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Subleading Contributions in Rare Semileptonic $B^+ \to \pi^+ \ell^+ \ell^-$ Decay

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In the Standard Model, the $b \to s$ and $b \to d$ flavor-changing neutral currents (FCNC) are induced by loop effects. Rare semileptonic B-meson decays originated by these currents are standard channels for testing the Standard Model precisely and in searching for possible physics beyond it. Here, we consider the rare $B^+ \to \pi^+ \ell^+ \ell^-$ decay, where $\ell = \mu, \tau$ is a charged lepton, and present its dilepton invariant-mass spectrum and decay rate based on the effective electroweak Hamiltonian approach for the $b \to d \ell^+ \ell^-$ transitions in the Standard Model, taking into account weak annihilation and long-distance contributions, of which the later is from the light vector mesons. Our prediction for the total branching fraction of $B^+ \to \pi^+ \mu^+ \mu^-$ agrees with the latest LHCb result within the experimental and theoretical uncertainties. Moreover, including the annihilation diagrams and contributions from the ρ^0- and ω-resonances together gives better agreement with the experimental data in the low q^2-part of the dimuon mass spectrum. We also present theoretical predictions for the total and partial branching fractions for $B^+ \to \pi^+ \tau^+ \tau^-$ in the Standard Model. These results are potentially useful in testing the lepton flavor universality in the FCNC $B \to \pi \ell^+ \ell^-$ decays.

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1. Introduction

The physics of bottom hadrons plays a fundamental role both in the precision tests of the Standard Model (SM) and in searches of New Physics (NP). Theoretical interest in B-physics is greatly stimulated by the LHCb, CMS, and ATLAS experiments at the LHC, which provide a huge amount of new and accurate experimental data concerning the production and decays of B-mesons and bottom baryons. In the future, we also anticipate a wealth of data from the Belle-II experiment at SuperKEKB, which would provide a complementary information to the hadron collider experiments. Of major interests are the flavour changing neutral current (FCNC) processes induced by the $b \to s$ and $b \to d$ transitions. Currently, the $b \to s$ FCNC processes, $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$, are at the center of theoretical interest as a lot of precise data on these decays is available, showing signs of deviations from the SM. For the decays induced by the $b \to d$ FCNC transition, the $B^+ \to \pi^+\mu^+\mu^-$ decay is so far the only decay mode observed [1, 2]. Although, experimental data are in good agreement with the short-distance theoretical predictions for the dimuon invariant-mass distribution on $B^+ \to \pi^+\mu^+\mu^-$ decay [3, 4], the lowest q^2-part of the spectrum, where q^2 is the dilepton invariant mass, is seen substantially higher compared to these estimates. In this paper we present the numerical analysis of the $B^+ \to \pi^+\ell^+\ell^-$ decays, where $\ell = \mu, \tau$, and calculate the dimuon invariant mass distribution taking into account weak-annihilation and long-distance contributions, improving the comparison with the LHCb data on the $B^+ \to \pi^+\mu^+\mu^-$ decay.

2. The differential branching fraction of the $B \to P\ell^+\ell^-$ decay

The theory of rare semileptonic B-meson decays is presented in [5, 6]. including subleading contributions, the $B \to P\ell^+\ell^-$ differential branching fraction, where P is a pseudoscalar meson, can be written as follows:

$$\frac{d\text{Br}\,(B \to P\ell^+\ell^-)}{dq^2} = S_P \frac{2G_F^2\alpha_{\text{em}}^2\tau_B}{3(4\pi)^5 m_B^3}|V_{tb}V_{tp}^*|^2\lambda^{3/2}(q^2)F^{BP}(q^2)\sqrt{1-4m_\ell^2/q^2}, \tag{1}$$

$$F^{BP}(q^2) = F_{97}^{BP}(q^2) + F_{10}^{BP}(q^2), \quad \lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2 \tag{2}$$

$$F_{97}^{BP}(q^2) = \left(1 + \frac{2m_\ell^2}{q^2}\right)\left|C_9^{\text{eff}}(q^2)\, f_+^{BP}(q^2) + \frac{2m_b C_7^{\text{eff}}(q^2)}{m_B + m_P}\, f_T^{BP}(q^2) + +L_A^{BP}(q^2) + \Delta C_V^{BP}(q^2)\right|^2 \tag{3}$$

$$F_{10}^{BP}(q^2) = \left(1 - \frac{4m_\ell^2}{q^2}\right)\left|C_{10}^{\text{eff}}\, f_+^{BP}(q^2)\right|^2 + \frac{6m_\ell^2}{q^2}\,\frac{\left(m_B^2 - m_P^2\right)^2}{\lambda(q^2)}\left|C_{10}^{\text{eff}}\, f_0^{BP}(q^2)\right|^2, \tag{4}$$

where S_P is an isospin factor of the final meson, in particular, $S_{\pi^\pm} = 1$ and $S_{\pi^0} = 1/2$ for the π-mesons, $C_{7,9,10}^{\text{eff}}$ are effective Wilson coefficients including the NLO QCD corrections [7], $L_A^{BP}(q^2)$ and $\Delta C_V^{BP}(q^2)$ are the Weak Annihilation (WA) and Long-Distance (LD) contributions, respectively. The WA contribution is calculated within the so-called Large Energy Effective Theory (LEET), and for the $B \to \pi\ell^+\ell^-$ decay, it depends on two terms [8]:

$$L_A^{B\pi(t)}(q^2) = Q_q \frac{\pi^2}{3}\frac{4f_B f_\pi}{m_b}\lambda_{B,-}^{-1}(q^2)\,C_{34}, \qquad L_A^{B\pi(u)}(q^2) = -Q_q \frac{\pi^2}{3}\frac{4f_B f_\pi}{m_b}\lambda_{B,-}^{-1}(q^2)\,C_{12}, \tag{5}$$

where Q_q is the spectator-quark electric charge, f_B and f_π are the B- and π-meson decay constants, respectively, $C_{34} = C_3 + 4\,(C_4 + 12C_5 + 16C_6)\,/3$ is a combination of the Wilson coefficients of the

QCD penguin operators, $C_{12} = 3C_2$, and $\lambda_{B,-}^{-1}(q^2)$ is the first inverse moment of the B-meson light-cone distribution amplitude. The LD contributions are coming from the two-body $B \to V(\to \bar{\ell}\ell)\pi$ decays, where $V = \rho^0, \omega, J/\psi, \psi(2S)$ are neutral vector mesons, and can be written as follows [9]:

$$\Delta C_V^{B\pi}(q^2) = -16\pi^2 \frac{V_{ub}V_{ud}^* H^{(u)}(q^2) + V_{cb}V_{cd}^* H^{(c)}(q^2)}{V_{tb}V_{td}^*}, \tag{6}$$

$$H^{(p)}(q^2) = (q^2 - q_0^2) \sum_V \frac{k_V f_V A_{BV\pi}^p}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})}, \tag{7}$$

where f_V and $\Gamma_V^{\rm tot}$ are the decay constant and total decay width of a vector meson, k_V is the valence quark content factor, $A_{BV\pi}^p$ $(p = u, c)$ are transition amplitudes, and the free parameter $q_0^2 = -1.0$ GeV^2 is chosen to achieve a better convergence in the denominator of (7).

The differential branching fraction (1) involves three $B \to P$ transition form factors. Among the available parameterizations of the $B \to \pi$ form factors in the literature, we use the following two: the Boyd-Grinstein-Lebed (BGL) [10] and modified Bourrely-Caprini-Lellouch (mBCL) [11].

3. Numerical analysis of $B^+ \to \pi^+\ell^+\ell^-$ decay

We calculate the dimuon invariant mass distributions for the $B^+ \to \pi^+\mu^+\mu^-$ decay in two cases: short-distance contribution only and with taking into account additionally WA and LD amplitudes. The resulting distributions are shown for the BGL parameterization in Fig. 1 and compared with the experimental data [2]. To show the impact of both WA and LD contributions, we plot the $B^+ \to \pi^+\mu^+\mu^-$ differential branching fractions in bins of the dimuon invariant mass squared in Fig. 2 for the BGL (left plot) and mBCL (right plot) parameterizations. As can be seen, our results (in green) are in better agreement with the LHCb data [2] in comparison with the known theoretical predictions [9] (in red), especially in the low-q^2 part of the entire range, where WA compensates the contribution from the ρ^0- and ω-resonances.

We also present the total branching fraction, $\mathrm{Br}_{\rm th}^{\rm BGL}$, (its short-distance part is determined as $\mathrm{Br}_{\rm pert}^{\rm BGL}$) for the BGL parameterization in comparison with experimental data [2]:

$$\mathrm{Br}_{\rm pert}^{\rm BGL} = \left(1.51^{+0.22}_{-0.12}\right) \times 10^{-8}, \quad \mathrm{Br}_{\rm th}^{\rm BGL} = \left(1.71^{+0.24}_{-0.14}\right) \times 10^{-8}, \quad \mathrm{Br}_{\rm exp} = (1.83 \pm 0.29) \times 10^{-8}. \tag{8}$$

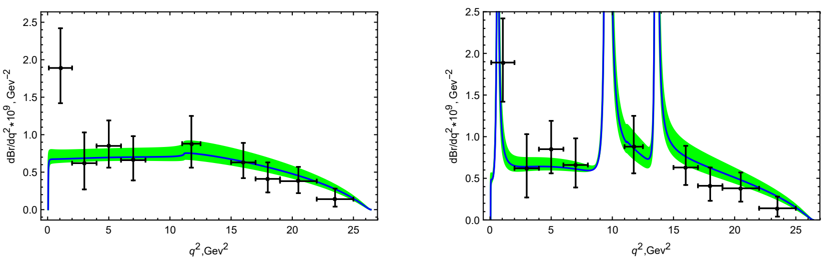

Figure 1: Dilepton invariant-mass distribution for $B^+ \to \pi^+\mu^+\mu^-$ decay without (left) and with (right) taking into account WA and LD contributions for the BGL parameterization of the form factors. The green areas indicate the uncertainty due to the factorization scale and CKM-matrix element V_{td}. The crosses show the existing experimental data from LHCb [2].

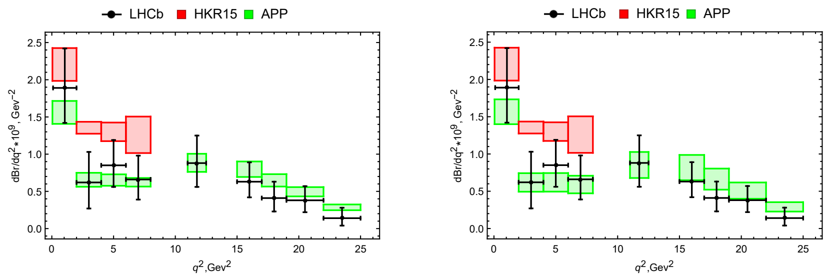

Figure 2: Differential branching fraction of the $B^+ \to \pi^+\mu^+\mu^-$ decay in bins of dimuon invariant mass squared by taking into account the WA and LD contribution for the BGL (left) and mBCL (right) parameterizations. The red bins indicate theoretical predictions from [9] (called HKR15), the green bins are our predictions (APP), and the experimental data from LHCb [2] are shown by crosses.

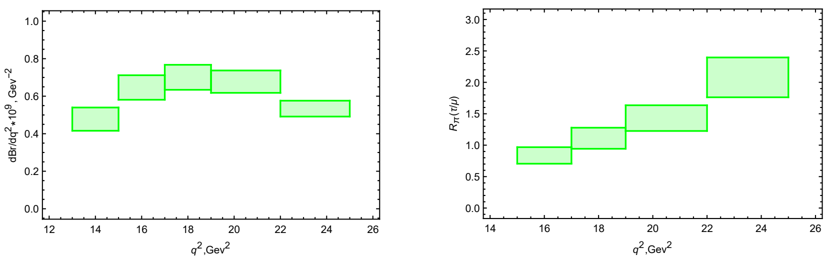

Figure 3: Differential branching fraction of the $B^+ \to \pi^+\tau^+\tau^-$ decay in bins of the di-tauon invariant mass squared (left) and branching ratio $R^{\pi}_{\tau/\mu}$ (right) for the BGL parameterization.

As can be seen, the value for the total branching fraction taking into account the WA and LD contributions, $\mathrm{Br}^{\mathrm{BGL}}_{\mathrm{th}}$, agrees well with the experimental data within uncertanties.

For $B^+ \to \pi^+\tau^+\tau^-$, we present the differential branching fraction in bins of the di-tauon invariant mass squared and the ratio of the fractional branching ratios, $R^{\pi}_{\tau/\mu}$, for the BGL parameterization in Fig. 3, where $R^{\pi}_{\tau/\mu}$ is defined as follows:

$$R_{\pi}^{(\tau/\mu)}(q_1^2, q_2^2) = \int_{q_1^2}^{q_2^2} d\mathrm{Br}(B^+ \to \pi^+\tau^+\tau^-)/dq^2 \Bigg/ \int_{q_1^2}^{q_2^2} d\mathrm{Br}(B^+ \to \pi^+\mu^+\mu^-)/dq^2 \,. \quad (9)$$

Our prediction for the $B^+ \to \pi^+\tau^+\tau^-$ total branching fraction is consistent with the estimate [3]:

$$\mathrm{Br}^{\mathrm{BGL}}_{\mathrm{th}} = \left(7.56^{+1.04}_{-0.75}\right)\times 10^{-9}, \quad \mathrm{Br}^{\mathrm{mBCL}}_{\mathrm{th}} = \left(6.28^{+0.88}_{-0.78}\right)\times 10^{-9}, \quad \mathrm{Br}^{\mathrm{FG}}_{\mathrm{th}} = (7.00 \pm 0.70)\times 10^{-9}. \quad (10)$$

4. Summary and outlook

Taking into account the weak-annihilation and long-distance contributions, theoretical predictions for the dimuon invariant-mass spectrum in the $B^+ \to \pi^+\mu^+\mu^-$ decay, obtained in the effective electroweak Hamiltonian framework based on the SM, agree well with the existing experimental data in the entire q^2-region, including low-q^2. With the updated measurements from LHCb, including data collected at 13 TeV, as well as forthcoming data anticipated from the Belle-II experiment at SuperKEKB, precise tests of the Standard Model in the exclusive FCNC $b \to d$ transitions will be carried out. As for the semitauonic $B^+ \to \pi^+\tau^+\tau^-$ decay, theoretical predictions in the SM give a factor 2 suppression in comparison with the $B^+ \to \pi^+\mu^+\mu^-$ decay, which will be targeted by the Belle-II and LHCb. This would allow tests of the lepton flavour universality in $b \to d$ transitions, not yet undertaken experimentally.

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