

# Interaction of the pseudoscalar glueball with (pseudo)scalar mesons and their first excited states

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We study the interaction of the pseudoscalar glueball with scalar and pseudoscalar quark-antiquark meson fields as well as their first excited states. We introduce a corresponding chiral Lagrangian that describes the two- and three-body decays of a pseudoscalar glueball,  $J^{PC} = 0^{-+}$ , with a mass of 2.6 GeV as predicted by lattice QCD simulations. We compute the decay of the pseudoscalar glueball into (pseudo)scalar and their excited states. The various branching ratios are parameters-free predictions. These states and channels are in reach of the ongoing BESIII, Belle II, LHCb, and NICA experiments and the upcoming PANDA experiment at the FAIR facility.

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## 1. Introduction

In recent years, the investigation of the properties of glueballs, bound states of gluons, and exotic states has been the focus of many experimental and theoretical hadronic physics studies [1, 2] for a deeper understanding of the non-perturbative behavior of Quantum Chromodynamics (QCD). The hadronic properties of pseudoscalar glueball and its exotic states have been also widely investigated [3, 4, 5, 6, 7] and References therein because they contain an important feature of QCD, the chiral anomaly [8]. Lattice QCD simulations calculated the glueball spectrum [9, 10, 11], and predicted the pseudoscalar glueball state,  $J^{PC} = 0^{-+}$ , with a mass of about 2.6 GeV. The decay widths of the lightest pseudoscalar glueball were computed into light mesons [4], excited mesons [12] and into two nucleons [5]. In addition, the two- and three-body decays of the first excited pseudoscalar glueball have been calculated and were presented as the branching ratio, as seen in Refs. [7, 12]. Which obtained that the first excited pseudoscalar glueball could decay into the pseudoscalar charmed meson  $\eta_c$  as  $\Gamma_{\tilde{G} \rightarrow \eta_c \pi \pi}$ .

The present study of the pseudoscalar glueball is based on the eLSM [13], the effective chiral model of low-energy QCD. The model implements the symmetries of the QCD and their breaking and contains all quark-antiquark mesons with (pseudo)scalar and (axial)vector as well as a scalar and a pseudoscalar glueball. The eLSM played an important role in the study of hadron phenomenology, which has been successfully used to study the vacuum properties of light mesons in the cases of  $N_f = 2$  [14],  $N_f = 3$  [15], glueballs [4, 5, 7, 16, 12], baryons [17], excited mesons [13], hybrids [18], and charmed mesons [19, 20, 21].

In this study, we investigate the pseudoscalar glueball [4] through its decay channels by introducing an interaction chiral Lagrangian which describe the decays of the pseudoscalar glueball into (pseudo)scalar mesons and their excited states. We obtain within the present work new channel resonances for the pseudoscalar glueball which lead experimentalists for searching glueballs by measuring the proposed channels.

## 2. Interaction with (pseudo-)scalar and excited (pseudo-)scalar mesons

The effective Lagrangian which couples the pseudoscalar glueball  $\tilde{G} \equiv |gg\rangle$  with quantum numbers  $J^{PC} = 0^{-+}$  to the ordinary (pseudo-)scalar and the first excited (pseudo-)scalar mesons [12].

$$\mathcal{L}_{\tilde{G}\Phi\Phi_E}^{int} = c_{\tilde{G}\Phi\Phi_E} \tilde{G} \left[ \left( \det\Phi - \det\Phi_E^\dagger \right)^2 + \left( \det\Phi^\dagger - \det\Phi_E \right)^2 \right], \quad (2.1)$$

where  $c_{\tilde{G}\Phi\Phi_E}$  is a dimensionless coupling constant,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}, \quad (2.2)$$

and

$$\Phi_E = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_{NE} + a_{0E}^0) + i(\eta_{NE} + \pi_E^0)}{\sqrt{2}} & a_{0E}^+ + i\pi_E^+ & K_{SE}^+ + iK_E^+ \\ a_{0E}^- + i\pi_E^- & \frac{(\sigma_{NE} - a_{0E}^0) + i(\eta_{NE} - \pi_E^0)}{\sqrt{2}} & K_{SE}^0 + iK_E^0 \\ K_{SE}^- + iK_E^- & \bar{K}_{SE}^0 + i\bar{K}_E^0 & \sigma_{SE} + i\eta_{SE} \end{pmatrix}, \quad (2.3)$$

are multiplets containing the (pseudo-)scalar mesons [15] and the excited (pseudo-)scalar mesons [13], respectively. Under  $SU_L(3) \times SU_R(3)$  chiral transformations the multiplets  $\Phi$  and  $\Phi_E$  transform as  $\Phi \rightarrow U_L \Phi U_R^\dagger$  and  $\Phi_E \rightarrow U_L \Phi_E U_R^\dagger$ , respectively, whereas  $U_{L(R)} = e^{-i\Theta_{L(R)}^a}$  are  $U(3)_{L(R)}$  matrices, and transform under the charge conjugation  $C$  as  $\Phi \rightarrow \Phi^T$ ,  $\Phi_E \rightarrow \Phi_E^T$  as well as under the parity  $P$  as  $\Phi(t, \vec{x}) \rightarrow \Phi^\dagger(t, \vec{x})$ ,  $\Phi_E(t, \vec{x}) \rightarrow \Phi_E^\dagger(t, \vec{x})$ , respectively. The determinants of the multiplets  $\Phi$  and  $\Phi_E$  are invariant under  $SU_L(3) \times SU_R(3)$ . However, according to the chiral anomaly, these multiplets are not invariant under the axial  $U(1)_A$  transformation.

$$\det \Phi \rightarrow \det U_A \Phi U_A = e^{-i\Theta_A^0 \sqrt{2N_f}} \det \Phi \neq \det \Phi, \quad (2.4)$$

$$\det \Phi_E \rightarrow \det U_A \Phi_E U_A = e^{-i\Theta_A^0 \sqrt{2N_f}} \det \Phi_E \neq \det \Phi_E. \quad (2.5)$$

On the other hand, the pseudoscalar glueball field  $\tilde{G}$  is chirally invariant and transform under the parity  $P$  and under charge conjugation. Consequently the effective chiral Lagrangian (2.1) contains the symmetries of the QCD Lagrangian.

The assignment of the quark-antiquark fields in the present work is as follows: (i) In the pseudoscalar sector  $P$ , the fields  $\vec{\pi}$  and  $K$  represent the pion isotriplet and the kaon isodoublet respectively [22]. The bare quark-antiquark fields  $\eta_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$  and  $\eta_S \equiv |\bar{s}s\rangle$  are the non-strange and strangeness mixing components of the physical states  $\eta$  and  $\eta'$  which can be obtained by [22]:

$$\eta = \eta_N \cos \varphi + \eta_S \sin \varphi, \quad \eta' = -\eta_N \sin \varphi + \eta_S \cos \varphi, \quad (2.6)$$

where the mixing angle is  $\varphi \simeq -44.6^\circ$  [15]. (ii) In the scalar sector  $S$ , the field  $\vec{a}_0$  is assigned to the physical isotriplet state  $a_0(1450)$  and the scalar kaon field  $K_S$  to the physical isodoublet state  $K_0^*(1430)$ . In the scalar-isoscalar sector, the non-strange bare field  $\sigma_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$  can be assigned to the resonance  $f_0(1370)$  and the bare strange field  $\sigma_S$  corresponds to  $f_0(1500)$  [16], which the two resonances mix with the scalar glueball,  $G$ , which refers to  $f_0(1710)$ , where the mixing matrix constructed in Ref. [16]. (iii) In the excited pseudoscalar sector the excited pion  $\vec{\pi}_E$  and the excited kaon  $K_E$  are assigned to  $\pi(1300)$  and  $K(1460)$ , respectively. The excited non-strange bare fields  $\eta_{NE}$  and strange bare field  $\eta_{SE}$  correspond to the physical resonances  $\eta(1295)$  and  $\eta(1440)$ , respectively. (iv) In the excited scalar sector the excited field  $\vec{a}_0$  corresponds to the physical state  $a_0(1950)$  and the excited scalar kaon fields  $K_{SE}$  is assigned to the resonances  $K_0^*(1950)$ . The excited scalar-isoscalar sector, the excited non-strange bare field  $\sigma_{NE} \equiv |\bar{n}n\rangle$  is identified with the physical resonance  $f_0(1790)$  and the excited bare strange field  $\sigma_{SE} \equiv |\bar{s}s\rangle$  is assigned either to  $f_0(2020)$  or to  $f_0(2100)$  as has been discussed as a consequence of the model.

One has to shift the scalar-isoscalar fields by their vacuum expectation values  $\phi_N$  and  $\phi_S$  as [15]  $\sigma_N \rightarrow \sigma_N + \phi_N$  and  $\sigma_S \rightarrow \sigma_S + \phi_S$  to implement the effect of spontaneous symmetry breaking, which takes place. One has also shift the axial-vector fields to redefine the wave-function renormalization constants of the pseudoscalar fields, as  $\vec{\pi} \rightarrow Z_\pi \vec{\pi}$ ,  $K \rightarrow Z_K K$ ,  $\eta_{N,S} \rightarrow Z_{\eta_{N,S}} \eta_{N,S}$ , where  $Z_i$  refers to the wave function renormalization constants.

The two- and three-body decay of the pseudoscalar glueball,  $\tilde{G}$ , have been presented in Table I as the branching ratios.

Quantity	The theoretical result
$\Gamma_{\tilde{G} \rightarrow \eta\eta} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.002
$\Gamma_{\tilde{G} \rightarrow \eta\eta'} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.440
$\Gamma_{\tilde{G} \rightarrow \eta'\eta'} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.249
$\Gamma_{\tilde{G} \rightarrow \eta_{SE}\eta} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.0085
$\Gamma_{\tilde{G} \rightarrow \eta_{NE}\eta} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.0289
$\Gamma_{\tilde{G} \rightarrow \eta_{NE}\eta'} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.2082
$\Gamma_{\tilde{G} \rightarrow \pi\pi\sigma_{SE}} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.00016 for $\sigma_{SE} \equiv f_0(2020)$ 0.0000014 for $\sigma_{SE} \equiv f_0(2100)$
$\Gamma_{\tilde{G} \rightarrow a_0\pi\eta} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.0011
$\Gamma_{\tilde{G} \rightarrow \pi\pi f_0(1370)} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.0405
$\Gamma_{\tilde{G} \rightarrow \pi\pi f_0(1500)} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.0209
$\Gamma_{\tilde{G} \rightarrow \pi\pi f_0(1710)} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.0003
$\Gamma_{\tilde{G} \rightarrow KK f_0(1370)} / \Gamma_{\tilde{G}\Phi\Phi_E}^{tot}$	0.00005

**Table 1:** Branching ratios for the two- and three-body decay of the pseudoscalar glueball  $\tilde{G}$ .

### 3. Conclusion

We have presented an interaction chiral Lagrangian, for the three flavour case  $N_f = 3$ , describing two- and three-body decays of a pseudoscalar glueball into (pseudo)scalar mesons and excited (pseudo)scalar mesons. We have calculated the decays of the pseudoscalar glueball into two-body ( $PP, PP_E$ ) and three-body ( $PPS_E, PPS$ ) which include the scalar-isoscalar states, as seen in Table I. We have chosen the mass of the pseudoscalar glueball 2.6 GeV which is in agreement with Lattice QCD in the quenched approximation. The results have been predicted as branching ratios because of unknown coupling constant. That thus determine the expectation of the dominant decay channels. The presented decay properties of the pseudoscalar glueball represent a useful guideline for NICA, BESIII, Belle II, LHCb experiments and for the corresponding upcoming experiments with the PANDA detector at FAIR. So, the present work is very interesting for the search of the pseudoscalar glueball.

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