PROCEEDINGS OF SCIENCE

$B_c \to J/\psi$ Form Factors and $R(J/\psi)$ using Lattice QCD

Judd Harrison*, **Christine T. H. Davies**
SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, UK
E-mail: judd.harrison@glasgow.ac.uk, christine.davies@glasgow.ac.uk

Andrew Lytle
INFN, Sezione di Roma Tor Vergata, Via della Ricerca Scientifica 1, 00133 Roma RM, Italy
E-mail: andrew.lytle@glasgow.ac.uk

We present a lattice QCD determination of the $B_c \to J/\psi$ vector and axial-vector form factors over the full physical q^2 range with non-perturbatively renormalised lattice currents. Our calculation uses the Highly Improved Staggered Quark (HISQ) action on the second generation MILC gluon ensembles including light, strange and charm sea quarks. We use HISQ heavy quarks, with a range of masses going up to the b on our finest lattices. Finally we use these form factors to compute the differential decay rates and construct the ratio of decay rates $R(J/\psi) = \Gamma(B_c \to J/\psi\tau^-\overline{\nu}_\tau)/\Gamma(B_c \to J/\psi\ell^-\overline{\nu}_\ell)$ where ℓ is either a muon or electron.

18th International Conference on B-Physics at Frontier Machines - Beauty2019 -
29 September / 4 October, 2019
Ljubljana, Slovenia

*Speaker.

PoS(Beauty2019)059

1. Introduction

Experimental measurements of flavor-changing B decays are a source of tension with the Standard Model, e.g.[1, 2, 3, 4]. Future experimental measurements of the differential decay rate for $B_c^- \to J/\psi\mu\overline{\nu}$ motivate the precise calculation of the relevant form factors in the Standard Model. The recent discussion [5] of the reliability of the CLN and BGL parameterisations with regards to their extrapolation to zero recoil also suggests that predictions for the full q^2 range would be valuable not just for $B_c \to J/\psi$ but also for $B_{(s)} \to D^*_{(s)}$, for which this calculation serves as a prototype. The $B_c \to J/\psi$ decay is a more attractive starting point for a lattice QCD calculation than $B_{(s)} \to D^*_{(s)}$ as the charm propagators are numerically less expensive to compute, the statstical accuracy is higher and the finite volume effects are smaller due to the absence of valence light quarks. The main obstacle faced by such a lattice calculation is the large heavy quark mass necessitating very fine lattice spacings. The method used here to avoid this problem, referred to as Heavy-HISQ, involves using unphysically light HISQ heavy quarks and extrapolating to the physical b mass. Recent HPQCD results have demonstrated the efficacy of this approach applied to related semileptonic processes [6, 7]. This calculation will also serve as a prototype for the extension to non-zero recoil of the $B_s \to D_s^*$ form factors as well as for the eventual calculation of $B \to D^*$ form factors across the full q^2 range. These future calculations would also allow for the lattice calculation of $R(D^*)$, the branching ratio between τ and μ final lepton states for $B \to D^*$, for which there currently exists a 3σ tension between the Standard Model prediction and experiment [8]

Using the full differential decay rate, [9], and assuming the J/ψ decay is purely electromagnetic and summing over $\mu^+\mu^-$ helicities, we write the partial rate

$$\frac{d\Gamma}{dq^2} = \frac{G^2}{(2\pi)^3}|V_{cb}|^2 \frac{(q^2-M_\ell^2)^2|\vec{p}'|}{12M_{B_c}^2 q^2}\mathscr{B}(J/\psi \to \mu^+\mu^-)\Big[\big(H_-{}^2+H_0{}^2+H_+{}^2\big) + \frac{M_\ell^2}{2q^2}\big(H_-{}^2+H_0{}^2+H_+{}^2+3H_t{}^2\big)\Big], \quad (1.1)$$

where p' is the momentum of the J/ψ, $|\vec{p}'|$ is the magnitude of the J/ψ spatial momentum in the B_c rest frame, p is the momentum of the B_c, $q = p - p'$ and M_ℓ is the lepton mass. The helicity amplitudes are defined as

$$\begin{aligned}
H_\pm(q^2) =& (M_{B_c}+M_{J/\psi})A_1(q^2) \mp \frac{2M_{B_c}|\vec{p}'|}{M_{B_c}+M_{J/\psi}}V(q^2),\\
H_0(q^2) =& \frac{1}{2M_{J/\psi}\sqrt{q^2}}\Big(-4\frac{M_{B_c}^2|\vec{p}'|^2}{M_{B_c}+M_{J/\psi}}A_2(q^2)\\
&+ (M_{B_c}+M_{J/\psi})(M_{B_c}^2-M_{J/\psi}^2-q^2)A_1(q^2)\Big),\\
H_t(q^2) =& \frac{2M_{B_c}|\vec{p}'|}{\sqrt{q^2}}A_0(q^2),
\end{aligned} \quad (1.2)$$

and correspond to the nonzero components of $\overline{\varepsilon}^*_\mu\langle J/\psi(\varepsilon)|\bar{c}\gamma^\mu(1-\gamma^5)b|B_c^-\rangle$ with respect to the J/ψ and W^- polarisation vectors ε and $\overline{\varepsilon}$. The form factors in (1.2) are the standard Lorentz invariant ones, their relation to the matrix elements is given by [10]

$$\langle J/\psi(p',\varepsilon)|\bar{c}\gamma^\mu b|B_c^-(p)\rangle = \frac{2iV(q^2)}{M_{B_c}+M_{J/\psi}}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p'_\rho p_\sigma$$
$$\langle J/\psi(p',\varepsilon)|\bar{c}\gamma^\mu\gamma^5 b|B_c^-\rangle = 2M_{J/\psi}A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu$$
$$+(M_{B_c}+M_{J/\psi})A_1(q^2)\Big[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\Big]$$
$$-A_2(q^2)\frac{\varepsilon^*\cdot q}{M_{B_c}+M_{J/\psi}}\Big[p^\mu+p'^\mu-\frac{M_{B_c}^2-M_{J/\psi}^2}{q^2}q^\mu\Big]. \tag{1.3}$$

2. Lattice Calculation

On the lattice we work with the H_c^-, the heavy-charm pseudoscalar with heavy quark mass m_h, at rest. We compute correlation functions

$$\begin{aligned} C_{\mathrm{2pt}}^{J/\psi}(t,0) =&\langle 0|\bar{c}\gamma^\nu c(t)\left(\bar{c}\gamma^\nu c(0)\right)^\dagger|0\rangle,\\ C_{\mathrm{2pt}}^{H_c}(t,0) =&\langle 0|\left(\bar{b}\gamma^5 c(t)\right)^\dagger\bar{b}\gamma^5 c(0)|0\rangle,\\ C_{\mathrm{3pt}}(T,t,0) =&\langle 0|\bar{c}\gamma^\nu c(T)\ \bar{c}\Gamma b(t)\ \bar{b}\gamma^5 c(0)|0\rangle \end{aligned} \tag{2.1}$$

where repeated indices are not summed over. We fit these to

$$\begin{aligned} \tilde{C}_{\mathrm{2pt}}^{J/\psi}(t,0) =& \sum_n\Big((A^n)^2e^{-tE_n}+(-1)^t(A^n_o)^2e^{-tE_n^o}\Big),\\ \tilde{C}_{\mathrm{2pt}}^{H_c}(t,0) =& \sum_n\Big((B^n)^2e^{-tM_n}+(-1)^t(B^n_o)^2e^{-tM_n^o}\Big),\\ \tilde{C}_{\mathrm{3pt}}(T,t,0) =& \sum_{n,m}\Big(A^nB^mJ^{nm}e^{-(T-t)E_n-tM_m}\\ &+(-1)^{T+t}A^n_oB^mJ^{nm}_{oe}e^{-(T-t)E^o_n-tM_m}\\ &+(-1)^tA^nB^m_oJ^{nm}_{eo}e^{-(T-t)E_n-tM^o_m}\\ &+(-1)^TA^n_oB^m_oJ^{nm}_{oo}e^{-(T-t)E^o_n-tM^o_m}\Big) \end{aligned} \tag{2.2}$$

respectively, where n, m are on shell particle states with quantum numbers resulting in nonzero amplitudes and the o labels indicate energies and amplitudes corresponding to the time-doubled states typically present with HISQ quarks. The lowest energy, non-oscillating contributions, from which we extract matrix elements, give:

$$A^0 = \frac{N_{J/\psi}}{\sqrt{2E_{J/\psi}}}\left(1+\frac{\vec{p}'^2_{(\nu)}}{M^2_{J/\psi}}\right)^{1/2}, \qquad B^0=\frac{N_{H_c}}{\sqrt{2M_{H_c}}}. \tag{2.3}$$

Where $\vec{p}_{(\nu)}$ is the ν component of the J/ψ spatial momentum, with ν corresponding to the choice of polarisation in (2.1). The ground state matrix element which we extract from our lattice fits is

Set	w_0/a	$N_x \times N_t$	am_{l0}	am_{s0}	am_{c0}	am_h^{val}	am_c^{val}	n_{configs}
1	1.9006(20)	32×96	0.0074	0.037	0.440	$0.6, 0.65, 0.8$	0.449	980
2	2.896(6)	48×144	0.0048	0.024	0.286	$0.427, 0.525, 0.65, 0.8$	0.274	500
3	3.892(12)	64×192	0.00316	0.0158	0.188	$0.5, 0.65, 0.8$	0.194	374
4	1.9518(7)	64×96	0.00316	0.0158	0.188	$0.5, 0.65, 0.8$	0.433	300

Table 1: Details of the gauge field configurations used in our calculation [11, 12]. Set 1 is referred to as ‘fine’, set 2 as ‘superfine’, set 3 as ‘ultrafine’ and set 4 as ‘physical fine’. The values of the relative scale-setting quantity w_0/a are taken from [13, 14, 7], while the physical value $w_0 = 0.1715(9)$ fm is determined in [15]. The charm and heavy valence masses given were taken from [6].

Set	θ	T
1	$0, 0.361, 0.723, 1.084, 1.446, 1.807$	$14, 17, 20$
2	$0, 0.826, 1.651, 2.477, 3.302, 4.128$	$22, 25, 28$
3	$0, 1.241, 2.483, 3.724, 4.966, 6.207$	$31, 36, 41$
4	$0, 0.361, 0.723, 1.084, 1.446, 1.807$	$14, 17, 20$

Table 2: Details of the masses, twists and three point ranges, T. The twists are given in units of π/L and are applied along both the x and y direction.

then, for spatial J/ψ operator component ν and with current $\bar{c}\Gamma^\mu b$,

$$J^{00} = \sum_\lambda \frac{\varepsilon^\nu(\lambda)\langle J/\psi(\lambda)|\bar{c}\Gamma b|H_c^-\rangle}{\sqrt{2E_{J/\psi}2M_{H_c}\left(1+\vec{p}^{\prime 2}_{(\nu)}/M^2_{J/\psi}\right)}}. \tag{2.4}$$

We use the the second generation MILC gluon ensembles including light, strange and charm sea quarks [11, 12] and compute HISQ charm and heavy quark propagators. The details of these gauge configurations are given in table 1.

We give momentum to the c quark propagator via a momentum twist. The twists were chosen to evenly span the physical q^2 range for the largest value of am_h, approximated from aM_{H_c} values given in [6] and the physical J/ψ mass. The values of twists and T are given in table 2. The renormalisation factors relating the matrix elements of our lattice current operators were computed in [6] for the axial-vector and in [7] for the vector current using the PCAC and PCVC relations respectively and include mass dependent discretisation correction terms for the HISQ-quark tree level wavefunction renormalisation computed in [16], though these discretisation corrections are numerically tiny. The correlator fits discussed in this section were done simultaneously on each set using the **corrfitter** python package [17].

3. Extrapolation to the Physical Point

In order to fit the q^2 dependence we use the z-expansion [18] and map the physical q^2 range to within the unit circle via the change of variables

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \tag{3.1}$$

where

$$t_\pm = (M_{H_c} \pm M_{J/\psi})^2. \tag{3.2}$$

Physical particles with $\bar{b}c$ quark content, masses between the pair production threshold and $t_-^{1/2}$ and the appropriate quantum numbers to couple to the current operator result in the appearance of a pole in the corresponding form factor. Following [19] we include these poles in our fit form using the form

$$P(q^2) = \left(\frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - M^2_{\text{pole}}}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - M^2_{\text{pole}}}} \right)^{-1}. \tag{3.3}$$

For the pole masses we use the well known relation $M_V^2 = M_P^2 + \Delta_{PV}$, where we take $\Delta_{PV} = 0.56(10)\text{GeV}^2$ [20], and $M_{H_c^A} = M_{H_c^V} + \Delta_{AV}$ where $\Delta_{AV} = 0.410(13)\text{GeV}$ [21]. Following [7] we take our fit function to be

$$F(q^2) = P(q^2) \sum_{n=0}^{3} a_n z^n \left(1 + \delta_n\right) \tag{3.4}$$

where

$$a_n = \sum_{j,k,l=0}^{3} b_n^{jkl} \Delta_h^{(j)} \left(\frac{am_c^{\text{val}}}{\pi}\right)^{2k} \left(\frac{am_h^{\text{val}}}{\pi}\right)^{2l}, \tag{3.5}$$

$\Delta_h^{(0)} = 1$ and

$$\Delta_h^{(j\neq 0)} = \left(\frac{2\Lambda_{\text{QCD}}}{M_{\eta_h}}\right)^j - \left(\frac{2\Lambda_{\text{QCD}}}{M_{\eta_b}}\right)^j. \tag{3.6}$$

δ_n captures mistuning effects following [6] and we take $\Lambda_{\text{QCD}} = 0.5\text{GeV}$. We also impose the kinematical constraint at $q^2 = 0$, $2M_{J/\psi}A_0(0) = (M_{J/\psi} + M_{H_c})A_1(0) - (M_{J/\psi} - M_{H_c})A_2(0)$. We take priors of $0(1)$ for each b_n, except those terms of order $\mathscr{O}(a^2)$ for which we take $0(0.5)$ encompassing the HISQ 1-loop improvement. All remaining priors were taken as $0(1)$. The fit is done simultaneously across all form factors in order to preserve correlations important for constructing helicity amplitudes. The results for A_1 and V, whose contributions to the decay rate are dominant, are given in Figure 1.

4. Results

Using the form factors we construct the helicity amplitudes and differential decay rates using (1.2). Where an integration over q^2 is necessary we use a simple trapezoidal interpolation in order to ensure covariaces are carried through correctly, taking sufficiently many points that the results are insensitive to using additional points. The differential rate $d\Gamma/dq^2$ is plotted in figure 2 where we plot the rate for $\ell = \mu$ and $\ell = \tau$.

We also compute the total decay rates for the case $\ell = \mu$ and $\ell = \tau$. We find

$$\Gamma(B_c^- \to J/\psi \mu^- \overline{\nu}_\mu) = |V_{cb}|^2 1.70(18) \times 10^{13} s^{-1} \text{ (PRELIMINARY)}, \tag{4.1}$$

$$\Gamma(B_c^- \to J/\psi \tau^- \overline{\nu}_\tau) = |V_{cb}|^2 4.40(36) \times 10^{12} s^{-1} \text{ (PRELIMINARY)}, \tag{4.2}$$

and the ratio

$$R(J/\psi) = 0.2592(92) \text{ (PRELIMINARY)}. \tag{4.3}$$

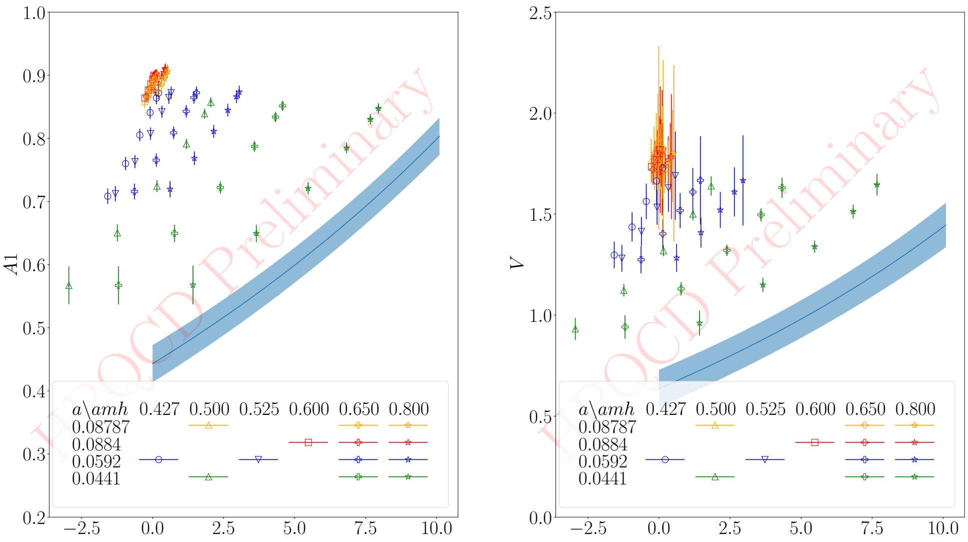
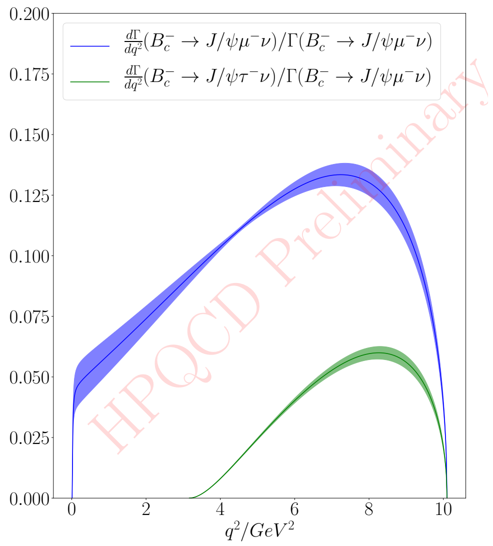


Figure 1: q^2 dependence of A_1, V and heavy-HISQ extrapolated curves.

Figure 2: The differential rate $d\Gamma/dq^2$, normalised by the total decay rate Γ.



References

[1] BABAR collaboration, *Evidence for an excess of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ decays*, *Phys. Rev. Lett.* **109** (2012) 101802 [1205.5442].

[2] BELLE collaboration, *Measurement of the Differential Branching Fraction and Forward-Backword Asymmetry for $B \to K^{(*)}\ell^+\ell^-$*, *Phys. Rev. Lett.* **103** (2009) 171801 [0904.0770].

[3] BABAR collaboration, *Measurement of Branching Fractions and Rate Asymmetries in the Rare Decays $B \to K^{(*)}l^+l^-$*, *Phys. Rev.* **D86** (2012) 032012 [1204.3933].

[4] BABAR collaboration, *Measurement of an Excess of* $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ *Decays and Implications for Charged Higgs Bosons*, *Phys. Rev.* **D88** (2013) 072012 [1303.0571].

[5] D. Bigi, P. Gambino and S. Schacht, *A fresh look at the determination of* $|V_{cb}|$ *from* $B \to D^*\ell\nu$, *Phys. Lett.* **B769** (2017) 441 [1703.06124].

[6] E. McLean, C. T. H. Davies, A. T. Lytle and J. Koponen, *Lattice QCD form factor for* $B_s \to D_s^* l\nu$ *at zero recoil with non-perturbative current renormalisation*, *Phys. Rev.* **D99** (2019) 114512 [1904.02046].

[7] E. McLean, C. T. H. Davies, J. Koponen and A. T. Lytle, $B_s \to D_s\ell\nu$ *Form Factors for the full* q^2 *range from Lattice QCD with non-perturbatively normalized currents*, 1906.00701.

[8] P. Gambino, M. Jung and S. Schacht, *The* V_{cb} *puzzle: An update*, *Phys. Lett.* **B795** (2019) 386 [1905.08209].

[9] T. D. Cohen, H. Lamm and R. F. Lebed, *Tests of the standard model in BD , BD* and BcJ/*, *Phys. Rev.* **D98** (2018) 034022 [1807.00256].

[10] J. D. Richman and P. R. Burchat, *Leptonic and semileptonic decays of charm and bottom hadrons*, *Rev. Mod. Phys.* **67** (1995) 893 [hep-ph/9508250].

[11] MILC collaboration, *Lattice QCD Ensembles with Four Flavors of Highly Improved Staggered Quarks*, *Phys. Rev.* **D87** (2013) 054505 [1212.4768].

[12] MILC collaboration, *Scaling studies of QCD with the dynamical HISQ action*, *Phys. Rev.* **D82** (2010) 074501 [1004.0342].

[13] B. Chakraborty, C. T. H. Davies, P. G. de Oliviera, J. Koponen, G. P. Lepage and R. S. Van de Water, *The hadronic vacuum polarization contribution to* a_μ *from full lattice QCD*, *Phys. Rev.* **D96** (2017) 034516 [1601.03071].

[14] B. Chakraborty, C. T. H. Davies, B. Galloway, P. Knecht, J. Koponen, G. C. Donald et al., *High-precision quark masses and QCD coupling from* $n_f = 4$ *lattice QCD*, *Phys. Rev.* **D91** (2015) 054508 [1408.4169].

[15] R. J. Dowdall, C. T. H. Davies, G. P. Lepage and C. McNeile, *Vus from pi and K decay constants in full lattice QCD with physical u, d, s and c quarks*, *Phys. Rev.* **D88** (2013) 074504 [1303.1670].

[16] C. Monahan, J. Shigemitsu and R. Horgan, *Matching lattice and continuum axial-vector and vector currents with nonrelativistic QCD and highly improved staggered quarks*, *Phys. Rev.* **D87** (2013) 034017 [1211.6966].

[17] G. P. Lepage, *corrfitter*, *Corrfitter Version 6.0.7* (github.com/gplepage/corrfitter) .

[18] E. Gustafson and Y. Meurice, *Determining the Efficacy of Different Parameterizations of the z-expansion*, *PoS* **LATTICE2018** (2019) 291 [1807.10257].

[19] R. J. Hill, *The Modern description of semileptonic meson form factors*, *eConf* **C060409** (2006) 027 [hep-ph/0606023].

[20] X. T. Song, *Vector - Pseudoscalar Mass Splitting and* $\Upsilon - \eta_b$ *Mass Difference*, *Phys. Rev.* **D40** (1989) 3655.

[21] R. J. Dowdall, C. T. H. Davies, T. C. Hammant and R. R. Horgan, *Precise heavy-light meson masses and hyperfine splittings from lattice QCD including charm quarks in the sea*, *Phys. Rev.* **D86** (2012) 094510 [1207.5149].

PoS(Beauty2019)059