Rare kaon decays and CP violation

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Owing to the strong suppression of flavour-changing neutral-current transitions in the Standard Model, rare kaon decays constitute a superb tool to constrain hypothetical new-physics interactions. At the same time, they provide many interesting tests of the Standard Model itself, being sensitive both to short-distance electroweak scales and to the long-distance QCD dynamics. A brief overview of the current status is presented. The Standard Model prediction for the direct CP-violating ratio ε'/ε is also discussed.

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1. Introduction

The investigation of kaon decays has uncovered many fundamental ingredients of the electroweak theory, such as flavour quantum numbers, meson-antimeson mixing, parity violation, CP violation, quark mixing, and the GIM mechanism [1]. The kaon decay amplitudes provide very interesting tests of the Standard Model (SM), since they involve an intricate interplay between weak, electromagnetic and strong interactions. Moreover, rare kaon decays are sensitive to short-distance scales (c, t, $W^{\pm}$, Z) and have the potential to unravel new physics (NP) beyond the SM. In particular, high-precision searches for lepton-flavour violation (LFV) beyond the 10^{-10} level [$\mathrm{Br}(K_L \to e^{\pm}\mu^{\mp}) < 4.7 \cdot 10^{-12}$ [2], $\mathrm{Br}(K_L \to e^{\pm}e^{\pm}\mu^{\mp}\mu^{\mp}) < 4.12 \cdot 10^{-11}$ [3], $\mathrm{Br}(K^+ \to \pi^+\mu^+e^-) < 1.3 \cdot 10^{-11}$ [4], $\mathrm{Br}(K^+ \to \pi^+\mu^-e^+) < 5.2 \cdot 10^{-10}$ [5] (90% CL)] are actually exploring energy scales above the 10 TeV region. In addition, the mechanism of CP violation can be accurately tested, both in the observed kaon decay modes and through still undetected processes such as $K_L \to \pi^0 \nu\bar{\nu}$.

The fundamental flavour-changing transitions among the constituent quarks are characterized by the electroweak scale, but the corresponding hadronic amplitudes are governed by the long-distance behaviour of the strong interactions, *i.e.*, the confinement regime of QCD. In kaon decays, the presence of widely separated mass scales ($m_\pi < m_K \ll M_W$) amplifies the QCD corrections with logarithms of the large mass ratios. Using the operator product expansion (OPE) and the renormalization group to integrate out the heavy fields all the way down from M_W to scales $\mu < m_c$, one gets an effective Lagrangian, defined in the three-flavour theory [6],

$$\mathscr{L}_{\mathrm{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_i C_i(\mu)\, Q_i\,, \qquad C_i(\mu) = z_i(\mu) - y_i(\mu)\, \frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}\,, \qquad (1.1)$$

which contains local four-fermion operators Q_i, constructed with the light degrees of freedom (u, d, s; e, μ, ν_ℓ), modulated by Wilson coefficients $C_i(\mu)$ that are functions of the heavy (Z, W, t, b, c, τ) masses and encode the short-distance logarithmic corrections. The violations of the CP symmetry originate in the $y_i(\mu)$ components, which are proportional to the top-quark mixing factors.

The coefficients $C_i(\mu)$ are currently known at the next-to-leading order (NLO) [7, 8, 9, 10], which includes all corrections of $\mathscr{O}(\alpha_s^n t^n)$ and $\mathscr{O}(\alpha_s^{n+1}t^n)$, with $t \equiv \log{(M_1/M_2)}$ the logarithm of any ratio of heavy mass scales ($M_{1,2} \geq \mu$). The long-distance contributions from scales below the renormalization scale μ are contained in the non-perturbative matrix elements of the operators Q_i between the initial and final hadronic states. These contributions should cancel exactly the renormalization scale (and scheme) dependence of the Wilson coefficients. Unfortunately, a rigorous analytic evaluation of the hadronic matrix elements, keeping full control of the QCD renormalization conventions, remains still a challenging task.

At low energies, one can use symmetry considerations to define another effective field theory in terms of the QCD Goldstone bosons (π, K, η). Chiral Perturbation Theory (χPT) [11, 12] describes the dynamics of the pseudoscalar octet through a perturbative expansion in powers of momenta and quark masses over the chiral symmetry-breaking scale $\Lambda_\chi \sim 1$ GeV. Chiral symmetry determines the allowed operators, while all short-distance information is encoded in their low-energy couplings (LECs) [13, 14]. At LO the most general effective Lagrangian, with the same $SU(3)_L \otimes SU(3)_R$
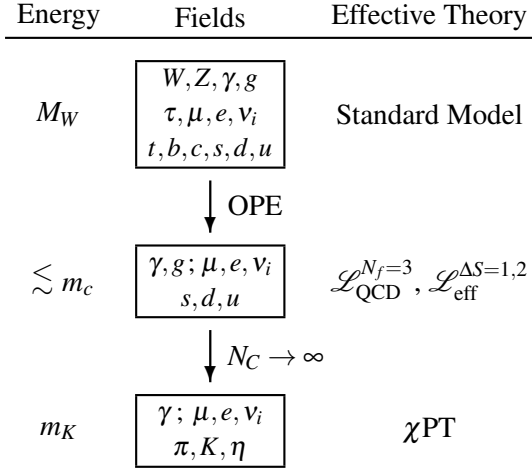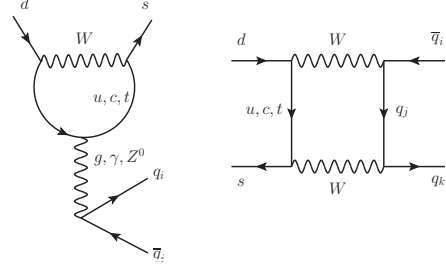
Figure 1: Evolution from M_W to m_K.

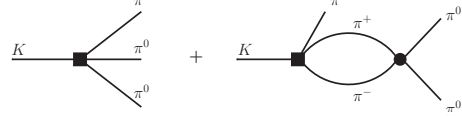**Figure 2:** Short-distance diagrams.

Figure 3: Long-distance diagrams.

transformation properties as the short-distance Lagrangian (1.1), contains three terms [1]:

$$\mathscr{L}_2^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \langle \lambda L_\mu L^\mu \rangle + g_{27} \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + e^2 g_8 g_{ew} F^6 \langle \lambda U^\dagger Q U \rangle \right\},$$
(1.2)

where the $SU(3)$ matrix field $U \equiv \exp(i\vec{\lambda}\vec{\phi}/F)$ parametrizes the octet of Goldstone bosons, $L_\mu = iF^2 U^\dagger D_\mu U$ represents the $V-A$ currents, $\lambda \equiv (\lambda_6 - i\lambda_7)/2$ projects onto the $\bar{s} \to \bar{d}$ transition, $Q = \frac{1}{3} \text{diag}(2,-1,-1)$ is the quark charge matrix and $\langle \rangle$ denotes a 3-dimensional flavour trace. The $\mathscr{O}(p^2)$ LECs g_8 and g_{27} measure the strength of the two parts of $\mathscr{L}_{eff}^{\Delta S=1}$ transforming as $(8_L, 1_R)$ and $(27_L, 1_R)$, respectively, under chiral rotations, while the $g_8 g_{ew}$ term is of $\mathscr{O}(e^2 p^0)$ and accounts for the $(8_L, 8_R)$ piece induced by the electromagnetic penguin operators.

The χPT framework determines the most general form of the K decay amplitudes, compatible with chiral symmetry, in terms of the LECs multiplying the relevant chiral operators. A first-principle calculation of these LECs would require to perform a non-perturbative matching between χPT and the underlying SM. While some lattice information is already available in the strong sector, the LECs can be determined phenomenologically and/or calculated in the limit of a large number of QCD colours N_C. Fig. 1 shows schematically the procedure used to evolve down from the electroweak scale, where the underlying flavour-changing processes take place (Fig. 2), to m_K. The short-distance logarithmic corrections $\log(M/\mu)$ are summed up with the OPE and the resulting effective Lagrangian $\mathscr{L}_{eff}^{\Delta S=1}$ is then matched into the low-energy χPT formalism.

At NLO in the χPT expansion, one must consider tree-level contributions from additional operators of $\mathscr{O}(p^4)$ and $\mathscr{O}(e^2 p^2)$, with their corresponding LECs, and quantum loops with the LO Lagrangian (1.2). These chiral loops (Fig. 3) generate non-polynomial contributions, with logarithms and threshold factors as required by unitarity. The loop corrections contain large infrared logarithms, $\log(\mu/m_\pi)$, and Goldstone re-scattering contributions (final-state interactions) that play a very important role in the kaon decay dynamics.

2. The CP-violating ratio ε'/ε

A tiny difference between the CP-violating ratios $\eta_{nm} \equiv \mathcal{M}[K_L^0 \to \pi^n \pi^m]/\mathcal{M}[K_S^0 \to \pi^n \pi^m] \approx \varepsilon \approx 2.2 \times 10^{-3}\, e^{i\pi/4}$, where $nm = +-, 00$ denote the final pion charges, was first measured by the CERN NA31 experiment [15] and later confirmed at the 7.2σ level with the full data samples of NA31, NA48 and the Fermilab experiments E731 and KTeV [16]:

$$\text{Re}(\varepsilon'/\varepsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right) = (16.6 \pm 2.3) \times 10^{-4}. \tag{2.1}$$

This important measurement established the presence of direct CP violation in the decay amplitudes, confirming that CP violation is associated with a $\Delta S = 1$ transition as predicted by the SM.

The first NLO theoretical predictions gave values of ε'/ε one order of magnitude smaller than (2.1), but it was soon realised that they were missing the important role of the final pion dynamics [17, 18, 19]. Once long-distance contributions are properly taken into account, the theoretical SM prediction turns out to be in good agreement with the experimental value, although the uncertainties are unfortunately large [20, 21]:

$$\text{Re}(\varepsilon'/\varepsilon)_{\text{SM}} = (14 \pm 5) \times 10^{-4}. \tag{2.2}$$

The underlying physics can be easily understood from the kaon data themselves. Owing to Bose symmetry, the two pions in the final state must be in a $I = 0$ or $I = 2$ configuration. In the absence of QCD corrections, the corresponding $K \to \pi\pi$ decay amplitudes $\mathcal{A}_I \equiv A_I\, e^{i\delta_I}$ are predicted to differ only by a $\sqrt{2}$ factor. However, their measured ratio is 16 times larger than that (a truly spectacular enhancement generated by the strong forces):

$$\omega \equiv \text{Re}(A_2)/\text{Re}(A_0) \approx 1/22, \qquad\qquad \delta_0 - \delta_2 \approx 45°. \tag{2.3}$$

Moreover, they exhibit a huge phase-shift difference that manifests the relevance of final-state interactions and, therefore, the presence of large absorptive contributions to the $K \to \pi\pi$ amplitudes, specially to the isoscalar one. Writing $\mathcal{A}_I = \text{Dis}(\mathcal{A}_I) + i\,\text{Abs}(\mathcal{A}_I)$ and neglecting the small CP-odd components, the measured $\pi\pi$ scattering phase shifts at $\sqrt{s} = m_K$ imply that

$$\text{Abs}(\mathcal{A}_0)/\text{Dis}(\mathcal{A}_0) \approx 0.82, \qquad\qquad \text{Abs}(\mathcal{A}_2)/\text{Dis}(\mathcal{A}_2) \approx -0.15. \tag{2.4}$$

The direct CP-violating effect involves the interference between the two isospin amplitudes,

$$\text{Re}(\varepsilon'/\varepsilon) = -\frac{\omega}{\sqrt{2}\,|\varepsilon|}\left[\frac{\text{Im}A_0}{\text{Re}A_0} - \frac{\text{Im}A_2}{\text{Re}A_2}\right] = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\text{Im}A_0^{(0)}}{\text{Re}A_0^{(0)}}(1 - \Omega_{\text{eff}}) - \frac{\text{Im}A_2^{\text{emp}}}{\text{Re}A_2^{(0)}}\right]. \tag{2.5}$$

It is suppressed by the small ratio ω and, moreover, it is very sensitive to isospin-breaking (IB) corrections [22, 23, 24], parametrized by $\Omega_{\text{eff}} = 0.11 \pm 0.09$ [21], because small IB corrections to A_0 feed into the small amplitude A_2 enhanced by the large factor $1/\omega$. In the right-hand side of Eq. (2.5), the (0) superscript indicates the isospin limit, $\omega_+ = \text{Re}(A_2^+)/\text{Re}(A_0)$ is directly extracted from $K^+ \to \pi^+\pi^0$ and A_2^{emp} contains the electromagnetic-penguin contribution to A_2 (the remaining contributions are included in Ω_{eff}).

Claims of too small SM values for ε'/ε usually originate from perturbative calculations that are unable to generate the physical phase shifts, *i.e.*, they predict $\delta_I = 0$ and, therefore, $\text{Abs}(\mathscr{A}_I) = 0$, failing completely to understand the empirical ratios (2.4). This unitarity pitfall implies also incorrect predictions for the dispersive components, since they are related by analyticity with the absorptive parts: a large absorptive contribution generates a large dispersive correction that is obviously missed in those calculations. This perturbative problem is more severe in ε'/ε because Eq. (2.5) involves a delicate numerical balance among the three contributing terms, and naive predictions sit precisely on a nearly-exact cancellation (a 40% positive correction to the first term enhances the whole result by one order of magnitude).

The ε'/ε anomaly was recently resurrected by the lattice RBC-UKQCD collaboration that reported $\text{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$ [25, 26]. The uncertainties are still large, but the quite low central value implies a 2.1σ deviation from the experimental measurement. This has triggered a revival of the old naive estimates [27, 28], some of them making also use of the lattice data [29, 30], and a large amount of NP explanations (a list of references is given in Refs. [20, 21]). However, it is premature to derive physics implications from the current lattice simulations, since they are still unable to reproduce the known phase shifts. While the lattice determination of δ_2 is only 1σ away from its physical value, δ_0 disagrees with the experimental result by 2.9σ, a much larger discrepancy that the one quoted for ε'/ε. Obviously, nobody suggests a NP contribution to the $\pi\pi$ elastic scattering phase shifts. The RBC-UKQCD collaboration is actively working in order to improve the present situation.

3. Rare kaon decays in the SM

Kaon decays mediated by flavour-changing neutral currents (FCNCs) are strongly suppressed in the SM and, therefore, are very sensitive to NP effects. In the SM, most of these processes are dominated by long-distance contributions, making quite challenging their precise theoretical understanding. However, there are also decays governed by short-distance amplitudes, such as $K \to \pi\nu\bar{\nu}$.

3.1 $\mathbf{K^0 \to \gamma\gamma}$ and $\mathbf{K^0 \to \ell^+\ell^-}$

At $\mathscr{O}(p^4)$ in the χPT expansion, the symmetry constraints do not allow any local $K_1^0 \gamma\gamma$ vertex ($K_{1,2}^0$ denote the CP-even and CP-odd neutral kaon states). The decay $K_S^0 \to \gamma\gamma$ proceeds then through a one-loop amplitude, with virtual $\pi^+\pi^-$ or K^+K^- pairs (Fig. 4), which is necessarily finite because there are no counterterms to renormalize divergences. The resulting prediction, $\text{Br}(K_S^0 \to \gamma\gamma) = 2.0 \times 10^{-6}$ [31, 32], is slightly lower than the experimental value $\text{Br}(K_S^0 \to \gamma\gamma) = (2.63 \pm 0.17) \times 10^{-6}$ [16]. Full agreement is obtained at $\mathscr{O}(p^6)$, once rescattering corrections ($K_S^0 \to \pi\pi \to \pi^+\pi^- \to \gamma\gamma$) are included [33].

The 2-loop amplitude $K_S^0 \to \gamma^*\gamma^* \to \ell^+\ell^-$ (Fig. 5) is also finite [34] because chiral symmetry forbids any CP-invariant local contribution at this order. The predicted rates, $\text{Br}(K_S^0 \to e^+e^-) = 2.1 \times 10^{-14}$ and $\text{Br}(K_S^0 \to \mu^+\mu^-) = 5.1 \times 10^{-12}$ [34], are well below the experimental upper bounds $\text{Br}(K_S^0 \to e^+e^-) < 9 \times 10^{-9}$ [35] and $\text{Br}(K_S^0 \to \mu^+\mu^-) < 2.1 \times 10^{-10}$ [36, 37] (90% CL). This calculation allows us to compute the longitudinal polarization P_L of either muon in the decay
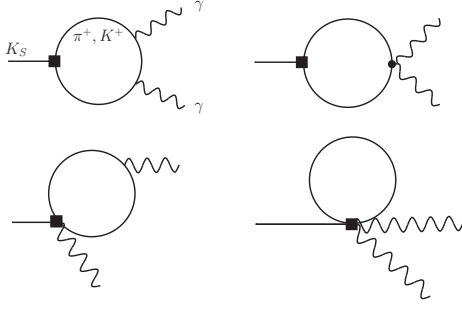
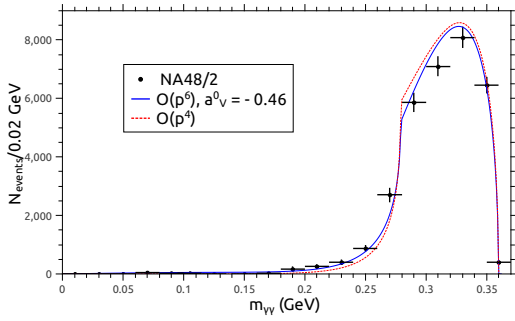**Figure 4:** Lowest-order contributions to $K_S \to \gamma\gamma$.

Figure 5: 2γ contribution to $K^0 \to \ell^+\ell^-$.

Figure 6: $K_L \to \pi^0\gamma\gamma$ spectra at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ in χPT. The data are from Ref. [41].

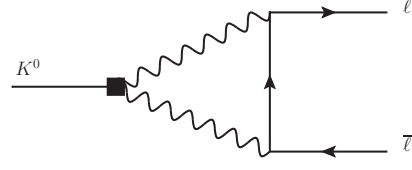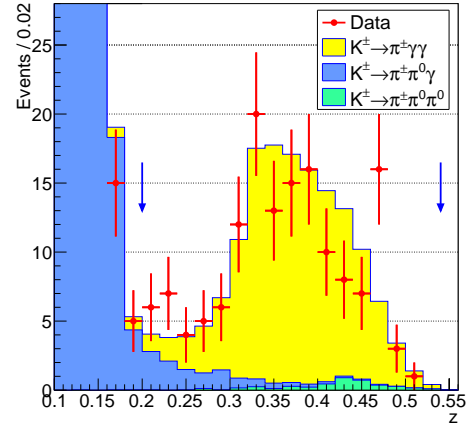**Figure 7:** Measured $K^\pm \to \pi^\pm\gamma\gamma$ spectrum and estimated signal from an $\mathcal{O}(p^6)$ χPT fit [47].

$K_L^0 \to \mu^+\mu^-$, a CP-violating observable which in the SM is dominated by indirect CP violation from K^0–$\bar{K}^0$ mixing. One finds $|P_L| = (2.6 \pm 0.4) \times 10^{-3}$ [34].

3.2 K → πγγ

Again, the symmetry constraints do not allow any tree-level contribution to $K_2^0 \to \pi^0\gamma\gamma$ from $\mathcal{O}(p^4)$ terms in the χPT Lagrangian. The decay amplitude is therefore determined by a finite loop calculation [38, 39, 40]. Due to the large absorptive $\pi^+\pi^-$ contribution, the spectrum in the invariant mass of the two photons is predicted to have a very characteristic behaviour (dotted line in Fig. 6), peaked at high values of $m_{\gamma\gamma}$. The agreement with the measured distribution [41] is remarkably good. However, the $\mathcal{O}(p^4)$ prediction for the rate, $\mathrm{Br}(K_L \to \pi^0\gamma\gamma) = 6.8 \times 10^{-7}$ [38], is significantly smaller than the present PDG average, $\mathrm{Br}(K_L^0 \to \pi^0\gamma\gamma) = (1.27 \pm 0.03) \times 10^{-6}$ [16], indicating that higher-order corrections are sizeable. Unitarity corrections from $K_L^0 \to \pi^+\pi^-\pi^0$ [42, 43] and local vector-exchange contributions [42, 44] restore the agreement at $\mathcal{O}(p^6)$.

A quite similar spectrum is predicted [45] for the charged mode $K^\pm \to \pi^\pm\gamma\gamma$, but in this case there is a free LEC already at $\mathcal{O}(p^4)$. Corrections of $\mathcal{O}(p^6)$ have been also investigated [46]. Both the measured spectrum and the rate can be correctly reproduced [47], as illustrated in Fig. 7.

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3.3 $K_L^0 \to \pi^0 e^+ e^-$

This decay is an interesting process in looking for new CP-violating signatures, because $K_2^0 \to \pi^0 \gamma^*$ violates CP [45, 48]. The CP-conserving amplitude proceeds through a 2γ intermediate state and is suppressed by an additional power of α. Using the $K_L^0 \to \pi^0 \gamma\gamma$ data, the CP-conserving rate is found to be below 10^{-12} [1]. The $K_L^0 \to \pi^0 e^+ e^-$ transition is then dominated by the $\mathcal{O}(\alpha)$ CP-violating contributions [45], both from K^0–$\bar{K}^0$ mixing and direct CP violation. The estimated rate, $\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = (3.1 \pm 0.9) \times 10^{-11}$ [1, 49, 50], is only a factor 10 smaller than the present (90% CL) upper bound of 2.8×10^{-10} [51] and should be reachable in the near future.

3.4 $K \to \pi \nu \bar{\nu}$

Long-distance effects play a negligible role in $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L^0 \to \pi^0 \nu \bar{\nu}$. These processes are dominated by short-distance loops (Z penguin, W box), involving the heavy top quark. The K^+ decay mode receives also sizeable contributions from internal charm-quark exchanges. The decay amplitudes are proportional to the hadronic matrix element of the $\Delta S = 1$ vector current, which (assuming isospin symmetry) can be obtained from $K_{\ell 3}$ decays:

$$T(K \to \pi \nu \bar{\nu}) \sim \sum_{i=c,t} F(V_{id} V_{is}^*; x_i) \, (\bar{\nu}_L \gamma_\mu \nu_L) \, \langle \pi | \bar{s}_L \gamma^\mu d_L | K \rangle , \qquad x_i \equiv m_i^2 / M_W^2 . \qquad (3.1)$$

The small long-distance and isospin-violating corrections can be estimated within χPT. The $K_L^0 \to \pi^0 \nu \bar{\nu}$ transition violates CP and is completely dominated by direct CP violation, the contribution from K^0–$\bar{K}^0$ mixing being only of the order of 1%. Taking the CKM inputs from global fits, one predicts $\mathrm{Br}(K_L^0 \to \pi^0 \nu \bar{\nu}) = (2.9 \pm 0.3) \times 10^{-11}$ and $\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu}) = (8.5 \pm 0.6) \times 10^{-11}$ [52, 53, 54]. The uncertainties are largely parametrical, due to CKM input, m_c, m_t and $\alpha_s(M_Z)$.

The current (90% CL) upper bounds on the charged [55] and neutral [56] modes are

$$\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu}) < 1.85 \times 10^{-10} , \qquad \mathrm{Br}(K_L^0 \to \pi^0 \nu \bar{\nu}) < 3.0 \times 10^{-9} . \qquad (3.2)$$

The ongoing CERN NA62 experiment aims to reach $\mathcal{O}(100)$ $K^+ \to \pi^+ \nu \bar{\nu}$ events (assuming SM rates), while increased sensitivities on the $K_L^0 \to \pi^0 \nu \bar{\nu}$ mode are expected to be achieved by the KOTO experiment at J-PARC.

4. Constraints on scalar leptoquarks from rare kaon decays

Rare kaon decays put strong constraints on NP interactions with non-trivial flavour dynamics. As an illustration, and motivated by the flavour anomalies reported recently in B decays [57], let us consider the implications of kaon data on generic couplings of hypothetical scalar leptoquarks (LQs) to the SM fermions [58]:

$$\mathscr{L}_{\mathrm{LQ}} = \overline{Q^c} \, i\tau_2 \, y_{S_1} L \, S_1 + \overline{d_R^c} \, y_{\tilde{S}_1} \ell_R \, \tilde{S}_1 + \overline{\ell_R} \, y_{R_2} R_2^\dagger Q - \overline{d_R} \, y_{\tilde{R}_2} \tilde{R}_2^T \, i\tau_2 L + \overline{Q^c} \, y_{S_3} \, i\tau_2 \, \boldsymbol{\tau} \cdot \mathbf{S_3} L + \mathrm{h.c.} . \quad (4.1)$$

We have included the five possible types of scalar LQs, coupling to SM particles, with the following $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers [59, 60]: S_1 ($\bar{\mathbf{3}}$, $\mathbf{1}$, $1/3$), $\tilde{S}_1$ ($\bar{\mathbf{3}}$, $\mathbf{1}$, $4/3$), R_2 ($\mathbf{3}$, $\mathbf{2}$, $7/6$), $\tilde{R}_2$ ($\mathbf{3}$, $\mathbf{2}$, $1/6$) and S_3 ($\bar{\mathbf{3}}$, $\mathbf{3}$, $1/3$). Q and L are the left-handed quark and lepton doublets, d_R and ℓ_R the corresponding right-handed singlets and $f^c \equiv \mathscr{C} \bar{f}^T$ indicates the charge-conjugated field of the
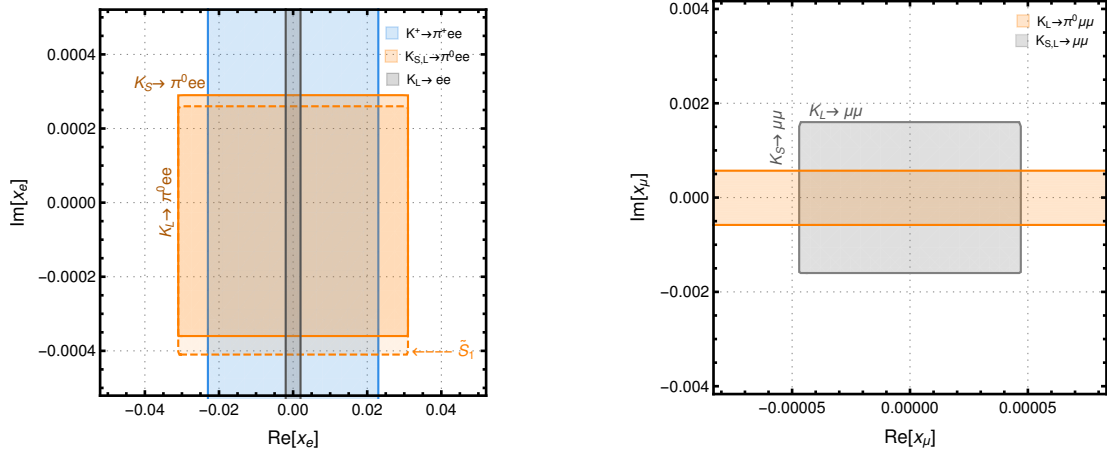
Figure 8: Allowed regions in the plane $(\mathrm{Re}[x_\ell], \mathrm{Im}[x_\ell])$, arising from leptonic and rare semileptonic kaon decays, for the electron (left panel) and muon (right panel) channels [58].

fermion f. All fermion fields carry flavour indices and y_{LQ} are arbitrary Yukawa matrices in flavour space. Eq. (4.1) only displays those couplings relevant for kaon decays.

The exchange of a heavy LQ between two fermionic currents induces tree-level contributions to the FCNC transitions $K^0 \to \ell^+ \ell^-$ and $K \to \pi \ell^+ \ell^-$. They are governed by the following combinations of LQ parameters:

$$
x_e = \left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \begin{cases} y_{\mathrm{LQ}}^{11}\,(y_{\mathrm{LQ}}^{12})^* \\ y_{\mathrm{LQ}}^{11}\,(y_{\mathrm{LQ}}^{21})^* \end{cases}, \qquad
x_\mu = \left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \begin{cases} y_{\mathrm{LQ}}^{21}\,(y_{\mathrm{LQ}}^{22})^* \\ y_{\mathrm{LQ}}^{12}\,(y_{\mathrm{LQ}}^{22})^* \end{cases}, \qquad (4.2)
$$

where x_e and x_μ correspond to the electron and muon modes, respectively. The first line in the brackets corresponds to R_2, while the second line refers to $\tilde{R}_2$, $\tilde{S}_1$ and $(4\times)\,S_3$ (the LQ S_1 does not contribute at tree level to these processes). The current constraints on x_ℓ from different kaon decay modes are displayed in Fig. 8 for the electron (left panel) and muon (right panel) final states. The K_S^0 and K_L^0 decays are complementary, providing separate access to both the real and imaginary parts of the NP couplings, while the decays of the charged kaon restrict their absolute value. The strongest constraints come from $K_L \to \mu^+\mu^-$ [$\mathrm{Re}(x_\mu)$], $K_L \to \pi^0\mu^+\mu-$ [$\mathrm{Im}(x_\mu)$], $K_L \to \pi^0 e^+ e^-$ [$\mathrm{Im}(x_e)$] and $K_L \to e^+ e^-$ [$\mathrm{Re}(x_e)$].

More stringent constraints can be derived from the current experimental limits on LFV transitions. The 90% CL upper bounds $\mathrm{Br}(K_L \to e^\pm \mu^\mp) < 4.7 \times 10^{-12}$ [2] and $\mathrm{Br}(K_L \to \pi^0 e^\pm \mu^\mp) < 7.6 \times 10^{-11}$ [61] imply [58]

$$
\left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \begin{cases} |y_{\mathrm{LQ}}^{21}\,(y_{\mathrm{LQ}}^{12})^* + y_{\mathrm{LQ}}^{22}\,(y_{\mathrm{LQ}}^{11})^*| \\ |y_{\mathrm{LQ}}^{21}\,(y_{\mathrm{LQ}}^{12})^* + y_{\mathrm{LQ}}^{11}\,(y_{\mathrm{LQ}}^{22})^*| \end{cases} < 1.9 \times 10^{-5}, \qquad (4.3)
$$

$$
\left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \begin{cases} |y_{\mathrm{LQ}}^{21}\,(y_{\mathrm{LQ}}^{12})^* - y_{\mathrm{LQ}}^{22}\,(y_{\mathrm{LQ}}^{11})^*| \\ |y_{\mathrm{LQ}}^{21}\,(y_{\mathrm{LQ}}^{12})^* - y_{\mathrm{LQ}}^{11}\,(y_{\mathrm{LQ}}^{22})^*| \end{cases} < 2.9 \times 10^{-4}, \qquad (4.4)
$$

respectively, while $\mathrm{Br}(K^+ \to \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11}$ [4] leads to

$$\left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \left(\left|y_{\mathrm{LQ}}^{21}\,(y_{\mathrm{LQ}}^{12})^*\right|,\, \left|y_{\mathrm{LQ}}^{11}\,(y_{\mathrm{LQ}}^{22})^*\right|\right) < 1.9 \times 10^{-4} \tag{4.5}$$

for the four LQ types.

The $K \to \pi \nu \bar{\nu}$ decay modes only receive tree-level contributions from S_1, S_3 and $\tilde{R}_2$. For identical neutrino flavours $\nu_\ell \bar{\nu}_\ell$ in the final state, the corresponding constraints are shown in Fig. 9 [58]. The relevant combination of LQ parameters is in this case

$$x_\nu = \left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \hat{y}_{\mathrm{LQ}}^{1\ell}\,(\hat{y}_{\mathrm{LQ}}^{2\ell})^*, \tag{4.6}$$

where $\hat{y}_{\mathrm{LQ}} = y_{\mathrm{LQ}} U$ with U the PMNS neutrino mixing matrix. Notice that the neutral mode only constrains $\mathrm{Im}(x_\nu)$, while $K^+ \to \pi^+ \nu \bar{\nu}$ puts limits on both the real and imaginary parts of x_ν.

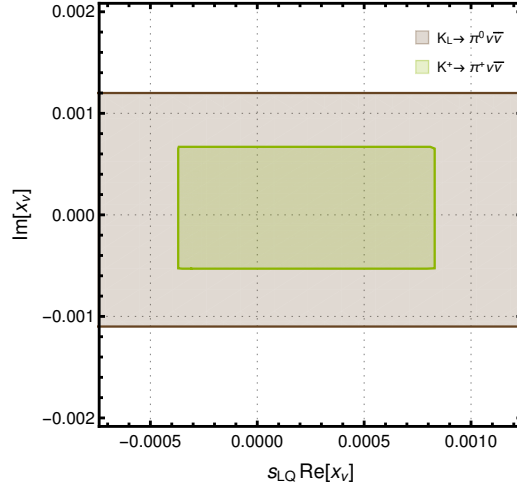
Figure 9: Allowed regions in the plane $(s_{\mathrm{LQ}}\,\mathrm{Re}[x_\nu], \mathrm{Im}[x_\nu])$, arising from $K \to \pi \nu \bar{\nu}$ decays [58]. The sign factor $s_{\mathrm{LQ}} = +1$ for $S_{1,3}$, while $s_{\mathrm{LQ}} = -1$ for $\tilde{R}_2$.

The three LQs induce also $K \to \pi \nu_m \bar{\nu}_n$ decay modes with different neutrino flavours, which should not evade the experimental limits in Eq. (3.2). The K^+ decay implies the upper bound [58]

$$\left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \Big[\sum_{m\neq n}|\hat{y}_{\mathrm{LQ}}^{1m}\,(\hat{y}_{\mathrm{LQ}}^{2n})^*|^2\Big]^{1/2} < 6.0 \times 10^{-4}, \tag{4.7}$$

while the neutral decay mode puts the constraint [58]

$$\left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{LQ}}}\right)^2 \times \Big[\sum_{m\neq n}|\hat{y}_{\mathrm{LQ}}^{1m}\,(\hat{y}_{\mathrm{LQ}}^{2n})^* - \hat{y}_{\mathrm{LQ}}^{2m}\,(\hat{y}_{\mathrm{LQ}}^{1n})^*|^2\Big]^{1/2} < 1.1 \times 10^{-3}. \tag{4.8}$$

The KOTO collaboration has recently reported the observation of four $K_L^0 \to \pi^0 \nu \bar{\nu}$ events, with an expected background of only 0.05 ± 0.02 events [62]. Removing one of the events that is suspected to originate in underestimated upstream activity background, the quoted single event

sensitivity of 6.9×10^{-10} would correspond to $\mathrm{Br}(K_L^0 \to \pi^0 \nu \bar{\nu}) \sim 2 \times 10^{-9}$, well above the new Grossman-Nir limit [63] implied by the NA62 upper bound on $\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})$:

$$\mathrm{Br}(K_L^0 \to \pi^0 \nu \bar{\nu}) \, < \, 4.2 \times \mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu}) \, < \, 7.8 \times 10^{-10} \,. \qquad (4.9)$$

This limit is valid under quite generic assumptions, provided the lepton flavour is conserved, and it can be directly inferred from the predicted LQ-induced decay amplitudes [58], if there are only identical neutrino flavours in the final state.

In order to reach the KOTO signal, one needs a sizeable decay amplitude into neutrinos with different flavours ($n \neq m$). This could be easily achieved within the S_1, S_3 and $\tilde{R}_2$ LQ scenarios. A confirmation of the KOTO events would just imply that the combination of LQ couplings in Eq. (4.8) takes a non-zero value quite close to its current upper bound, indicating a violation of lepton flavour. Other possible NP interpretations have been already considered in Refs. [64, 65, 66, 67, 68, 69, 70].

Acknowledgements

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