Measurements of the CKM angle $\gamma$ at LHCb

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These proceedings describe two LHCb measurements of the CKM angle $\gamma$: a measurement with $B^0 \rightarrow DK^{*0}$ decays where the $D$ meson decays into two- and four-body final states, using proton-proton collision data corresponding to an integrated luminosity of 5 fb$^{-1}$, and a measurement using $B^{\pm} \rightarrow DK^{\pm}$ decays where the $D$ decays into the final states $K^0_S\pi^+\pi^-$ and $K^0_SK^+K^-$, using data corresponding to an integrated luminosity of 2 fb$^{-1}$. Furthermore, the combination of all LHCb measurements of $\gamma$ is described. This combination constitutes the world-leading, single-experiment direct determination of $\gamma$ and obtains the value $\gamma = (74.0^{+5.0}_{-5.8})^\circ$. 

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1. Introduction

One of the main physics goals of the LHCb experiment is to perform high precision measurements of CKM [1, 2] parameters of the Standard Model (SM). Measurements of the CKM angle $\gamma \equiv \arg \left(-V_{ub}^\ast V_{cb}/V_{ud}^\ast V_{cd}\right)$ play a key role, because $\gamma$ can be measured in tree-level processes, with negligible theoretical uncertainties [3]. Thus, these measurements are simple to interpret within the SM, and less likely to be affected by potential New Physics effects than measurements relying on loop-level processes. A consistency test of the SM can be obtained by comparing direct measurements of $\gamma$ with indirect determinations, based on observables measured in loop-level processes and related to $\gamma$ assuming the SM to hold true. Recent averages of both direct measurements and indirect determinations of $\gamma$ are shown in Fig. 1, where it is clear that this consistency test is currently limited by the precision of existing direct measurements. The LHCb experiment is currently world-leading in improving this precision.

Experimentally, $\gamma$ is probed via observables sensitive to interference between $b \rightarrow uW$ and $b \rightarrow cW$ quark-level transitions. The majority of LHCb measurements of $\gamma$ rely on interference between $B \rightarrow D^0K$ and $B \rightarrow \bar{D}^0\bar{K}$ type decays, the classical example being the $B^\pm \rightarrow DK^\pm$ channel, where the $D$ meson is reconstructed in a final state common to $D^0$ and $\bar{D}^0$ mesons.\(^1\) The $CP$-violating phase between the two $B$ decays is $\pm \gamma$, where the sign depends on the $B$ charge (or flavour), and can be measured along with a $CP$-conserving phase difference, $\delta_B$, and the ratio of the amplitude magnitudes, $r_B$. For a $D$ final state $f$ common to both $D^0$ and $\bar{D}^0$ mesons, as well as its $CP$ conjugate state $\bar{f}$, four decay rates sensitive to $\gamma$ can be considered:

\[
\begin{align*}
\Gamma(B^- \rightarrow D(\rightarrow f)K^-) &\sim 1 + r_B^2 \frac{\alpha}{2} + 2r_B\frac{\delta_D}{2} \cos [\delta_B + \delta_D - \gamma], \\
\Gamma(B^+ \rightarrow D(\rightarrow f)\bar{K}^+) &\sim r_D^2 + r_B^2 + 2r_B\frac{\delta_D}{2} \cos [\delta_B - \delta_D + \gamma], \\
\Gamma(B^- \rightarrow D(\rightarrow \bar{f})\bar{K}^-) &\sim r_D^2 + r_B^2 + 2r_B\frac{\delta_D}{2} \cos [\delta_B - \delta_D - \gamma], \\
\Gamma(B^+ \rightarrow D(\rightarrow \bar{f})K^+) &\sim 1 + r_D^2 + 2r_B\frac{\delta_D}{2} \cos [\delta_B + \delta_D + \gamma].
\end{align*}
\]

1LHCb has also published measurements based on time-dependent analyses of $B^0 \rightarrow D^\mp K^{\pm}$ [13] and $B^0 \rightarrow D^\pm \pi^\mp$ [14] decays, but they are not treated in these proceedings.
Here, the decay amplitudes for $D^0 \rightarrow f$ decays and $D^0 \rightarrow f$ decays are related via $A(D^0 \rightarrow f) = r_D e^{i \delta_0} A(D^0 \rightarrow f)$, where $r_D < 1$ if $f$ is chosen so the former decay is suppressed. The effect of $CP$ violation in the $D$ decay is not included, and is negligible compared to the precision of current measurements. These equations (or equivalent versions for $B^0 \rightarrow D K^{*0}$ and other, similar decays) form the basis of a number of related measurement strategies, the exact analysis approach depending on the chosen $D$ final state. Naturally, the best sensitivity to $\gamma$ is achieved by combining measurements in many different channels. These proceedings cover two such measurements, as well as the combination of all LHCb results.

2. ADS/GLW measurement with $B^0 \rightarrow D K^{*0}$ decays

LHCb has recently published a measurement using $B^0 \rightarrow D K^{*0}$ decays [15], where the $D$ mesons decay in the so-called GLW [16, 17] modes $D \rightarrow \pi^+ \pi^- \pi^0$ and $D \rightarrow K^+ K^-$, which are $CP$ eigenstates, the quasi-GLW [18, 19] mode $D \rightarrow \pi^+ \pi^- \pi^0$, as well as the so-called ADS [20, 21] modes $D \rightarrow \pi^\pm K^\mp$ and $D \rightarrow \pi^0 K^\pm \pi^-$. Two charge associations of the $K \pi$ pair are considered in the latter cases, denoted the favoured mode, $D^0 \rightarrow K^- \pi^+(\pi^0)$, and suppressed mode, $D^0 \rightarrow K^- \pi^+(\pi^0)$. The measurement uses 3 fb$^{-1}$ of data taken in Run 1, and 2 fb$^{-1}$ of data taken in 2015 and 2016.

In the GLW channels, where the $D$ mesons are reconstructed in $CP$ eigenstates, Eqs. (1.1) simplify since $f = \bar{f}$, $r_D = 1$, and $\delta_B = 0$ (for $CP$-even eigenstates). The $CP$ asymmetry

$$A^{hh}_{CP} = \frac{\Gamma(B^0 \rightarrow D_C K^{*0}) - \Gamma(B^0 \rightarrow D_{CP} K^{*0})}{\Gamma(B^0 \rightarrow D_C K^{*0}) + \Gamma(B^0 \rightarrow D_{CP} K^{*0})} = \frac{2 \kappa_B^{DK^{*0}} \sin \delta_B^{DK^{*0}} \sin \gamma}{R_{CP}}$$

(2.1)

is measured. It can be obtained from the raw yield asymmetry, if it is corrected for any potential production asymmetry between $B^0$ and $\bar{B}^0$, and $CP$-violating asymmetries for the final state products. As shown, it can be related to the physics parameters $(\gamma, \kappa_B^{DK^{*0}}, \delta_B^{DK^{*0}})$ via another observable, the ratio

$$R^{hh}_{CP} = \frac{\Gamma(B^0 \rightarrow D_{CP} K^{*0}) + \Gamma(B^0 \rightarrow D_{CP} K^{*0})}{\Gamma(B^0 \rightarrow D_C K^{*0}) + \Gamma(B^0 \rightarrow D_{CP} K^{*0})}.$$  

(2.2)
The relation can be derived via equations equivalent to those in Eq. (1.1), with the inclusion of a coherence factor, \( \kappa = 0.958^{+0.005}_{-0.046} \) [22], in the interference term that accounts for the fact that \( B^0 \to D K^{*0} \) is not the only resonance contributing to the full \( B \to D \pi \kappa \) amplitude. The ratio \( R_{CP} \) can be obtained from the yield ratio between the GLW mode in question and the favoured ADS mode, using known branching fractions of the \( D^0 \) meson. The \( D \to \pi^+ \pi^- \pi^+ \pi^- \) mode can be analysed analogously to the pure \( CP \) eigenstates because the \( CP \)-even fraction of the decay \( F^+_{\pi\pi} = 0.769 \pm 0.023 \) has been measured in quantum-correlated \( D^0 \bar{D}^0 \) decays [23]. For this mode, all interference terms of Eqs. (1.1) obtain an additional factor of \( (2F^+_{\pi\pi} - 1) \). The \( B \) mass spectra that have been fit to obtain the yields are shown for this channel in Fig. 2. The obtained results from all GLW-like measurements are

\[
\begin{align*}
A^{KK}_{CP} &= -0.05 \pm 0.10 \pm 0.01, \\
A^{\pi\pi}_{CP} &= -0.18 \pm 0.14 \pm 0.01, \\
R^{KK}_{CP} &= 0.92 \pm 0.10 \pm 0.02, \\
R^{\pi\pi}_{CP} &= 1.32 \pm 0.19 \pm 0.03, \\
A^{4\pi}_{CP} &= -0.03 \pm 0.15 \pm 0.01, \\
R^{4\pi}_{CP} &= 1.01 \pm 0.16 \pm 0.04,
\end{align*}
\]

(2.3)

where the first uncertainty is statistical, and the second arises due to systematic effects. Furthermore, this measurement constitutes a first observation of the decay mode \( B^0 \to D(\to \pi^+ \pi^- \pi^+ \pi^-)K^{*0} \) with a significance of 8.4\( \sigma \).

In the ADS modes, the measured observables are the ratios

\[
\begin{align*}
R^+_{\pi K(\pi\pi)} &= \frac{\Gamma(B^0 \to D(\to \pi^+ K^- (\pi^+ \pi^-))K^{*0})}{\Gamma(B^0 \to D(\to K^+ \pi^- (\pi^+ \pi^-))K^{*0})}, \\
R^-_{\pi K(\pi\pi)} &= \frac{\Gamma(B^0 \to D(\to \pi^+ K^- (\pi^+ \pi^-))\bar{R}^{*0})}{\Gamma(B^0 \to D(\to K^+ \pi^- (\pi^+ \pi^-))\bar{R}^{*0})}.
\end{align*}
\]

(2.4)

These ratios can be related to the underlying physics parameters of interest \( \langle \gamma_B^{DK^{*0}}, \delta_B^{DK^{*0}} \rangle \) via equations equivalent to those in Eq. (1.1), again including the coherence factor \( \kappa \), and, in the \( D \to K^\pm \pi^+ \pi^- \pi^- \) case, a second coherence factor, \( \kappa^{K3\pi} = 0.43^{+0.17}_{-0.13} \) [24] that arises due to interference between the \( D^0 \) and \( \bar{D}^0 \) amplitudes over the phase space of the \( D \) decay. The obtained results are

\[
\begin{align*}
R^+_{\pi K} &= 0.064 \pm 0.021 \pm 0.002, \\
R^-_{\pi K} &= 0.095 \pm 0.021 \pm 0.003, \\
R^+_{\pi K \pi \pi} &= 0.074 \pm 0.026 \pm 0.002, \\
R^-_{\pi K \pi \pi} &= 0.072 \pm 0.025 \pm 0.003,
\end{align*}
\]

(2.5)

where the first uncertainty is statistical, and the second arises due to systematic effects. \( CP \) asymmetries analogous to that in Eq. (2.1) are measured in the favoured ADS channels, where they are expected to be very small. The obtained values are \( A^{K\pi}_{ADS} = 0.047 \pm 0.027 \pm 0.010 \) and \( A^{K\pi\pi\pi}_{ADS} = 0.037 \pm 0.032 \pm 0.010 \). Finally, this measurement constitutes a first observation of the suppressed ADS decay mode \( B^0 \to D(\to \pi^- K^+)K^{*0} \) with a significance of 5.8\( \sigma \), and obtains evidence of the suppressed four-body mode with a significance of 4.4\( \sigma \).
Contours contain the 68.3%, 95.5% and 99.7% C.L.

The measured yield asymmetries and ratios can be used to constrain the underlying physics parameters \((\gamma, r_B^{DK^{*0}}, \delta_B^{DK^{*0}})\), resulting in the two-dimensional confidence regions of Fig. 3. The measurement of \(r_B^{DK^{*0}} = 0.265 \pm 0.023\) has an uncertainty 50% lower than that of the current LHCb combination [11], which was performed including only Run 1 results for this set of modes, showcasing the increased precision to come with the inclusion of the Run 2 data set in other decay channels.

3. GGSZ measurement with \(B^\pm \to DK^\pm\) decays

LHCb has recently published a measurement using \(B^\pm \to DK^\pm\) decays, with the self-conjugate three-body final states \(K_S^0\pi^+\pi^-\) and \(K_S^0K^+K^-\), based on a data set corresponding to an integrated luminosity of 2 fb\(^{-1}\) taken in 2015 and 2016 [25]. This is an update of an analysis also performed on Run 1 data [26]. In this case, \(r_D\) and \(\delta_D\) of Eqs. (1.1) vary over the phase-space of the \(D\) decay, and the equations simplify again since \(f = \bar{f}\). This phase-space variation can be exploited to make a measurement with sensitivity to \(\gamma\) by measuring the decay yields in multiple bins of the phase space [27, 28]. Defining the (real) \(CP\)-violation observables of interest \((x_\pm, y_\pm)\) via the relation

\[
x_\pm + iy_\pm = r_B \exp[i(\delta_B \pm \gamma)],
\]

the signal yield from \(B^\pm\) decays in bin \(i\) of the Dalitz plot, \(N_\pm^i\), can be expressed [29, 30]

\[
N_+^i = h^+ \left(F_{+,i} + r_B^2 F_i + 2r_B \sqrt{F_i F_{+,i}} (x_i + c_i + s_i)\right),
\]

\[
N_-^i = h^- \left(F_{-,i} + r_B^2 F_i - 2r_B \sqrt{F_i F_{-,i}} (x_i - c_i - s_i)\right)
\]

if the bins are chosen to be symmetric around the \(m^2(K_S^0\pi^+) = m^2(K_S^0\pi^-)\) diagonal of the Dalitz plot, numbered so that bins above the diagonal have \(i > 0\), and for a given bin \(+i\), the symmetric bin below the diagonal has number \(-i\). The binning schemes used in this measurement are shown in Fig. 4. In Eq. (3.2), the \(F_i\) parameters denote the fraction of \(D^0\) decays in bin \(i\), and are measured in

\[
\begin{align*}
\end{align*}
\]
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Figure 4: Binning schemes of the (left) $D \to K^0\pi^+\pi^-$ and (right) $D \to K^0K^+K^-$ Dalitz plots used in the GGSZ measurements.

a control channel of double-flavour tagged $D^0$ decays: $B \to D^{*\pm}(\to D^0\pi^\pm)\mu^-\bar{\nu}_\mu X$. Corrections are applied to take into account that there are unavoidable differences in the reconstruction efficiency profile over the Dalitz plot between signal and control channels. The parameters $c_i$ and $s_i$ are the average cosine and sine of $\delta_i$ over bin $i$. These are external inputs, measured independently by the CLEO collaboration [31] using quantum-correlated $D^0$-$\bar{D}^0$ pairs produced via decay of the $\psi(3770)$ resonance.

The observables of interest, $x_\pm$ and $y_\pm$, are determined directly in the fit to data, via a simultaneous fit to the $B$ mass spectrum in all bins, where the signal yields are parameterised using Eqs. (3.2). The normalisation constants $h^\pm$ are determined independently of each other in the measurement, which leaves it insensitive to production and detection asymmetries, as well as the leading-order effects of $CP$-violation and material interaction of the $K^0_s$ [32]. The obtained values are combined with the results obtained from analysing the Run 1 data set [26], and used to constrain the physics parameters of interest $(\gamma, r_B^{DK^\pm}, \delta_B^{DK^\pm})$. The confidence regions for these parameters are shown in Fig. 5. The Run 1 and 2015+16 results yield a combined measurement of $\gamma = (80^{+10}_{-9})^\circ$, which is the most precise measurement of $\gamma$ in a single decay channel to date.

4. Combination of LHCb measurements

LHCb has published a combination of many independent measurements, in order to obtain the most sensitive overall determination of $\gamma$ [11]. The combination includes measurements based on decays of $B^\pm$ and $B^0$ decays analysed using several different ADS, GLW, and GGSZ-type $D$ final states. Furthermore, time-dependent analyses of $B^0 \to D^\pm K^\mp$ [13] and $B^0 \to D^\pm \pi^\pm$ [14] decays are included. These rely on interference between mixing and decay, are sensitive to both $\gamma$ and $\beta(s)$, and allow for unambiguous solutions for $\gamma$. The combined value is $\gamma = (74.0^{+5.0}_{-3.8})^\circ$, which dominates the current world averages. The confidence limits for $\gamma$ obtained in the combination are shown in Fig. 6, for both the overall combination and individually for measurements that use a given $B$. 

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Figure 5: Constraints on (left) $(\gamma, \rho_{DK}^{+})$ and (right) $(\gamma, \delta_{DK}^{+})$ from the LHCb GGSZ measurements [25, 26] of $B^\pm \to DK^\pm$ decays, where $D \to K_S^0 h^+ h^-$. 

Figure 6: (Left) Overall confidence limit for $\gamma$ in the LHCb combination, and (right) split by $B$ decay mode (the results of Section 2 have not yet been included in the constraints from $B^0$ decays).

meson species. The results of Section 2 have been published after the latest LHCb combination, but will be included in the next version.

The overall uncertainty on the LHCb $\gamma$ combination is driven by the GLW analysis of $B^\pm \to D(\to hh)K^\pm$ decays. All systematic uncertainties in this channel can be studied with the LHCb data set, and therefore continuous improvement can be expected as data sets corresponding to 50 fb$^{-1}$ of integrated luminosity will have been collected after the LHCb Upgrade phase, and 300 fb$^{-1}$ or more after the proposed Upgrade Phase II [33]. Still, GGSZ measurements are crucial, as they provide a single, unambiguous solution for $\gamma$, and serve as an important consistency check of the LHCb combination. The obtainable sensitivity on these measurements depend on the precision of external strong-phase inputs, as described in Section 3, which currently contribute a systematic uncertainty of about 4$^\circ$. Forthcoming, and much anticipated, measurements from BESIII will reduce this number to approximately 1$-$2$^\circ$ in the near future. However, further strong-phase measurements will be necessary to obtain the best possible precision on $\gamma$ in model-independent GGSZ measurements, both for LHCb and Belle II.

5. Conclusions and outlook

These proceedings have detailed two of the latest $\gamma$ measurements made by the LHCb collabo-
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an ADS/GLW measurement made with $B^0 \to DK^{*0}$ decays and a GGSZ measurement made with $B^\pm \to DK^{\pm}$ decays, as well as the LHCb combination of $\gamma$ measurements, which provides the world-leading, single-experiment determination $\gamma = (74.0^{+5.0}_{-4.8})^\circ$. Significant updates to the LHCb combination can be expected in the near future, including the inclusion of the $B^0 \to DK^{*0}$ measurement described in Section 2, which was published after the current LHCb combination, and a number of results with the full Run 1 and 2 data set. As the full Run 2 data set is analysed the precision on $\gamma$ will be reduced to $4^\circ$, and may surpass this expectation.

The uncertainty on $\gamma$ will reach sub-degree levels within the next 5–10 years, reaching a world average of $0.3^\circ$ by $\sim 2035$. This will be achieved with data corresponding to an integrated luminosity of 50 ab$^{-1}$ collected by Belle II, and 300 fb$^{-1}$ with the LHCb upgrades, and will allow for many interesting, ultra-high precision tests of the Standard Model in the years to come.

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