$B_c \rightarrow J/\psi$ Form Factors and $R(J/\psi)$ using Lattice QCD

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We present a lattice QCD determination of the $B_c \rightarrow J/\psi$ vector and axial-vector form factors over the full physical $q^2$ range with non-perturbatively renormalised lattice currents. Our calculation uses the Highly Improved Staggered Quark (HISQ) action on the second generation MILC gluon ensembles including light, strange and charm sea quarks. We use HISQ heavy quarks, with a range of masses going up to the $b$ on our finest lattices. Finally we use these form factors to compute the differential decay rates and construct the ratio of decay rates $R(J/\psi) = \Gamma(B_c \rightarrow J/\psi \tau^- \bar{\nu}_\tau)/\Gamma(B_c \rightarrow J/\psi \ell^- \bar{\nu}_\ell)$ where $\ell$ is either a muon or electron.

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1. Introduction

Experimental measurements of flavor-changing $B$ decays are a source of tension with the Standard Model, e.g.[1,2,3,4]. Future experimental measurements of the differential decay rate for $B_c^- \to J/\psi \mu \bar{\nu}$ motivate the precise calculation of the relevant form factors in the Standard Model. The recent discussion [5] of the reliability of the CLN and BGL parameterisations with regards to their extrapolation to zero recoil also suggests that predictions for the full $q^2$ range would be valuable not just for $B_c \to J/\psi$ but also for $B_{(s)} \to D^{*}_{(s)}$, for which this calculation serves as a prototype. The $B_c \to J/\psi$ decay is a more attractive starting point for a lattice QCD calculation than $B_{(s)} \to D^{*}_{(s)}$ as the charm propagators are numerically less expensive to compute, the statistical accuracy is higher and the finite volume effects are smaller due to the absence of valence light quarks. The main obstacle faced by such a lattice calculation is the large heavy quark mass necessitating very fine lattice spacings. The method used here to avoid this problem, referred to as Heavy-HISQ, involves using unphysically light HISQ heavy quarks and extrapolating to the physical mass. Recent HPQCD results have demonstrated the efficacy of this approach applied to related semileptonic processes [6,7]. This calculation will also serve as a prototype for the extension to non-zero recoil of the $B_c \to D^{*}_{s}$ form factors as well as for the eventual calculation of $B \to D^{*}$ form factors across the full $q^2$ range. These future calculations would also allow for the lattice calculation of $R(D^{*})$, the branching ratio between $\tau$ and $\mu$ final lepton states for $B \to D^{*}$, for which there currently exists a $3\sigma$ tension between the Standard Model prediction and experiment [8].

Using the full differential decay rate, [9], and assuming the $J/\psi$ decay is purely electromagnetic and summing over $\mu^+\mu^-$ helicities, we write the partial rate

$$\frac{d\Gamma}{dq^2} = \frac{G^2}{(2\pi)^3} |V_{cb}|^2 \left| q^2 - M_{J/\psi}^2 \right|^2 |\vec{p}'|^2 \frac{1}{12M_{B_c} q^2} \mathcal{B}(J/\psi \to \mu^+\mu^-) \left[ (H_-^2 + H_0^2 + H_+^2) \right. $$

$$\left. + \frac{M_{J/\psi}^2}{2q^2} (H_-^2 + H_0^2 + H_+^2 + 3H_r^2) \right], \hspace{1cm} (1.1)$$

where $p'$ is the momentum of the $J/\psi$, $|\vec{p}'|$ is the magnitude of the $J/\psi$ spatial momentum in the $B_c$ rest frame, $p$ is the momentum of the $B_c$, $q = p - p'$ and $M_\ell$ is the lepton mass. The helicity amplitudes are defined as

$$H_\pm(q^2) = (M_{B_c} + M_{J/\psi})A_1(q^2) \mp \frac{2M_{B_c} |\vec{p}'|}{M_{B_c} + M_{J/\psi}} V(q^2),$$

$$H_0(q^2) = \frac{1}{2M_{J/\psi} \sqrt{q^2} } \left[ -4 \frac{M_{B_c}^2 |\vec{p}'|^2}{M_{B_c} + M_{J/\psi}} A_2(q^2) \right. $$

$$\left. + (M_{B_c} + M_{J/\psi})(M_{B_c}^2 - M_{J/\psi}^2 - q^2)A_1(q^2) \right],$$

$$H_1(q^2) = \frac{2M_{B_c} |\vec{p}'|}{\sqrt{q^2}} A_0(q^2), \hspace{1cm} (1.2)$$

and correspond to the nonzero components of $\bar{c} \gamma_\mu (J/\psi(\epsilon)) |c \gamma^\mu (1 - \gamma^5)b|B_c^-$ with respect to the $J/\psi$ and $W^-$ polarisation vectors $\epsilon$ and $\bar{\epsilon}$. The form factors in (1.2) are the standard Lorentz invariant
ones, their relation to the matrix elements is given by \[ \langle J/\psi(p', \epsilon)|\bar{c}\gamma^{\mu}b|B_c^- (p) \rangle = \frac{2iV(q^2)}{M_{B_c} + M_{J/\psi}} e^{\mu\nu\rho\sigma} \epsilon_{\sigma}^{\ast} p'_{\rho} p_{\nu} \langle J/\psi(p', \epsilon)|\bar{c}\gamma^{\mu}b|B_c^- (p) \rangle = 2M_{J/\psi} A_0(q^2) \frac{\epsilon^{\ast} \cdot q}{q^2} q^\mu + (M_{B_c} + M_{J/\psi}) A_1(q^2) \left[ \epsilon^{\ast} \cdot q - \frac{\epsilon^{\ast} \cdot q}{q^2} q^\mu \right] - A_2(q^2) \frac{\epsilon^{\ast} \cdot q}{M_{B_c} + M_{J/\psi}} \left[ p^\mu + p'^\mu - \frac{M_{B_c}^2 - M_{J/\psi}^2}{q^2} q^\mu \right]. \tag{1.3} \]

2. Lattice Calculation

On the lattice we work with the \( H^- \), the heavy-charm pseudoscalar with heavy quark mass \( m_h \), at rest. We compute correlation functions

\[
C_{2\text{pt}}^{J/\psi}(t, 0) = \langle 0| \bar{c} \gamma^\nu c(t) (\bar{c} \gamma^\nu c(0))^\dagger | 0 \rangle, \\
C_{2\text{pt}}^{H_c}(t, 0) = \langle 0| (\bar{b} \gamma^5 c(t))^\dagger \bar{b} \gamma^5 c(0) | 0 \rangle, \\
C_{3\text{pt}}(T, t, 0) = \langle 0| \bar{c} \gamma^\nu c(T) \bar{c} T b(t) \bar{b} \gamma^5 c(0) | 0 \rangle
\]

(2.1)

where repeated indices are not summed over. We fit these to

\[
C_{2\text{pt}}^{J/\psi}(t, 0) = \sum_n \left( A^n e^{-iE_n t} + (-1)^n \bar{A}^n e^{-iE_n t} \right), \\
C_{2\text{pt}}^{H_c}(t, 0) = \sum_n \left( B^n e^{-iM_n t} + (-1)^n \bar{B}^n e^{-iM_n t} \right), \\
\tilde{C}_{3\text{pt}}(T, t, 0) = \sum_{n,m} \left( A^n B^m \epsilon_{n,m} e^{-i(T-T')E_n t} \right),
\]

(2.2)

respectively, where \( n, m \) are on shell particle states with quantum numbers resulting in nonzero amplitudes and the \( o \) labels indicate energies and amplitudes corresponding to the time-doubled states typically present with HISQ quarks. The lowest energy, non-oscillating contributions, from which we extract matrix elements, give:

\[
A^0 = \frac{N_{J/\psi}}{\sqrt{2E_{J/\psi}}} \left( 1 + \bar{p}_{(\psi)}^2 \right)^{1/2}, \\
B^0 = \frac{N_{H_c}}{\sqrt{2M_{H_c}}}. \tag{2.3}
\]

Where \( \bar{p}_{(\psi)} \) is the \( \psi \) component of the \( J/\psi \) spatial momentum, with \( \psi \) corresponding to the choice of polarisation in (2.1). The ground state matrix element which we extract from our lattice fits is
then, for spatial $J/\psi$ operator component $\nu$ and with current $\epsilon \Gamma^{a} b$,
\begin{equation}
\langle \bar{c}\gamma^{\alpha}\Gamma_{\nu}c\rangle \sim \sum_{\lambda} \frac{\epsilon^{\nu}(\lambda)|J/\psi(\lambda)|\epsilon \Gamma^{a} b|H_{c}^{\nu}\rangle}{\sqrt{2E_{J/\psi}2M_{H_{c}}\left(1+\frac{P^{2}(\nu)}{M_{J/\psi}^{2}}\right)}} \tag{2.4}
\end{equation}

We use the second generation MILC gluon ensembles including light, strange and charm sea quarks [1,2] and compute HISQ charm and heavy quark propagators. The details of these gauge configurations are given in Table 1.

We give momentum to the $c$ quark propagator via a momentum twist. The twists were chosen to evenly span the physical $q^{2}$ range for the largest value of $am_{h}$, approximated from $aM_{H_{c}}$ values given in [3] and the physical $J/\psi$ mass. The values of twists and $T$ are given in table 2. The renormalisation factors relating the matrix elements of our lattice current operators were computed in [3] for the axial-vector and in [4] for the vector current using the PCAC and PCVC relations respectively and include mass dependent discretisation correction terms for the HISQ-quark tree level wavefunction renormalisation computed in [5], though these discretisation corrections are numerically tiny. The correlator fits discussed in this section were done simultaneously on each set using the corrfit package [6].

### 3. Extrapolation to the Physical Point

In order to fit the $q^{2}$ dependence we use the $z$-expansion [8] and map the physical $q^{2}$ range to within the unit circle via the change of variables
\begin{equation}
z = \frac{\sqrt{t_{+} - q^{2}} - \sqrt{t_{+} - t_{0}}}{\sqrt{t_{+} - q^{2}} + \sqrt{t_{+} - t_{0}}} \tag{3.1}
\end{equation}

Table 1: Details of the gauge field configurations used in our calculation [1,2]. Set 1 is referred to as ‘fine’, set 2 as ‘superfine’, set 3 as ‘ultrafine’ and set 4 as ‘physical fine’. The values of the relative scale-setting quantity $w_{0}/a$ are taken from [4,5,7], while the physical value $w_{0} = 0.1715(9)$ fm is determined in [8]. The charm and heavy quark masses given were taken from [9].

<table>
<thead>
<tr>
<th>Set</th>
<th>$w_{0}/a$</th>
<th>$N_{x} \times N_{t}$</th>
<th>$am_{h0}$</th>
<th>$am_{s0}$</th>
<th>$am_{c0}$</th>
<th>$am_{h}^{\text{val}}$</th>
<th>$am_{c}^{\text{val}}$</th>
<th>$n_{\text{configs}}$</th>
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<tr>
<td>1</td>
<td>1.9006(20)</td>
<td>32 × 96</td>
<td>0.0074</td>
<td>0.037</td>
<td>0.440</td>
<td>0.6, 0.65, 0.8</td>
<td>0.449</td>
<td>980</td>
</tr>
<tr>
<td>2</td>
<td>2.896(6)</td>
<td>48 × 144</td>
<td>0.0048</td>
<td>0.024</td>
<td>0.286</td>
<td>0.427, 0.525, 0.65, 0.8</td>
<td>0.274</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>3.892(12)</td>
<td>64 × 192</td>
<td>0.00316</td>
<td>0.0158</td>
<td>0.188</td>
<td>0.5, 0.65, 0.8</td>
<td>0.194</td>
<td>374</td>
</tr>
<tr>
<td>4</td>
<td>1.9518(7)</td>
<td>64 × 96</td>
<td>0.00316</td>
<td>0.0158</td>
<td>0.188</td>
<td>0.5, 0.65, 0.8</td>
<td>0.433</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 2: Details of the masses, twists and three point ranges, $T$. The twists are given in units of $\pi/L$ and are applied along both the $x$ and $y$ direction.
where
\[ t_\pm = (M_{H_L} \pm M_{J/\psi})^2. \] (3.2)

Physical particles with $\bar{b}c$ quark content, masses between the pair production threshold and $t^{1/2}$ and the appropriate quantum numbers to couple to the current operator result in the appearance of a pole in the corresponding form factor. Following \cite{19} we include these poles in our fit form using the form
\[ P(q^2) = \left( \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - M_{\text{pole}}^2}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - M_{\text{pole}}^2}} \right)^{-1}. \] (3.3)

For the pole masses we use the well known relation $M_\beta^2 = M_p^2 + \Delta_{PV}$, where we take $\Delta_{PV} = 0.56(10)\text{GeV}^2$ \cite{20}, and $M_{H_L} = M_{H_T} + \Delta_{AV}$ where $\Delta_{AV} = 0.410(13)\text{GeV}$ \cite{21}. Following \cite{22} we take our fit function to be
\[ F(q^2) = P(q^2) \sum_{n=0}^{3} a_n e^n (1 + \delta_n) \] (3.4)
where
\[ a_n = \sum_{j,k,l=0} \beta_n^{jkl} \Delta_h^{(j)} \left( \frac{am_{\pi}}{\pi} \right)^{2k} \left( \frac{am_{h_{10}}}{\pi} \right)^{2l}. \] (3.5)

\[ \Delta_h^{(0)} = 1 \quad \text{and} \quad \Delta_h^{(j \neq 0)} = \left( \frac{2\Lambda_{\text{QCD}}}{M_{H_{10}}} \right)^j - \left( \frac{2\Lambda_{\text{QCD}}}{M_{H_{10}}} \right)^j. \] (3.6)

$\delta_n$ captures mistuning effects following \cite{23} and we take $\Lambda_{\text{QCD}} = 0.5\text{GeV}$. We also impose the kinematical constraint at $q^2 = 0$, $2M_{J/\psi}A_0(0) = (M_{J/\psi} + M_{H_L})A_1(0) - (M_{J/\psi} - M_{H_L})A_2(0)$. We take priors of $0(1)$ for each $b_n$, except those terms of order $O(a^2)$ for which we take $0(0.5)$ encompassing the HISQ 1-loop improvement. All remaining priors were taken as $0(1)$. The fit is done simultaneously across all form factors in order to preserve correlations important for constructing helicity amplitudes. The results for $A_1$ and $V$, whose contributions to the decay rate are dominant, are given in Figure \cite{22}.

4. Results

Using the form factors we construct the helicity amplitudes and differential decay rates using \cite{19}. Where an integration over $q^2$ is necessary we use a simple trapezoidal interpolation in order to ensure covariates are carried through correctly, taking sufficiently many points that the results are insensitive to using additional points. The differential rate $d\Gamma/dq^2$ is plotted in figure \cite{19} where we plot the rate for $\ell = \mu$ and $\ell = \tau$.

We also compute the total decay rates for the case $\ell = \mu$ and $\ell = \tau$. We find
\[ \Gamma(B_c^- \to J/\psi \mu^- \nu_\mu) = |V_{cb}|^2 1.70(18) \times 10^{13} \text{s}^{-1} \text{ (PRELIMINARY)}, \] (4.1)
\[ \Gamma(B_c^- \to J/\psi \tau^- \bar{\nu}_\tau) = |V_{cb}|^2 4.40(36) \times 10^{12} \text{s}^{-1} \text{ (PRELIMINARY)}, \] (4.2)
and the ratio
\[ R(J/\psi) = 0.2592(92) \space \text{ (PRELIMINARY)}. \] (4.3)
$\bar{B}_c \to J/\psi$ Form Factors and $R(J/\psi) \text{ using Lattice QCD}$

Judd Harrison

Figure 1: $q^2$ dependence of $A_1$, $V$ and heavy-HISQ extrapolated curves.

Figure 2: The differential rate $d\Gamma/dq^2$, normalised by the total decay rate $\Gamma$.

References

[1] BABAR collaboration, Evidence for an excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ decays, *Phys. Rev. Lett.* 109 (2012) [1205.5442].


B_\bar{c} \rightarrow J/\psi Form Factors and R(J/\psi) using Lattice QCD

Judd Harrison


[7] E. McLean, C. T. H. Davies, J. Koponen and A. T. Lytle, $B_s \rightarrow D_{s}\ell \nu$ Form Factors for the full $q^2$ range from Lattice QCD with non-perturbatively normalized currents, [1906.00701].

