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Spectrum of anomalous dimensions in hypercubic theories

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Models invariant under the hypercubic symmetry group H_N have interesting applications in describing the physical world, the most important being in the physics of magnets with a cubic structure. In this report, we present the explicit construction of the operator spectrum in hypercubic theories and the results for the 1-loop anomalous dimensions of all the H_N composite operators with no derivatives.

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1. Introduction

Without any doubt, conformal field theories (CFTs) are of great importance for the description of nature. They play a fundamental role in the theory of phase transitions and are crucial for our understanding of quantum field theory [1]. Moreover, CFTs are very important in string theory and constitute one side of the celebrated AdS/CFT correspondence [2]. To solve a CFT amounts to compute its *CFT data*, i.e. the scaling dimensions of all primary operators and the full set of three-point function coefficients of the theory.

In this work, we focus on conformal theories invariant under the group of symmetries of an N-dimensional hypercube, namely the hypercubic group H_N. In nature, they appear in the description of critical cubic magnets [3], structural phase transitions in crystals [4], and of the randomly dilute Ising model [5]. In molecular physics, the hypercubic group appears in the study of non-rigid water clusters and non-rigid molecules [6].

For these reasons, hypercubic models have been investigated for a long time and present-day results include the computation of beta function and critical exponents up to six loops order in the ε-expansion [7, 8, 9]. Recently, this theory has been also explored non-perturbatively via the conformal bootstrap method [10, 11, 12, 13, 14]. Despite these impressive investigations, before the present work there was little knowledge about the full operator spectrum of the theory. Our main goal is, therefore, to find the operator content in hypercubic theories. We focus on composite operators with an arbitrary number of fields n but no derivatives (no spin). As an application, we compute the spectrum of 1-loop anomalous dimensions for such operators. This is done by making use of a recently-developed method [15, 16] which combines the equation of motion (EOM) with the powerful constraints of conformal symmetry. This report is based on the work in [17].

2. $O(N)$ vector model with cubic anisotropy

The theory we investigate is the $O(N)$ vector model with cubic anisotropy in $d = 4-\varepsilon$, described below

$$S = \int D^d x \left(\frac{(\partial \phi_i)^2}{2} + \tfrac{1}{4!} V_{ijkl}\phi_i\phi_j\phi_k\phi_l \right) \tag{2.1}$$

where

$$V_{ijkl} = \frac{g_1}{3}(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj} + \delta_{ik}\delta_{jl}) + g_2\delta_{ijkl} \tag{2.2}$$

with

$$\delta_{ijkl} = \begin{cases} 1, & \text{when } \ i=j=k=l \\ 0, & \text{otherwise .} \end{cases} \tag{2.3}$$

The $g_2\delta_{ijkl}$ term explicitly breaks the $O(N)$ symmetry so that the action is invariant only under the hypercubic symmetry group $H_N \subset O(N)$.

At the 1-loop level, the renormalized model exhibits four fixed points (FPs). The fixed point couplings are determined by the dimension of the space-time and, at the leading order in the ε-

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expansion, they read

$$
\begin{aligned}
&(g_1^G, g_2^G) = (0,0)\,, && (g_1^I, g_2^I) = (4\pi)^2\left(0, \frac{\varepsilon}{3}\right), \\
&(g_1^O, g_2^O) = (4\pi)^2\left(\frac{3\,\varepsilon}{N+8}, 0\right), && (g_1^H, g_2^H) = (4\pi)^2\left(\frac{\varepsilon}{N}, \frac{(N-4)}{3N}\varepsilon\right).
\end{aligned} \tag{2.4}
$$

The first is a Gaussian FP while the second and third are, respectively, an Ising-like FP (describing N decoupled Ising models) and a FP described by an $O(N)$-invariant CFT. The last one, usually called "cubic fixed point", corresponds to a theory invariant under the hypercubic group H_N. It is important to note that there are values of N at which different FPs coincide. This happens for the cubic FP and the $O(N)$ one at $N = N_c \underset{1-loop}{=} 4$ and for the cubic FP and the Ising one at $N \to \infty$ and $N = 2$. Finally, for $N = 1$, the cubic FP reduces to the Gaussian theory. These limits will be used as a check for our results.

The value of N_c is physically of great importance and dictates the infrared physics of cubic magnets for which $N = 3$. In fact, for $N = 3 < N_c$, the latter is controlled by the $O(N)$ FP while, for $N = 3 > N_c$, the cubic critical regime is achieved. At present time, the most precise theoretical prediction for N_c arises from a 6-loops computation and gives $N_c \sim 2.9$ [9]. Being N_c very close to 3, the experimental determination of the universality class of cubic magnets is a non-trivial task which, at the time of writing, remains unsettled.

3. Composite operators in the H_N model

In this section, we present the construction of composite spin-0 operators in the H_N model, which represents our main achievement. Previous work in this direction has been made in [18, 19, 20]. We start by working out the irreducible representations of H_N. To this end, we use that $H_N = S_N \ltimes \mathscr{Z}_2^N$ and we start by constructing the irreps of $S_N \ltimes \mathscr{Z}_2^N$. These are easily obtained by taking the outer tensor products of the two irreducible representations of $\mathscr{Z}_2$ N times. Labelling the latter as [1] and [2], the irreps of $\mathscr{Z}_2^N$ are

$$
[2]^{\otimes\alpha} \otimes [1^2]^{\otimes\beta}\,, \qquad \alpha + \beta = N\,. \tag{3.1}
$$

The next step is to take the wreath product $S_N \ltimes \mathscr{Z}_2^N$. We, therefore, split the symmetric group S_N into direct products $S_\alpha \times S_\beta$ and we build the irreps of H_N by multiplying these products with the corresponding irreps of $\mathscr{Z}_2^N$ in Eq.(3.1) [21].

From this construction is now clear that we can represent the irreducible representations of H_N as double-partitions of N, (α, β). Pictorially, these are represented by ordered pairs of Young tableau with, respectively, α and β boxes [22, 20]. For instance, the ten irreducible representations of H_3 are

$$
\left([\mathbf{2}]^{\otimes\mathbf{3}} \otimes \mathbf{S_3}\right) : \left(\ydiagram{3}, \emptyset\right), \left(\ydiagram{2,1}, \emptyset\right), \left(\ydiagram{1,1,1}, \emptyset\right); \quad \left([\mathbf{2}]^{\otimes\mathbf{2}} \otimes [\mathbf{1^2}] \otimes \mathbf{S_2} \times \mathbf{S_1}\right) : \left(\ydiagram{2}, \ydiagram{1}\right), \left(\ydiagram{1,1}, \ydiagram{1}\right);
$$

$$
\left([\mathbf{1^2}]^{\otimes\mathbf{3}} \otimes \mathbf{S_3}\right) : \left(\emptyset, \ydiagram{3}\right), \left(\emptyset, \ydiagram{2,1}\right), \left(\emptyset, \ydiagram{1,1,1}\right); \quad \left([\mathbf{2}] \otimes [\mathbf{1^2}]^{\otimes\mathbf{2}} \otimes \mathbf{S_1} \times \mathbf{S_2}\right) : \left(\ydiagram{1}, \ydiagram{2}\right), \left(\ydiagram{1}, \ydiagram{1,1}\right)
$$

where "$\emptyset$" is the empty partition. The left (*right*) tableau represents α (β) objects, even (*odd*) under $\mathscr{Z}_2$ and transforming in an irreducible representation of S_α (S_β).

The dimension of a given irrep (α, β) is [22]

$$dim(\alpha,\beta) = \binom{N}{\alpha} \times dim(\alpha) \times dim(\beta) \tag{3.2}$$

where $dim(\alpha)$ ($dim(\beta)$) is the dimension of the corresponding representation of S_α (S_β) which can be easily obtained via the hook's rule.

The defining N-dimensional representation is

$$\phi_i = (N-1,1) = (\underbrace{\square\square\ldots\square}_{N-1}\,,\ \boxed{i}\,) \tag{3.3}$$

and the decomposition of the tensor product of an arbitrary representation with the defining representation above reads

$$(N-1,1)\otimes(\alpha,\beta) = \sum_{\alpha^+,\,\beta^-}(\alpha^+,\beta^-) \oplus \sum_{\alpha^-,\,\beta^+}(\alpha^-,\beta^+) \tag{3.4}$$

In terms of Young tableaux, α^+ (α^-) are obtained by moving one box from β to α (α to β). β^+ and β^- are obtained in the same way. For example, for $\phi_i \otimes \phi_j$ we have

$$(\underbrace{\square\square\ldots}_{N-1},\square)\otimes(\underbrace{\square\square\ldots}_{N-1},\square) = (\underbrace{\square\square\ldots}_{N},\emptyset)\oplus(\underbrace{\begin{array}{l}\square\square\ldots\\ \square\end{array}}_{N},\emptyset)\oplus(\underbrace{\square\square\ldots}_{N-2},\square\square)\oplus(\underbrace{\square\square\ldots}_{N-2},\begin{array}{l}\square\\ \square\end{array}) \tag{3.5}$$

We can now use these results to construct composite H_N operators with n fields and no derivatives. The first step is to work out the corresponding H_N-irreps by computing n times the tensor product of the defining representation with itself. We divide the resulting bi-tableaux into two families. The first contains the bi-tableaux which do not appear at smaller n and which correspond to a *unique* composite operator of order n. Instead, in the second family fall all the bi-tableaux that already appeared at smaller n. In this case, it is possible to associate more than one operator of order n to the same irreducible representation and it is, therefore, necessary to solve the corresponding mixing problem.

The second step of our construction is to write down the composite operators corresponding to the obtained representations. To this end, we have developed an algorithmic procedure that is rooted in the H_N representation theory presented above.

We start by considering a *unique* composite operator, i.e. stemming from the first family of bi-tableaux, whose corresponding representation has ($\alpha = N,\ \beta = 0$). For instance

$$\left(\begin{array}{l}\square\square\ldots\square\\ \square\square\\ \square\end{array},\emptyset\right)\ .$$

Then we fill the left tableau with indices

$$\left(\begin{array}{l}\boxed{\mu_1}\boxed{\mu_2}\ldots\boxed{\mu_s}\\ \boxed{i}\,\boxed{j}\\ \boxed{k}\end{array},\emptyset\right)$$

and we impose the condition $i \neq j \neq k \neq \mu_1 \neq \mu_2 \neq ... \neq \mu_s$ ...

We then proceed by associating indexed fields raised to the *zeroth* power to the boxes in the first row; indexed fields raised to the *second* power to the boxes in the second row and so on, with increasing *even* powers of the fields for subsequent rows.

Finally, we apply the usual rules for Young tableaux (symmetrization over the boxes in the same row, and antisymmetrization over the column). For our example, this leads to ($n = 8$ in this case)

$$\left(\begin{array}{l} \boxed{\mu_1}\boxed{\mu_2}\cdots\boxed{\mu_s} \\ \boxed{\,i\,}\boxed{\,j\,} \\ \boxed{k} \end{array}, \emptyset\right) = (\phi^4_{[k}\phi^2_i\phi^0_{\mu_1]})\cdot(\phi^2_{[j}\phi^0_{\mu_2]})\cdot\phi^0_{\mu_3}\phi^0_{\mu_4}\dots\phi^0_{\mu_s} =$$
$$= (\phi_k^4\phi_i^2 + \phi_{\mu_1}^4\phi_k^2 + \phi_i^4\phi_{\mu_1}^2 - \phi_k^4\phi_{\mu_1}^2 - \phi_i^4\phi_k^2 - \phi_{\mu_1}^4\phi_i^2)\,(\phi_j^2 - \phi_{\mu_2}^2) \qquad i \neq j \neq k \neq \mu_1 \neq \mu_2$$

The same construction applies also to the right partition β, the crucial difference being that now we need to consider *odd* powers of the fields. This procedure can fail if a bi-tableau appears for the first time at a value of n too small to allow it. This simply means that the corresponding operator requires derivatives to be constructed. For instance, this happens for the last irrep in Eq.(3.5), which appears for the first time at $n = 2$. Consider the most general Young diagram; it will have k different types of columns (with a different number of boxes) that we label with the index i. Each of them can have multiplicity p_i. Thus the *unique* composite operators corresponding to the most general bi-tableau can be compactly written as

$$H_{n,\{m_i\},\{p_i\}} = \prod_{i=1}^{k}(\phi^{m_i}_{[\mu^i_1}\phi^{m_i-2}_{\mu^i_2}\phi^{m_i-4}_{\mu^i_3}\dots\phi^{M}_{\mu^i_{q_i}]})^{p_i} \qquad \mu^i_1 \neq \mu^i_2 \neq \dots \neq \mu^i_{q_i} \tag{3.6}$$

where we labeled the highest power of the field in a given column by m_i and

$$M = \begin{cases} 0 \ \text{ if } \ m_i \ \text{ is } \ \text{even} \\ 1 \ \text{ if } \ m_i \ \text{ is } \ \text{odd} \end{cases} . \tag{3.7}$$

Consider now the second family of bi-tableaux, i.e. those which "reappeared" at the level n. Considering that everything can mix will do it, the corresponding mixing space can be constructed as follows:

1. Write the corresponding *unique* composite operator, stemming from the procedure above, and multiply the result with the power of ϕ^2 needed to reach the level n. For example, for $n = 6$, we obtain
$$(\underbrace{\begin{array}{l}\square\square\square\square\cdots \\ \square\end{array}}_{N}, \emptyset) = (\phi^2)^2(\phi_i^2 - \phi_j^2) \qquad i \neq j\,.$$

2. Then "distribute" these ϕ^2 factors through the rest of the operator in all possible ways as follows
$$(\underbrace{\begin{array}{l}\square\square\square\square\cdots \\ \square\end{array}}_{N}, \emptyset) = \begin{cases} (\phi^2)^2(\phi_i^2 - \phi_j^2) \\ (\phi^2)(\phi_i^4 - \phi_j^4) \\ (\phi_i^6 - \phi_j^6) \end{cases} \qquad i \neq j\,.$$

3. The last step accounts for the mixing between powers of ϕ^2 and the other H_N-singlets with the same classical dimension. The H_N-scalars are formed by products and powers of all the operators of the form

$$\sum_i \phi_i^2 = \phi^2,\ \sum_i \phi_i^4,\ \sum_i \phi_i^6, \dots, \sum_i \phi_i^n \ . \tag{3.8}$$

In our example $(\phi^2)^2$ mixes with $\sum_i \phi_i^4$ and we have to add one extra operator to the mixing space

$$\Big(\underbrace{\ydiagram{4,1}\dots}_{N}\ ,\ \emptyset\ \Big) = \begin{cases} \sum_k \phi_k^4 (\phi_i^2 - \phi_j^2) \\ (\phi^2)^2 (\phi_i^2 - \phi_j^2) \\ (\phi^2)(\phi_i^4 - \phi_j^4) \\ (\phi_i^6 - \phi_j^6) \end{cases} \qquad i \neq j$$

This completes the construction of all the composite H_N operators with no derivatives. As an application, we proceed to the computation of all the corresponding 1-loop anomalous dimensions.

4. Computation

In this section, we sketch the procedure used to compute the scaling dimensions Δ_{S_n} to the leading order in the ε-expansion. To separate the 1-loop contribution, we write the scaling dimensions as

$$\Delta_{S_n} = n(1 - \frac{\varepsilon}{2}) + \gamma_{S_n}\varepsilon + \mathscr{O}(\varepsilon^2) \tag{4.1}$$

where S_n is a composite operator of order n and γ_{S_n} is its 1-loop anomalous dimension.

In order to compute the γs we resorted to a novel method [15, 16] which combines the use of the EOM with the conformal symmetry constraints. The key idea is to use the EOM in order to write the following equation

$$\langle \Box \phi_i S_n S_{n+1} \rangle = \frac{1}{3!} \langle (g_1 \phi_i \phi^2 + g_2 \phi_i^3) S_n S_{n+1} \rangle \tag{4.2}$$

Since conformal symmetry fixes entirely the form of the three-point functions, Eq.(4.2) leads to an eigenvalues equation for γ_{S_n}, which reads

$$\mathscr{D} S_n = \gamma_{S_n} S_n \tag{4.3}$$

where

$$\mathscr{D} = \frac{1}{3N} \left(\frac{\phi^2 \partial^2}{2} + (\phi \cdot \partial)^2 - \phi \cdot \partial + \frac{N-4}{2} \sum_i \phi_i^2 \partial_i^2 \right) \tag{4.4}$$

Notice that for operators which mix, Eq.(4.3) determines uniquely the mixing matrix.

5. Results

In this section, we present our results for the anomalous dimensions in the hypercubic model. In what follows, we will check our results by using the coalescence of cubic and $O(N)$ FPs at $N = N_c = 4$ and cubic and Ising FPs at $N \to \infty$, $N = 2$. Furthermore, interesting properties can be deduced by considering $N = 1$, where we have the free theory. Since we are considering spin 0 operators, the relevant $O(N)$ operators belong to the fully symmetric $O(N)$ space. The latter is composed by singlets and m-index traceless symmetric tensors, i.e by operators of the form $T^{(m)}_{(i_1 \dots i_m)} \phi^{2q}$. Clearly, we have $m + 2q = n$, being n the number of fields in the operator. The 1-loop anomalous dimensions of this family of $O(N)$ operators can be written compactly as [23, 24, 25]

$$\gamma_{m,q} = \frac{m(m-1) + q(N + 6(q+m) - 4)}{N+8} \tag{5.1}$$

In the decoupled Ising model, all these representations altogether simply reduce to the ϕ^n family of operators, for which the 1-loop anomalous dimension reads [15]

$$\gamma_n = \frac{n(n-1)}{6} \,. \tag{5.2}$$

5.1 The hypercubic tower

In full analogy with (5.1) and (5.2), there is an infinite family of H_N composite operators whose 1-loop anomalous dimensions can be written compactly trough a single expression. In fact, by plugging the family of operators defined in Eq.(3.6), $H_{n,\{m_i\},\{p_i\}}$, in the eigenvalue equation (4.3) one obtains

$$\boxed{\gamma_{n,\{m_i\},\{p_i\}} = \frac{1}{6N}\left(2n(n-1) + (N-4)\sum_{i=1}^{k} p_i[m_i(m_i-1) + (m_i-2)(m_i-3) + \ldots]\right)} \tag{5.3}$$

This is one of our main achievements. It is easy to check that for $N = 4$ Eq.(5.3) matches Eq.(5.1) with $q = 0$. This last condition reflects the fact that we are considering operators of order n which are in one-to-one correspondence with the irreducible representations of H_N. We stress that Eq.(5.3) gives the anomalous dimensions for *all* such operators.

We proceed by enlightening Eq.(5.3) with explicit examples. First, we consider the family of bi-tableaux with one row, which corresponds to $k = 1$, $m_i = 1$, and $p_i = p = n$. That is

$$\Big(\underbrace{\square\square\square\square\ldots}_{N-p}\,,\,\underbrace{\square\square\square\square\ldots}_{p}\Big) \tag{5.4}$$

The corresponding operators read

$$\phi_{\mu_1}\phi_{\mu_2}\ldots\phi_{\mu_p} \qquad \mu_1 \neq \mu_2 \neq \ldots \neq \mu_p \,. \tag{5.5}$$

Using Eqs.(3.2) and (5.3), one promptly obtains

$$dim = \binom{N}{n} \qquad\qquad \gamma_n = \frac{n(n-1)}{3N} \,. \tag{5.6}$$

Consider now the family of bi-tableau with two rows, i.e. $k = 3$, $m_i = \{1,2,3\}$ and $p_i = \{p_1, p_2, p_3\}$. Here the first and second rows of the left (*right*) partition have $N - p_1 - p_2 - p_3$ and

p_2 (p_1 *and* p_3) boxes, respectively. The corresponding operators can be written compactly as

$$\underbrace{\phi_{\mu_1}\phi_{\mu_2}...\phi_{\mu_{p_1}}}_{p_1 \text{ terms}}\underbrace{(\phi^2_{\mu_{p_1+1}}-\phi^2_{\mu_{p_1+2}})(\phi^2_{\mu_{p_1+3}}-\phi^2_{\mu_{p_1+4}})...(\phi^2_{\mu_{p_1+2p_2-1}}-\phi^2_{\mu_{p_1+2p_2}})}_{p_2 \text{ terms}}\times$$
$$\times\underbrace{(\phi^3_{\mu_{p_1+2p_2+1}}\phi_{\mu_{p_1+2p_2+2}}-\phi^3_{\mu_{p_1+2p_2+2}}\phi_{\mu_{p_1+2p_2+1}})....(\phi^3_{\mu_{q-1}}\phi_{\mu_q}-\phi^3_{\mu_q}\phi_{\mu_{q-1}})}_{p_3 \text{ terms}}, \quad \mu_1\neq\mu_2\neq...\neq\mu_q \tag{5.7}$$

where

$$q = p_1+2p_2+2p_3 \qquad\qquad n = p_1+2p_2+4p_3\ . \tag{5.8}$$

In this case, Eqs.(3.2) and (5.3) give

$$dim = \binom{N}{p_2}\binom{N-p_2}{2p_3+p_1}\binom{2p_3+p_1}{p_3+p_1}\frac{(N-p_1-2p_3-2p_2+1)(p_1+1)}{(N-p_1-2p_3-p_2+1)(p_1+p_3+1)} \tag{5.9}$$

and

$$\gamma_{n,p_1,p_2} = \frac{4n(n-1)+(N-4)(3n-2p_2-3p_1)}{12N}\ . \tag{5.10}$$

As emerges from the examples above, for every bi-tableau with an arbitrary number of rows, we can use Eq.(5.3) to compute all the 1-loop anomalous dimension of the corresponding *unique* composite operators of order n.

5.2 Explicit results

Aside from Eq.(5.3), the previous sections delineate an algorithmic procedure to compute the anomalous dimension of every composite operator with no derivatives. In this subsection we present explicit results for H_N operators with $2\le n\le 5$ fields. The indices of the operators in this subsection have to be understood to take all different values, e.g. if we write $\phi_i\phi_j$, we imply $i\neq j$.

For the $n=2$ (two fields) case, we have

$$dim(\underbrace{\square\square\square\square...}_{N-2}, \square\square) = \binom{N}{2} \qquad \phi_i\phi_j \qquad \gamma=\tfrac{2}{3N}$$
$$dim(\underbrace{\begin{smallmatrix}\square\square\square\square...\\ \square\end{smallmatrix}}_{N}, \emptyset) = (N-1) \qquad {\phi_i}^2-{\phi_j}^2 \qquad \gamma=\tfrac{N-2}{3N}$$
$$dim(\underbrace{\square\square\square\square...}_{N}, \emptyset) = 1 \qquad \phi^2 \qquad \gamma=\tfrac{2(N-1)}{3N}$$

The results above were already obtained in [14] through the conformal bootstrap approach.

The 1-loop anomalous dimension for operators with $n=3$ fields are

$$dim(\underbrace{\square\square\square\square...}_{N-3}, \square\square\square) = \binom{N}{3} \qquad \phi_i\phi_j\phi_k \qquad \gamma=\tfrac{2}{N}$$
$$dim(\underbrace{\begin{smallmatrix}\square\square\square\square...\\ \square\end{smallmatrix}}_{N-1}, \square) = N(N-2) \qquad \phi_i({\phi_j}^2-{\phi_k}^2) \qquad \gamma=\tfrac{2+N}{3N}$$
$$dim(\underbrace{\square\square\square\square...}_{N-1}, \square) = N \qquad \phi_i\phi^2-2\phi_i^3 \qquad \gamma=\tfrac{2(N-1)}{3N}$$
$$dim(\underbrace{\square\square\square\square...}_{N-1}, \square) = N \qquad \phi_i\phi^2+\tfrac{N-4}{3}\phi_i^3 \qquad \gamma=1$$

We are not aware of any explicit results for $n=3$ in the literature. As a check, we note that for $N=4$ we have the corresponding $O(N)$ results Eq.(5.1) with $q=0$ and $m=3$ for the first three operators and $q=m=1$ for the fourth one. For $N=2$ and $N=\infty$ only the last operator survives and we obtain the Ising model result Eq.(5.2). Finally, for $N=1$, all the operators vanish except the third one, for which we obtain the free theory result $\gamma=0$.

For the operators with $n=4$ fields, we have

$dim=\binom{N}{4}$	$\phi_i\phi_j\phi_k\phi_l$	$\gamma=\frac{4}{N}$
$dim=\frac{N(N-1)}{2}$	$(\phi_i\phi_j^3-\phi_j\phi_i^3)$	$\gamma=1$
$dim=\frac{N(N-3)}{2}$	$(\phi_i^2-\phi_j^2)(\phi_k^2-\phi_l^2)$	$\gamma=\frac{2(N+2)}{3N}$
$dim=\frac{N(N-1)(N-3)}{2}$	$\phi_i\phi_j(\phi_k^2-\phi_l^2)$	$\gamma=\frac{N+8}{3N}$
$dim=\frac{N(N-1)}{2}$	$\frac{12-N+\sqrt{N^2+24N-48}}{12}(\phi_i\phi_j\phi^2)+\phi_i\phi_j^3+\phi_j\phi_i^3$	$\gamma=\frac{12+5N+\sqrt{N^2+24N-48}}{6N}$
$dim=\frac{N(N-1)}{2}$	$\frac{12-N-\sqrt{N^2+24N-48}}{12}(\phi_i\phi_j\phi^2)+\phi_i\phi_j^3+\phi_j\phi_i^3$	$\gamma=\frac{12+5N-\sqrt{N^2+24N-48}}{6N}$
$dim=(N-1)$	$\frac{4-N}{2}(\phi_i^2-\phi_j^2)\phi^2+\phi_i^4-\phi_j^4$	$\gamma=1$
$dim=(N-1)$	$\frac{4}{3}(\phi_i^2-\phi_j^2)\phi^2+\phi_i^4-\phi_j^4$	$\gamma=\frac{2(3N-2)}{3N}$
$dim=1$	$(\phi^2)^2-2\sum_{i=1}^N\phi_i^4$	$\gamma=\frac{4(N-1)}{3N}$
$dim=1$	$(\phi^2)^2+\frac{N-4}{3}\sum_{i=1}^N\phi_i^4$	$\gamma=2$

The results above have been already obtained in [26]. Finally, the results for operators with $n=5$ fields are

$dim=\binom{N}{5}$	$\phi_i\phi_j\phi_k\phi_l\phi_m$	$\gamma=\frac{20}{3N}$
$dim=\frac{N(N-1)(N-4)}{2}$	$\phi_i(\phi_j^2-\phi_k^2)(\phi_l^2-\phi_m^2)$	$\gamma=\frac{2(N+6)}{3N}$
$dim=\frac{N(N-1)(N-2)}{3}$	$\phi_i(\phi_j^3\phi_k-\phi_k^3\phi_j)$	$\gamma=\frac{3N+8}{3N}$
$dim=\frac{N(N-1)(N-2)(N-4)}{6}$	$\phi_i\phi_j\phi_k(\phi_l^2-\phi_m^2)$	$\gamma=\frac{16+N}{3N}$
$dim=2\times\frac{N(N-1)(N-2)}{6}$	$\begin{pmatrix}\phi^2\phi_i\phi_j\phi_k\\ \phi_i^3\phi_j\phi_k+perm\end{pmatrix}$	$\gamma=\frac{30+5N\pm\sqrt{(46+N)(N-2)}}{6N}$
$dim=3\times N(N-2)$	$\begin{pmatrix}\phi^2\phi_i(\phi_j^2-\phi_k^2)\\ \phi_i^3(\phi_j^2-\phi_k^2)\\ \phi_i(\phi_j^4-\phi_k^4)\end{pmatrix}$	$\gamma=\frac{1}{3N}\begin{pmatrix}3(N+6) & 3 & 6\\ 2(N-4) & 4(N+1) & 0\\ 4(N-4) & 0 & 2(3N-2)\end{pmatrix}$
$dim=4\times N$	$\begin{pmatrix}\phi_i\left(\phi^2\right)^2\\ \phi_i^5\\ \phi^2\phi_i^3\\ \phi_i\sum_{j=1}^N\phi_j^4\end{pmatrix}$	$\gamma=\frac{1}{3N}\begin{pmatrix}4(N+5) & 0 & 3 & 6\\ 0 & 10(N-2) & 6(N-4) & 4(N-4)\\ 4(N-4) & 10 & 5(N+2) & 4\\ 4(N-4) & 0 & 0 & 2(3N-2)\end{pmatrix}$

The consistency checks for $N=1,2,4,\infty$ are passed as in the $n=3$ case. To the best of our knowledge, there are no results for $n\geq 5$ in the literature.

5.3 The weighted sum

We conclude with an intriguing feature of the spectrum of anomalous dimensions in the hypercubic model. Consider the sum of the anomalous dimensions of order n operators, S_n, weighted by the dimensions of the corresponding H_N-irreps d_{S_n}, that is

$$W_n=\sum_{S_n}d_{S_n}\gamma_{S_n} \tag{5.11}$$

where the sum runs over all the irreducible representations appearing at the level n. Clearly, the Gaussian limit requires $W_n \propto N-1$. Moreover, the values of W_n show an intriguing pattern, that is

$$W_2 = \frac{2}{3}(N-1), \qquad W_3 = \frac{2}{3}(N-1)(N+2), \tag{5.12}$$
$$W_4 = \frac{1}{3}(N-1)(N+2)(N+3), \qquad W_5 = \frac{1}{9}(N-1)(N+2)(N+3)(N+4)$$

which suggests the existence of a general formula for W_n.

6. Outlook

In the work presented in this report, we have found the spectrum of spin-0 composite operators in hypercubic theories. The result should be instrumental in future investigations of such models. As a first application, we have computed all the 1-loop anomalous dimensions for composite operators with an arbitrary number of fields and no derivatives. Another important application could be the generalization of the conformal bootstrap analyses of the H_3 model [11, 10, 12] to the H_N case. We have also examined some general features of the anomalous dimension spectrum. These include a compact formula for all the operators which do not mix and the interesting pattern of the values of W_n. It would be particularly interesting to extend our findings to operators with spin and to compute the anomalous dimension to higher orders in the ε-expansion.

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