligned} \tag{4.6}$$

[2] Improved Higgs mass calculations would yield a different interval, still compatible with current experimental data (see below).

Discussion of the model with a more detailed description can be found in [14–16]. The non-degenerate and isolated solutions to the vanishing of $\gamma_i^{(1)}$ are:

$$
\begin{aligned}
&(g_1^u)^2 = \frac{8}{5}\, g^2 \;,\; (g_1^d)^2 = \frac{6}{5}\, g^2 \;,\; (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5}\, g^2 \;,\\
&(g_2^d)^2 = (g_3^d)^2 = \frac{3}{5}\, g^2 \;,\; (g_{23}^u)^2 = \frac{4}{5}\, g^2 \;,\; (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5}\, g^2 \;,\\
&(g^\lambda)^2 = \frac{15}{7} g^2 \;,\; (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2}\, g^2 \;,\; (g_1^f)^2 = 0 \;,\; (g_4^f)^2 = 0 \;.
\end{aligned}
\tag{4.7}
$$

We have also the relation $h = -MC$, while the sum rules lead to:

$$
m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2 \;,\; m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} \;,\; m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3} \;. \tag{4.8}
$$

Therefore, we only have two free parameters, namely $m_{\mathbf{10}}$ and M in the dimensionful sector.

When $SU(5)$ breaks down to the MSSM, a suitable rotation in the Higgs sector [15, 16, 79–82], permits only a pair of Higgs doublets (coupled mostly to the third family) to remain light and acquire vev's. Avoiding fast proton decay is achieved with the usual doublet-triplet splitting, although different from the one applied to the minimal $SU(5)$ due to the extended Higgs sector of the finite model. Therefore, below the GUT scale we get the MSSM where the third generation is given by the finiteness conditions while the first two remain unrestricted.

4.3 The Finite $SU(N)^3$ Model

We proceed now to construct a FUT based on a product gauge group. Concider an $N = 1$ SUSY theory with $SU(N)_1 \times SU(N)_2 \times \cdots \times SU(N)_k$ having n_f families transforming as $(N, N^*, 1, \ldots, 1) + (1, N, N^*, \ldots, 1) + \cdots + (N^*, 1, 1, \ldots, N)$. Then, the first order coefficient of the β-function, for each $SU(N)$ group is:

$$
b = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N \,. \tag{4.9}
$$

Demanding finiteness, i.e. $b = 0$, we are led to the choice $n_f = 3$. Phenomenological reasons lead to the choice of the $SU(3)_C \times SU(3)_L \times SU(3)_R$ model, discussed in Ref. [83], while a detailed discussion of the general well known example can be found in Ref. [84–87]. The leptons and quarks transform as:

$$
q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \qquad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3), \qquad \lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*) \tag{4.10}
$$

where h are down-type quarks acquiring masses close to M_{GUT}. A cyclic Z_3 symmetry is imposed on the multiplets to achieve equal gauge couplings at the GUT scale and in that case the vanishing of the first-order β-function is satisfied. Continuing to the vanishing of the anomalous dimension of all the fields (see Eq.(3.2)), we note that there are two trilinear invariant terms in the superpotential, namely:

$$
f\; Tr(\lambda q^c q) + \frac{1}{6} f' \;\varepsilon_{ijk}\varepsilon_{abc}(\lambda_{ia}\lambda_{jb}\lambda_{kc} + q_{ia}^c q_{jb}^c q_{kc}^c + q_{ia} q_{jb} q_{kc}), \tag{4.11}
$$

with f and f' the corresponding Yukawa couplings. The superfields $(\tilde{N},\tilde{N}^c)$ obtain vev's and provide masses to leptons and quarks

$$m_d = f\langle\tilde{N}\rangle,\ \ m_u = f\langle\tilde{N}^c\rangle,\ \ m_e = f'\langle\tilde{N}\rangle,\ \ m_\nu = f'\langle\tilde{N}^c\rangle. \tag{4.12}$$

Having three families, 11 f couplings and 10 f' couplings are present in the most general superpotential. Demanding the vanishing of all superfield anomalous dimensions, 9 conditions are imposed

$$\sum_{j,k} f_{ijk}(f_{ljk})^* + \frac{2}{3}\sum_{j,k} f'_{ijk}(f'_{ljk})^* = \frac{16}{9}g^2\delta_{il}\,, \tag{4.13}$$

where

$$f_{ijk} = f_{jki} = f_{kij}, \qquad f'_{ijk} = f'_{jki} = f'_{kij} = f'_{ikj} = f'_{kji} = f'_{jik}\,. \tag{4.14}$$

The masses of leptons and quarks are acquired from the vev's of the scalar parts of the superfields $\tilde{N}_{1,2,3}$ and $\tilde{N}^c_{1,2,3}$.

At $M_{\rm GUT}$ the $SU(3)^3$ FUT breaks[3] to MSSM, where as was already mentioned, both Higgs doublets couple mostly to the third generation. The FUT breaking leaves its mark in the form of Eq.(4.13), i.e. boundary conditions on the gauge and Yukawa couplings, the relation $h = -Mf$ and finally the soft scalar mass sum rule at $M_{\rm GUT}$. In this specific model this rule takes the form:

$$m^2_{H_u} + m^2_{\tilde{t}^c} + m^2_{\tilde{q}} = M^2 = m^2_{H_d} + m^2_{\tilde{b}^c} + m^2_{\tilde{q}}\,. \tag{4.15}$$

The model is finite to all-orders if the solution of Eq.(4.13) is both *isolated* and unique. Then, $f' = 0$ and we have the relations

$$f^2 = f^2_{111} = f^2_{222} = f^2_{333} = \frac{16}{9}g^2\,. \tag{4.16}$$

Since all f' vanish, in one-loop order, the lepton masses vanish. Since these masses, even radiatively, cannot be produced because of the finiteness conditions, we are faced with a problem which needs further study. If the solution of Eq.(4.13) is unique but not isolated (i.e. parametric), we can have non zero f' leading to non-vanishing lepton masses and at the same time achieving two-loop finiteness. In that case the set of conditions restricting the Yukawa couplings read:

$$f^2 = r\left(\frac{16}{9}\right)g^2\,, \quad f'^2 = (1-r)\left(\frac{8}{3}\right)g^2\,, \tag{4.17}$$

where r parametrises the different solutions and as such is a free parameter. It should be noted that we use the sum rule as boundary condition for the soft scalar masses.

4.4 The Reduced MSSM

We end up with the application of the method of coupling reduction to a version of the MSSM, where a covering GUT is assumed. The original partial reduction can be found in ref. [90,91] where only the third fermionic generation is considered. Following this restriction, the superpotential reads:

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2\,, \tag{4.18}$$

[3] [88, 89] and refs therein discuss in detail the spontaneous breaking of $SU(3)^3$.

where $Y_{t,b,\tau}$ and $h_{t,b,\tau}$ refer only to the third family, and the SSB Lagrangian is given by

$$
\begin{aligned}
-\mathscr{L}_{\mathrm{SSB}} = \sum_{\phi} m_{\phi}^2 \hat{\phi}^* \hat{\phi} + \left[m_3^2 \hat{H}_1 \hat{H}_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + \mathrm{h.c} \right] \\
+ \left[h_t \hat{H}_2 \hat{Q} \hat{t}^c + h_b \hat{H}_1 \hat{Q} \hat{b}^c + h_\tau \hat{H}_1 \hat{L} \hat{\tau}^c + \mathrm{h.c.} \right],
\end{aligned}
\tag{4.19}
$$

We start with the dimensionless sector and consider initially the top and bottom Yuakwa couplings and the strong gauge one. The rest of the couplings will be treated as corrections. Their running behaviour, as corrections to the strong coupling, is understood [27]. If $Y^2_{(t,b)}/(4\pi) \equiv \alpha_{(t,b)}$, the REs and the Yukawa RGEs give

$$\alpha_i = G_i^2 \alpha_3, \quad \text{where} \quad G_i^2 = \frac{1}{3}, \qquad i = t, b.$$

If the tau Yukawa is included in the reduction, the corresponding G^2 coefficient for tau turns negative [92], explaining why this coupling is treated also as a correction.

We assume that the ratios of the top and bottom Yukawa to the strong coupling are constant on the GUT scale, i.e. they have negligible scale dependence,

$$\frac{d}{dg_3}\left(\frac{Y^2_{t,b}}{g_3^2}\right) = 0,$$

Then, including the corrections from the $SU(2)$, $U(1)$ and tau couplings, at the GUT scale, the coefficients $G^2_{t,b}$ become:

$$G_t^2 = \frac{1}{3} + \frac{71}{525}\rho_1 + \frac{3}{7}\rho_2 + \frac{1}{35}\rho_\tau, \qquad G_b^2 = \frac{1}{3} + \frac{29}{525}\rho_1 + \frac{3}{7}\rho_2 - \frac{6}{35}\rho_\tau \tag{4.20}$$

where

$$\rho_{1,2} = \frac{g^2_{1,2}}{g_3^2} = \frac{\alpha_{1,2}}{\alpha_3}, \qquad \rho_\tau = \frac{g_\tau^2}{g_3^2} = \frac{\frac{Y_\tau^2}{4\pi}}{\alpha_3} \tag{4.21}$$

We shall treat Eqs.(4.20) as boundary conditions at the GUT scale.

Going to the two-loop level, we assume that the corrections take the following form:

$$\alpha_i = G_i^2 \alpha_3 + J_i^2 \alpha_3^2, \qquad i = t, b\ .$$

Then, the two-loop coefficients, J_i, including the corrections from the gauge and the tau Yukawa couplings, are:

$$J_t^2 = \frac{1}{4\pi}\frac{N_t}{D}, \quad J_b^2 = \frac{1}{4\pi}\frac{N_b}{5D},$$

where D, N_t and N_b are known quantities which can be found in ref. [93].

Proceeding to the the SSB Lagrangian, Eq.(4.19), and the dimension-one parameters, i.e the trilinear couplings $h_{t,b,\tau}$, we first reduce $h_{t,b}$ and we get

$$h_i = c_i Y_i M_3 = c_i G_i M_3 g_3, \quad \text{where} \quad c_i = -1 \qquad i = t, b,$$

and M_3 is the gluino mass. Adding the corrections from the gauge and the tau couplings we have

$$c_t = -\frac{A_A A_{bb} + A_{tb} B_B}{A_{bt} A_{tb} - A_{bb} A_{tt}}, \qquad c_b = -\frac{A_A A_{bt} + A_{tt} B_B}{A_{bt} A_{tb} - A_{bb} A_{tt}}.$$

Again, A_{tt}, A_{bb} and A_{tb} can be found in ref. [93].

We end up with the soft scalar masses m^2_ϕ of the SSB Lagrangian. Assuming the relations $m^2_i = c_i M^2_3$ ($i = Q, u, d, H_u, H_d$), and adding the corrections from the gauge, the tau couplings and h_τ, we get

$$c_Q = -\frac{c_{Q\mathrm{Num}}}{D_m}, \quad c_u = -\frac{1}{3}\frac{c_{u\mathrm{Num}}}{D_m}, \quad c_d = -\frac{c_{d\mathrm{Num}}}{D_m}, \quad c_{H_u} = -\frac{2}{3}\frac{c_{Hu\mathrm{Num}}}{D_m}, \quad c_{H_d} = -\frac{c_{Hd\mathrm{Num}}}{D_m},$$

where D_m, $c_{Q\mathrm{Num}}$, $c_{u\mathrm{Num}}$, $c_{d\mathrm{Num}}$, $c_{Hu\mathrm{Num}}$, $c_{Hd\mathrm{Num}}$ and the complete analysis are again given in ref. [93].

If only the reduced system were used, i.e. the strong, top and bottom Yukawa couplings as well as the h_t and h_b, the coefficients turn to be

$$c_Q = c_u = c_d = \frac{2}{3}, \quad c_{H_u} = c_{H_d} = -1/3,$$

which clearly obey the sum rules

$$\frac{m^2_Q + m^2_u + m^2_{H_u}}{M^2_3} = c_Q + c_u + c_{H_u} = 1, \qquad \frac{m^2_Q + m^2_d + m^2_{H_d}}{M^2_3} = c_Q + c_d + c_{H_d} = 1.$$

We finish this section with an essential point for the gaugino masses. The application of the Hisano-Shiftman relation, Eq.(2.12), is made for each gaugino mass as a boundary condition with unified gauge coupling at M_{GUT} scale. Then, at one-loop level, the gaugino mass depends on the one-loop coefficient of the corresponding β-function and an arbitrary M_0, $\;M_i = b_i M_0$. This fact permits, with a suitable choice of M_0, to have the gluino mass equal to the unified gaugino mass, while the gauginos masses of the other two gauge groups are given by the gluino mass multiplied by the ratio of the appropriate one-loop β coefficient.

5. Phenomenological Constraints

In this section we shall briefly review several experimental constraints that were applied to our phenomenological analysis. The used values do not correspond to the latest experimental results. However, this has a negligible impact on our analysis.

In our models we evaluate the pole mass of the top quark while the bottom quark mass is evaluated at the M_Z scale (to avoid uncertainties to its pole mass). The experimental values, taken from ref. [94] are:

$$m_t^{\mathrm{exp}} = (173.1 \pm 0.9)\ \mathrm{GeV} \quad , \quad m_b(M_Z) = 2.83 \pm 0.10\ \mathrm{GeV}\ . \tag{5.1}$$

We interpret the Higgs-like particle discovered in July 2012 by ATLAS and CMS [19, 95] as the light $\mathscr{CP}$-even Higgs boson of the MSSM [96–98]. The (SM) Higgs boson experimental average mass is [94] [4]

$$M_H^{\mathrm{exp}} = 125.10 \pm 0.14\ \mathrm{GeV}\ . \tag{5.2}$$

[4] This is the latest available LHC combination. More recent measurements confirm this value.

The theoretical uncertainty [23,24], however, for the prediction of M_h in the MSSM dominates the total uncertainty, since it is much larger than the experimental one. In our follwing analyses we shall use the new `FeynHiggs` code [23–25] (Version 2.16.0) to predict the Higgs mass. `FeynHiggs` evaluates the Higgs masses based on a combination of fixed order diagrammatic calculations and resummation of the (sub)leading logarithmic contributions at all orders. This provides a reliable M_h even for a large SUSY scale. This new version gives a downward shift on the Higgs mass M_h of the order of $\mathcal{O}(2\text{ GeV})$ for large SUSY masses, while computes the Higgs mass uncertainty point by point. The theoretical uncertainty calculated is added linearly to the experimental error in Eq.(5.2).

We also consider the following four flavour observables where SUSY has non-negligible impact. For the branching ratio $\mathrm{BR}(b \to s\gamma)$ we take a value from [99, 100], while for the branching ratio $\mathrm{BR}(B_s \to \mu^+\mu^-)$ we use a combination of [101–105]:

$$\frac{\mathrm{BR}(b \to s\gamma)^{\mathrm{exp}}}{\mathrm{BR}(b \to s\gamma)^{\mathrm{SM}}} = 1.089 \pm 0.27 \quad , \quad \mathrm{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9} \ . \tag{5.3}$$

For the B_u decay to $\tau\nu$ we use [100, 106, 107] and forΔM_{B_s} we use [108, 109]:

$$\frac{\mathrm{BR}(B_u \to \tau\nu)^{\mathrm{exp}}}{\mathrm{BR}(B_u \to \tau\nu)^{\mathrm{SM}}} = 1.39 \pm 0.69 \quad , \quad \frac{\Delta M_{B_s}^{\mathrm{exp}}}{\Delta M_{B_s}^{\mathrm{SM}}} = 0.97 \pm 0.2 \ . \tag{5.4}$$

6. Numerical Analysis

In this section we review the results of the phenomenological analysis of the models reviewed above. The full analysis can be found in our recent work [110], where it was shown - among others - that for the Minimal $SU(5)$ model the bottom mass is in agreement with the experimental results only at the 4σ level. Furthermore, no model fulfills the current Cold Dark Matter (CDM) bounds, since the relic abundance is either too high (in all three GUTs) or too low (Reduced MSSM). Alternative ways are proposed for the two $SU(5)$ models in [110], while a similar R-parity violation should be considered for the other two as well.

6.1 Finite $N = 1$ $SU(5)$

As discussed in Subsection 4.2, we present the full particle spectrum predicted in the Finite $N = 1$ supersymmetric $SU(5)$ model. Conditions set by finiteness do not restrict the renormalization properties at low energies, so we are left with boundary conditions on the gauge and Yukawa couplings (4.7), the $h = -MC$ relation and the soft scalar-mass sum rule at M_{GUT}. In Fig. 1, $m_b(M_Z)$ and m_t are shown as functions of the unified gaugino mass M. The orange (blue) lines denote the 2σ (3σ) experimental uncertainties. The only phenomenologically viable option is to consider $\mu < 0$ (as shown in [110–117].

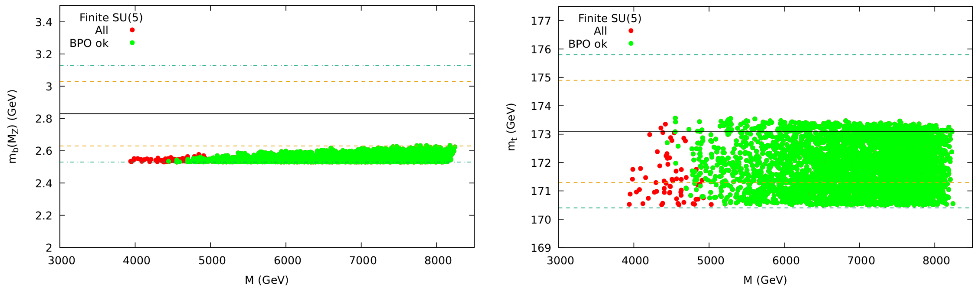

Figure 1: $m_b(M_Z)$ (left) and m_t (right) as a function of M for the Finite $N = 1$ $SU(5)$. Green points satisfy the B-physics constraints.

The light Higgs boson mass is given in Fig. 2 (left), while its theory uncertainty [26] is given in Fig. 2 (right). This point-by-point uncertainty (calculated with `FeynHiggs`) drops significantly (wrt past analyses) to $0.65 - 0.70$ GeV.

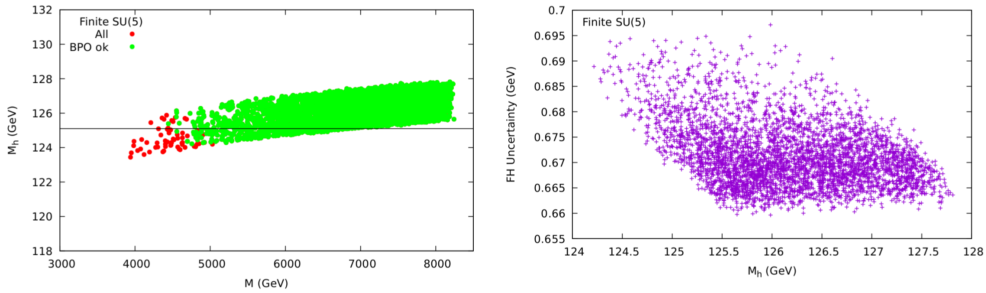

Figure 2: *Left: M_h as a function of M. Green points comply with B-physics constraints. Right: The lightest Higgs mass theoretical uncertainty calculated with* `FeynHiggs` *2.16.0 [26].*

	M_h	M_H	M_A	$M_{H^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{g}}$
lightest	124.4	5513	5513	5510	5940	6617	5888	6617	8819
heaviest	125.8	28121	28121	28120	10486	11699	10318	11686	15509
	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$\tan\beta$
lightest	2225	3123	3819	4801	2120	3811	4820	4811	50
heaviest	4215	5788	7108	8200	4019	7108	8227	8227	51

Table 1: *Spectra of the Finite $N = 1$ $SU(5)$. Masses are in GeV and rounded to 1 (0.1) GeV (for the light Higgs mass).*

In Tab. 1 we give the lightest and the heaviest spectrum. Compared to our previous analyses [110–117,120–122], the improved evaluation of M_h and its uncertainty prefers a heavier (Higgs) spectrum and thus allows only a heavy supersymmetric spectrum. Very heavy coloured SUSY particles are favoured (nearly independent of the M_h uncertainty), in agreement with the non-observation of those particles at the LHC [118]. Overall, the allowed coloured SUSY masses would remain unobservable at the (HL-)LHC, the ILC or CLIC. However, the coloured spectrum would be accessible at the FCC-hh [119], as could the full heavy Higgs boson spectrum.

The model has a high relic abundance for CDM. The CDM alternatives proposed for the Minimal $SU(5)$ model in [110] can also be applied here.

6.2 Two-Loop Finite $N = 1$ $SU(3) \otimes SU(3) \otimes SU(3)$

We continue our analysis with the two-loop finite $N = 1$ supersymmetric $SU(3) \otimes SU(3) \otimes SU(3)$ model, where again below $M_{\rm GUT}$ we get the MSSM. We take into account two new thresholds for the masses of the new particles at $\sim 10^{13}$ GeV and $\sim 10^{14}$ GeV resulting in a wider phenomenologically viable parameter space [123].

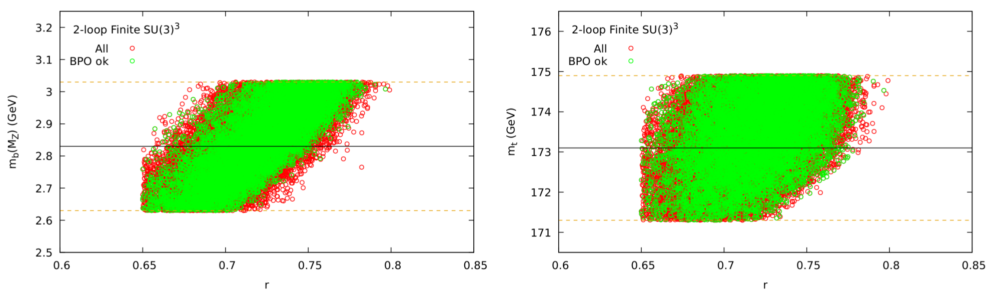

Figure 3: Bottom and top quark masses for the Finite $N = 1$ $SU(3) \otimes SU(3) \otimes SU(3)$ model, with $\mu < 0$, as functions of r. The black horizontal line is the experimental central value, and the dashed orange ones are the 2σ limits. Green points satisfy B-physics constraints.

Looking for the values of the parameter r (see Subsection 4.3) which comply with the experimental limits, we find (see Fig. 3) that both masses are in the experimental range for the same value of r between 0.65 and 0.80 (we singled out the $\mu < 0$ case as the most promising). The inclusion of the abovementioned thresholds gives an important improvement on the top mass from past versions of the model [83, 124–126].

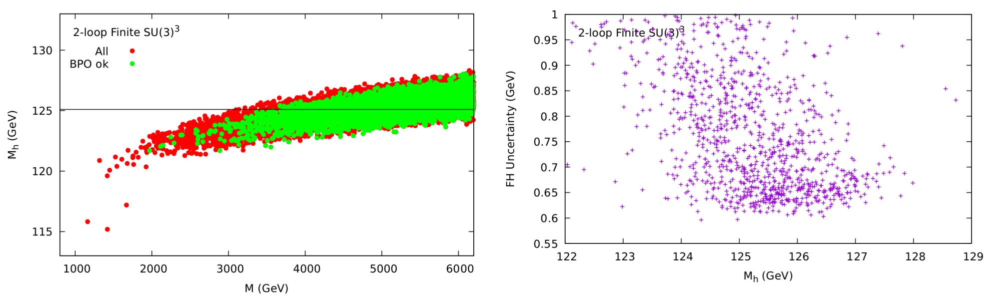

Figure 4: *Left: M_h as a function of M for the Finite $N = 1$ $SU(3) \otimes SU(3) \otimes SU(3)$. Right: The Higgs mass theoretical uncertainty [26].*

In Fig. 4 (left) we show the light Higgs boson mass, while the point-by-point calculated theoretical uncertainty is presented in Fig. 4 (right). Tab. 2 gives the lightest and heaviest spectrum. All constraints regarding quark masses, the light Higgs boson mass and B-physics are satisfied,

	M_h	M_H	M_A	$M_{H^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{g}}$
lightest	124.2	1918	1918	1917	4703	5480	4671	6013	6329
heaviest	125.9	12053	12053	12050	10426	10631	10426	11193	14550
	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$\tan\beta$
lightest	1774	2694	2736	5469	1517	2736	5480	5481	44
heaviest	5999	7113	6713	10522	3767	6703	10522	10523	53

Table 2: *Spectra of the Finite $N = 1$ $SU(3)\otimes SU(3)\otimes SU(3)$. Masses are in GeV and rounded to 1 (0.1) GeV (for the light Higgs mass).*

rendering making the model very successful. The accessibility of the heavier (coloured) spectrum will be subject to future colliders.

6.3 Reduced MSSM

For the analysis of our final model we choose the unification scale to apply the corrections to all these RGI relations. The full discussion on the selection of the free parameters of the model can be found in [110]. In total, we vary ρ_τ, ρ_{h_τ}, M and μ.

In Fig. 5 we see the model's predictions for the bottom and top mass respectively. The horizontal lines denote the 2σ level uncertainty. Next, M_h is shown in Fig. 6 (left),while the theory uncertainty given in Fig. 6 (right) has dropped below 1 GeV. The Higgs mass predicted by the model lies within the LHC range.

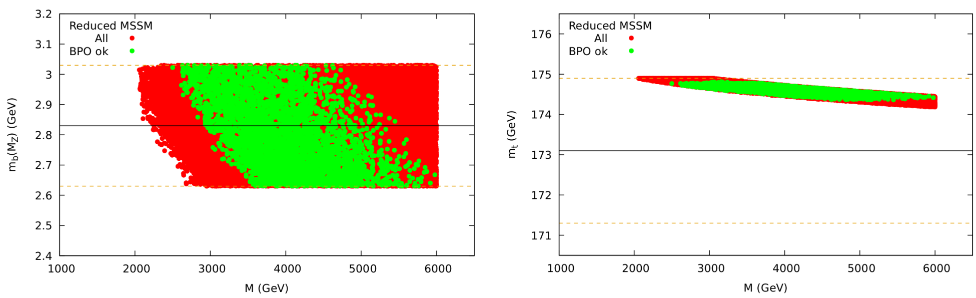

Figure 5: *The left (right) plot shows the bottom (top) quark mass for the Reduced MSSM.*

The M_h limits set a limit on the low-energy supersymmetric masses, rendering the Reduced MSSM highly predictive and testable. In Tab. 3 we show the lightest and heaviest value of each parameter of the supersymmetric spectrum. The HL-LHC [127] will be able to test the full Higgs spectrum. The lighter supersymmetric particles, which are given by the electroweak spectrum, will mostly remain unobservable at the LHC and at future e^+e^- colliders such as the ILC or CLIC. An exception are the lightest neutralino and chargino masses, which could be covered by CLIC3TeV. The coloured mass spectrum will remain unobservable at the (HL-)LHC, but could be accessible at the FCC-hh [119].

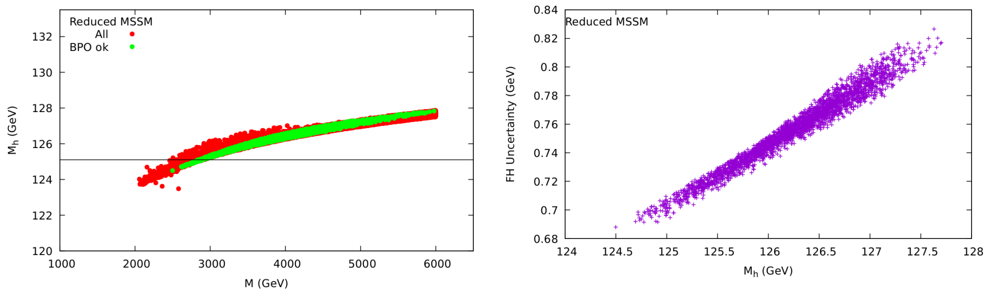

Figure 6: *Left: The lightest Higgs boson mass, M_h in the Reduced MSSM. The green points is the full model prediction. Right: the lightest Higgs mass theoretical uncertainty [26].*

	M_h	M_H	M_A	$M_{H^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{g}}$
lightest	124.5	1305	1305	1297	3851	4029	3699	4007	5126
heaviest	125.8	1801	1801	1780	5275	5564	5076	5502	7017
	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$\tan\beta$
lightest	1705	2536	843	1875	711	2579	3516	3517	40
heaviest	4288	6008	1004	2195	1001	3666	4814	4815	45

Table 3: *Spectra of the Reduced MSSM. All masses are in GeV and rounded to 1 (0.1) GeV (for the light Higgs mass).*

7. Conclusions

We briefly reviewed the ideas concerning the reduction of couplings, then new results were given for three specific models, namely the Finite $N = 1$ $SU(5)$, the Two-Loop Finite $N = 1$ $SU(3) \otimes SU(3) \otimes SU(3)$ and the Reduced MSSM. The three models are in natural agreement with all LHC measurements and searches and they predict relatively heavy spectra which evade detection in present and near-future colliders. An exception is the lighter part of the Reduced MSSM spectrum which could be covered by CLIC3TeV. FCC-hh will be able to test the predicted parameter spaces of all models.

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