CKM, mixing and $CP$ violation results at LHCb

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We review the most precise tests of the CKM model of flavour physics performed by the LHCb collaboration during 2021. These include world best measurements of both time-dependent and time-independent $CP$ violation in charm decays, the first observation of a nonzero mass difference of the neutral charm-meson eigenstates, and the most precise determination to date of the mass difference between the $B^0_s$ eigenstates. A new simultaneous combination of measurements of the CKM angle $\gamma$ and of charm decays allows to improve the precision on the decay-width difference of the neutral charm-meson eigenstates by a factor of 2 with respect to the combination of charm results only.
1. Introduction

The LHCb experiment has established itself as a main player in the study of beauty and charm hadrons, thanks to their large production cross-sections at the LHC and to its dedicated design [1]. In the following sections we review the most precise tests of the CKM model performed by the LHCb collaboration during 2021. Throughout the text, the first quoted uncertainties are statistical and the second are systematic.

2. Mixing and CP violation in charm decays

Two years after the first observation of CP violation in charm decays through the difference of the time-integrated CP asymmetries of $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$ decays [2], agreement on the dynamical origin of this phenomenon has not been achieved yet [3–7]. Measurements of CP violation in further decay channels could provide insights on the size of the QCD nonperturbative effects at play. The LHCb collaboration has recently measured the CP asymmetries of $D^0 \to K_S^0 K_S^0$ [8] and $D_{(s)}^0 \to h^+ h^0$ decays [9], where $h^+$ stands for a $\pi^+$ or $K^+$ meson, and $h^0$ for a $\pi^0$ or $\eta$ meson. The former has been proposed as a promising channel to measure CP violation in the decay, since only diagrams vanishing in the $U$-spin limit contribute to its leading-order decay amplitudes, while CP violating contributions do not cancel in this limit [10, 11]. The new measurement, $A_{CP}(D^0 \to K_S^0 K_S^0) = (-3.1 \pm 1.2 \pm 0.4 \pm 0.2)$%, where the third uncertainty is due to the precision with which the CP asymmetry of the normalisation channel is known [12], is the most precise to date and is compatible with zero within 2.4 standard deviations. The second measurement is the first of its kind performed at hadron colliders, and relies on rare Dalitz decays, $h^0 \to e^+ e^- \gamma$, or on two-photon decays where one of the photons converts into an electron-positron pair within the vertex detector, to allow triggering on the displaced $D$-meson decay vertex, which would be impossible to reconstruct using bare two-photon decays. The large charm-hadron production cross-section with respect to $B$ factories counterbalances the lower branching fraction of these final states, and allows to obtain results equally or more precise, all being consistent with zero within uncertainties ranging from 1 to 10%.

Crucial advances have been achieved also in time-dependent measurements. In the following we describe mixing and CP violation through the theoretical parametrisation [13], which parametrises the off-diagonal terms of the effective Hamiltonian governing the evolution of the $D^0$-$\bar{D}^0$ system, $H \equiv M - \frac{i}{2} \Gamma$, with the mixing parameters $x_{12} \equiv 2 |M_{12}/\Gamma|$ and $y_{12} \equiv |\Gamma_{12}/\Gamma|$ and the weak phases $\phi_2^M$ and $\phi_2^\Gamma$, defined as the phases of $M_{12}$ and $\Gamma_{12}$ relative to their dominant $\Delta U = 2$ contribution. The parameters $x_{12}$ and $\phi_2^M$ correspond to dispersive off-shell transitions, and are sensitive to interactions with new heavy particles, while $y_{12}$ and $\phi_2^\Gamma$ quantify absorptive on-shell transitions.

A new measurement of $D^0 \to K_S^0 \pi^+ \pi^-$ decays [14], based on the data sample collected during 2015–2018, increases tenfold the number of candidates with respect to the previous determination with 2011–2012 data thanks to improved trigger [15]. The variation of strong phases across the Dalitz plot ensures sensitivity to all mixing parameters and CP violation phases. The data are analysed with a model-independent method, in which the Dalitz plot is divided into 8 regions symmetric with respect to its bisector, each having an approximately constant strong-phase difference between the amplitudes in the two halves of the plane [16]. The ratios of the yields in the opposite halves of
Figure 1: (Top-left) Dalitz plot of background-subtracted $D^0 \rightarrow K^0_S \pi^+ \pi^-$ candidates; $m^2$ stands for $m^2(K^0_S \pi^+ \pi^-)$ ($m^2(K^0_S \pi^+ \pi^-)$) for $D^0 (\bar{D}^0)$ decays. (Bottom-left) Iso-$\Delta b$ binning of the Dalitz plane. (Right) CP-averaged yield ratios as a function of decay time for each Dalitz region, with fit projections overlayed.

the plane are measured for each Dalitz-plot region, labelled “$b$”, and in 13 intervals of decay time, labelled “$j$”; see fig. 1. The yields in the denominator mostly correspond to Cabibbo-favoured decays and are nearly constant as a function of decay time, while the fraction of Cabibbo-favoured decays following mixing in the numerator is as important as that of doubly Cabibbo-suppressed decays and increases as a function of decay time, thus providing sensitivity to the mixing parameters.

The ratios are equal to

$$R^b_{f,j} \equiv \frac{r_b + \sqrt{r_b} \Re \left[X^0_{f,j}(z_{CP} \pm \Delta z) \langle t \rangle_j + \frac{1}{2} \left| z_{CP} \pm \Delta z \right|^2 + r_b \Re (z_{CP}^2 - \Delta z^2) \langle t^2 \rangle_j \right]}{1 + \sqrt{r_b} \Re \left[X_b(z_{CP} \pm \Delta z) \langle t \rangle_j + \frac{1}{2} \Re (z_{CP}^2 - \Delta z^2) \langle t^2 \rangle_j \right]}$$

(1)

where $r_b$ is the ratio at zero decay time, $X_b \equiv \langle e^{i(\Delta \delta)} \rangle_b$ is the average of the exponential of the strong-phase difference, as measured at charm factories [17, 18], and the two complex parameters $z_{CP} \equiv -(y_{CP} + i x_{CP})$ and $\Delta z \equiv -(\Delta y + i \Delta x)$ are equal to

$$x_{CP} = x_{12} \cos \phi_z^M = (3.97 \pm 0.46 \pm 0.29) \times 10^{-3}, \quad \Delta x = -y_{12} \sin \phi_z^T = (-0.27 \pm 0.18 \pm 0.01) \times 10^{-3},$$

$$y_{CP} = y_{12} \cos \phi_z^T = (4.59 \pm 1.20 \pm 0.85) \times 10^{-3}, \quad \Delta y = x_{12} \sin \phi_z^M = (0.20 \pm 0.36 \pm 0.13) \times 10^{-3}.$$  

The results for $x_{CP}$ and $\Delta x$ improve the precision of their world average by a factor of 3, and the former constitutes the first observation of a nonzero mass difference between the neutral charm meson eigenstates, with a significance greater than 7 standard deviations.

The limits on the $\phi_z^M$ angle can be further improved by measuring the slope of the linear expansion of the time-dependent asymmetry of the decay rates of $D^0$ and $\bar{D}^0$ mesons into the CP-even final state $f$, where $f$ stands for $= K^+ K^-$ or $\pi^+ \pi^-$. 

$$A_{CP}(f,t) \equiv \frac{\Gamma(D^0 \rightarrow f,t) - \Gamma(\bar{D}^0 \rightarrow f,t)}{\Gamma(D^0 \rightarrow f,t) + \Gamma(\bar{D}^0 \rightarrow f,t)} \approx a_d^f - \Delta y - \frac{t}{\tau_{D^0}},$$

(2)

where $a_d^f$ is the CP asymmetry in the decay and $\Delta y$ has been defined above. A new measurement relying on the 2015–2018 data sample [19], $\Delta y = (1.0 \pm 1.1 \pm 0.3) \times 10^{-4}$, is the most precise
search for \( CP \) violation performed to date at a hadron collider and improves the precision of the world average by around a factor of 2.

3. Mixing and \( CP \) violation in beauty decays

Precise measurements of the mass difference between the \( B_s^0 \) eigenstates, \( \Delta m_s \), are needed to minimise the systematic uncertainty of measurements of the CKM angle \( \gamma \) through time-dependent analyses [20] and, if combined with measurements of its analogue for \( B^0 \) mesons, \( \Delta m_d \), provide constraints on the unitarity of the CKM matrix. The LHCb collaboration has recently measured \( \Delta m_s \) using \( B^0_s \rightarrow D_s^-\pi^+\) decays collected during 2015–2018, where the \( D_s^- \) meson decays into the \( K^+K^-\pi^- \) or \( \pi^+\pi^-\pi^- \) final states [21]; see fig. 2. The result is, in natural units,

\[
\Delta m_s = 17.7683 \pm 0.0051 \pm 0.0032 \text{ ps}^{-1}.
\]

Its relative precision, \( 3 \times 10^{-4} \), is lower than that of the previous world average by a factor of 2.

Further results include the first search for \( CP \) violation through an amplitude analysis of \( \Xi_b^- \rightarrow pK^-K^- \) decays [22] and a measurement of the CKM angle \( \gamma \) with \( A^0_b \rightarrow DpK^- \) decays [23]. Both are statistically limited and do not observe \( CP \) asymmetries differing significantly from zero.

4. First simultaneous combination of charm and beauty results

A recent combination of LHCb measurements of the CKM angle \( \gamma \) yields \( \gamma = (65.4^{+3.8}_{-4.2})^\circ \) [24]. Measurements base on time-independent and time-dependent analyses are compatible with each other at the level of 2 standard deviations, the precision of the latter being worse by a factor of 2; see fig. 2 centre. For the first time, measurements of charm decays are included in the combination and charm mixing parameters are fitted simultaneously to the angle \( \gamma \) and to the beauty hadronic observables. The precision on the strong-phase difference between \( D^0 \rightarrow K^-\pi^+ \) and \( D^0 \rightarrow K^+\pi^- \) decays, \( \delta^{K\pi}_D \), is improved by around a factor of 2 with respect to the previous world average, thanks to the improved precision of the ADS measurement in Ref. [25]. This allows to improve also the precision on the charm mixing parameter \( y_{12} \) by a factor of 2, thanks to the constraints set on \( -y_{12} \cos \delta^{K\pi}_D + x_{12} \sin \delta^{K\pi}_D \) by time-dependent \( D^0 \rightarrow K^+\pi^- \) measurements [26]; see fig. 2 right.

![Figure 2:](image)

**Figure 2:** (Left) Decay-time distribution of \( B^0_s \rightarrow D_s^-\pi^+ \) signal decays. The oscillations frequency is equal to \( \Delta m_s \); fit projections are overlaid. (Centre) One-dimensional 1 − CL profiles for the CKM angle \( \gamma \) from combinations using inputs from different \( B \)-meson species. (Right) Two-dimensional profile likelihood contours for the charm mixing parameters. The brown contours, drawn out from 1 to 5 standard deviations, show the improvement of this combination with respect to the previous world average, in blue.
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