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Four-body Faddeev-type calculation of $\bar{K}NNN$ system

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The theoretical investigation of $\bar{K}NNN$, which is an exotic system consisting of an antikaon and three nucleons, was performed. The four-body Faddeev-type equations were solved, and the binding energy and width of a quasi-bound state in the system were calculated.

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1. Introduction

The attractive nature of the $\bar{K}N$ interaction due to the existence of the subthreshold $\Lambda(1405)$ resonance has stimulated theoretical and experimental searches for K^- bound states in different systems. Many theoretical and experimental efforts were devoted to the lightest possible system $\bar{K}NN$. Calculations, performed using different methods, such as Faddeev equations with coupled channels, variational methods, and some others, see [2] and references therein, - agree that a quasi-bound state in the K^-pp system exists, but they yield quite diverse binding energies and widths.

The experimental situation is unsettled as well: several candidates for the K^-pp state were reported by different experiments, but the measured K^- binding energies and widths of such a state differ from each other and, moreover, are far from all theoretical predictions. Because of this new experiments are being planned and performed.

The detection of the heavier four-body $\bar{K}NNN$ system could be easier than in the case of $\bar{K}NN$ since direct scattering of K^- on three-body nuclei (such as ^{3}He or ^{3}H) can be performed. Some theoretical works were devoted to the question of the quasi-bound state in the $\bar{K}NNN$ system with different quantum numbers [3–6], but more accurate calculations within Faddeev-type equations are needed. The reason is that only these dynamically exact equations in momentum representation can treat energy dependent $\bar{K}N$ potentials, necessary for this system, exactly.

We solved the four-body Faddeev-type AGS equations [1] in order to study the quasi-bound $\bar{K}NNN$ systems. We used our experience with the three-body AGS calculations, described in [2], and our two-body potentials, constructed for them.

2. Four-body Faddeev-type AGS equations

Dynamically exact four-body Faddeev-type AGS equations [1] written for separable potentials are of the form

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})(\bar{G_0}^{-1})_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta}(1-\delta_{\sigma\tau})\bar{T}^{\tau}_{\alpha\gamma}(z)(\bar{G}_0)_{\gamma\delta}(z)\bar{U}^{\tau\rho}_{\delta\beta}(z), \tag{1}$$

were

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = \langle g_\alpha|G_0(z)U^{\sigma\rho}_{\alpha\beta}(z)G_0(z)|g_\beta\rangle, \tag{2}$$

$$\bar{T}^{\tau}_{\alpha\beta}(z) = \langle g_\alpha|G_0(z)U^{\tau}_{\alpha\beta}(z)G_0(z)|g_\beta\rangle, \tag{3}$$

$$(\bar{G}_0)_{\alpha\beta}(z) = \delta_{\alpha\beta}\tau_\alpha(z). \tag{4}$$

Here G_0 is the free Green function, $U^{\tau}_{\alpha\beta}$ and $U^{\sigma\rho}_{\alpha\beta}$ are three-body and four-body operators, correspondingly. The high indices τ,ρ define a partition of the four-body system, which can be $3+1$ or $2+2$ type. The low indices α,β define two-body subsystems of the partition, denoted by the high index. $|g_\alpha\rangle$ and τ_α in Eqs.(2,3,4) are form-factors and energy-dependent parts of the separable T-matrices corresponding to the separable potentials. If the "effective three-body potentials" $\bar{T}^{\tau}_{\alpha\beta}(z)$ in Eq. (1) are also presented in a separable form

$$\bar{T}^{\tau}_{\alpha\beta}(z) = |\bar{g}^{\tau}_\alpha\rangle\bar{\tau}^{\tau}_{\alpha\beta}(z)\langle\bar{g}^{\tau}_\alpha|, \tag{5}$$

the four-body equations can be written as [7]

$$\bar{X}^{\sigma\rho}_{\alpha\beta}(z) = \bar{Z}^{\sigma\rho}_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}^{\sigma\tau}_{\alpha\gamma}(z)\bar{\tau}^{\tau}_{\gamma\delta}(z)\bar{X}^{\tau\rho}_{\delta\beta}(z) \tag{6}$$

with new transition $\bar{X}^{\sigma\rho}$ and kernel $\bar{Z}^{\sigma\rho}$ operators, defined by

$$\bar{X}^{\sigma\rho}_{\alpha\beta}(z) = \langle \bar{g}^{\sigma}_{\alpha}|\bar{G}_0(z)_{\alpha\alpha}\bar{U}^{\sigma\rho}_{\alpha\beta}(z)\bar{G}_0(z)_{\beta\beta}|\bar{g}^{\rho}_{\beta}\rangle, \tag{7}$$

$$\bar{Z}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})\langle \bar{g}^{\sigma}_{\alpha}|\bar{G}_0(z)_{\alpha\beta}|\bar{g}^{\rho}_{\beta}\rangle. \tag{8}$$

We used the Energy Dependent Pole Expansion/ Approximation (EDPE/ EDPA) method, suggested in [8] specially for the four-body AGS equations, for the construction of separable versions of three-body and 2+2 amplitudes. The first step of the method is the calculation of the eigenvalues λ_n and the eigenfunctions $g_{n\alpha}(p;z)$ of the system of equations for a fixed energy z, which usually is chosen to be the binding energy $z = E_B$. After that the energy dependent form-factors and propagators for arbitrary energy are calculated by evaluation of integrals, containing the obtained eigenvalues and eigenfunctions at the fixed energy.

Since the EDPE method needs only one solution of the eigenvalue equations and calculations of the integrals after that, it saves computing time, when the three-body amplitudes have to be solved during four-body calculations. According to the authors, the method is accurate already with one term, i.e. EDPA.

3. Input and partitions

3.1 Two-body potentials

We used our potentials, describing the antikaon-nucleon interaction, constructed and used for our three-body $\bar{K}NN$ an $\bar{K}\bar{K}N$ calculations, see Ref. [2]. In particular, three models of the $\bar{K}N$ interaction were used: the separable potentials having one- $V^{1,\mathrm{SIDD}}_{\bar{K}N}$ or two-pole $V^{2,\mathrm{SIDD}}_{\bar{K}N}$ structure of the $\Lambda(1405)$ resonance and a chirally motivated model $V^{\mathrm{Chiral}}_{\bar{K}N}$. All three potentials describe low-energy K^-p scattering and $1s$ level shift of kaonic hydrogen with equally high accuracy. This allows us to study the dependence of the four-body results on the two-body input.

In our three-body calculations [2] we took the coupling between the $\bar{K}N$ and $\pi\Sigma$ channels into account explicitly and solved the AGS equations with coupled $\bar{K}NN$ and $\pi\Sigma N$ channels. But a reliable calculation of a four-body problem is a much harder task than a three-body one. That is why we used the exact optical $\bar{K}N$ potential, which is energy-dependent, corresponding to our antikaon-nucleon potentials with coupled channels. We showed [2] that the one-channel three-body calculation with such a potential is a very good approximation and assume that it is true for the four-body case as well.

As for the nucleon-nucleon interaction, we constructed a new version of the two-term separable NN potential, its form and parameters are described in [9]. The potential reproduces Argonne v18 NN phase shifts at low energies up to 500 MeV, so that it is repulsive at short distances. The new nucleon-nucleon potential provides the correct singlet and triplet NN scattering lengths and deuteron binding energy $E_{deu} = 2.225$ MeV. It reproduces the Argonne v18 NN phase shifts of pp scattering better than the previously used one.

3.2 Three-body subsystems and $2+2$ partition

We study the $\bar{K}NNN$ system with the lowest value of the four-body isospin $I^{(4)} = 0$, which can be defined as $K^- ppn$ or $\bar{K}^0 nnp$ system. Its total spin is equal to one half, while the orbital momentum is zero, since all two-body interactions are chosen to be zero. The $\bar{K}NNN$ system with these quantum numbers contains the following three-body subsystems:

- $\bar{K}NN$ with $I^{(3)} = 1/2, S^{(3)} = 0$ ($K^- pp$) or $S^{(3)} = 1$ ($K^- d$).
- NNN with $I^{(3)} = 1/2, S^{(3)} = 1/2$ (^{3}H or ^{3}He).

The three-body antikaon-nucleon system $\bar{K}NN$ with different quantum numbers was studied in our previous works, see Ref.[2] an references therein. The three-body AGS equations were solved with three separable $\bar{K}N$ potentials, mentioned above. In particular, a quasi-bound state exists in the $K^- pp$ system ($\bar{K}NN$ with spin zero), its binding energy and width were evaluated in our previous works and recalculated recently with the new model of the NN interaction [9].

No quasi-bound states were found in the $\bar{K}NN$ with the other set of quantum numbers: $I^{(3)} = 1/2$, $S^{(3)} = 1$ ($K^- d$) in our previous three-body calculations. However, the new NN potential changed the situation. The quasi-bound state caused by strong interactions only (in addition to an atomic state, kaonic deuterium) can exist in the $K^- d$ system similarly to the $K^- pp$ system with a much smaller binding energy($1 - 2$ MeV) than the binding energy of the $K^- pp$ quasi-bound state. The widths of the $K^- d$ and $K^- pp$ states are comparable, see [9].

As for the three-nucleon system NNN, we solved the system of AGS equations with the new two-term NN potential. The calculated binding energy was found to be 9.95 MeV for both ^{3}H and ^{3}He since the Coulomb interaction was not take into account.

Finally, $\bar{K}N + NN$ with isospin zero and spin one half is a $2+2$ partition of the four-body system. It is a special system with two non-interacting pairs of particles, which is described by a special three-body AGS system of equations.

4. Results and conclusions

The results of our four-body calculations are presented in Table 1. The four-body Faddeev-type equations Eq.(6) were solved using a direct pole search in the complex energy plane with three exact optical antikaon-nucleon potentials and the new NN interaction model. One-term separable versions of the amplitudes of the three-body subsystems and $2+2$ partition were constructed and used in the calculations. The binding energy is counted from the four-body break-up energy of the $\bar{K}NNN$ system $z_{\bar{K}NNN} = m_{\bar{K}} + 3\,m_N$.

The results obtained with phenomenological antikaon-nucleon potentials $V^{1,\mathrm{SIDD}}_{\bar{K}N}$ and $V^{2,\mathrm{SIDD}}_{\bar{K}N}$ differ from preliminary results [10] presented at 24th European Conference on Few-Body Problems in Physics (Guildford, UK, 1–6 September, 2019) since a new, more reliable numerical method of separable three-body and $2+2$ amplitudes construction, was used.

It is seen from the Table that the binding energy and width of the four-body state strongly depends on the particular model of the $\bar{K}N$ interaction. The largest difference is between results obtained with phenomenological and chiral models of $V_{\bar{K}N}$, while the difference between results with one- and two-pole phenomenological models is small. The predicted quasi-bound state in the

Table 1: Binding energy B_{K^-ppn} (MeV) and width Γ_{K^-ppn} (MeV) of the quasi-bound state in the K^-ppn system calculated with three $\bar{K}N$ potentials and new two-term separable V_{NN}.

	$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	$V_{\bar{K}N}^{\mathrm{Chiral}}$
B_{K^-ppn}	35.5	34.8	16.0
Γ_{K^-ppn}	56.7	60.3	67.2

$K^-ppn - \bar{K}^-nnp$ system has a smaller binding energy (16 − 36 MeV) than the similar state in the three-body K^-pp system [9]. The width of the four-body K^-ppn state is larger than the width of K^-pp and K^-d quasi-bound states obtained with two-pole $\bar{K}N$ potentials $V_{\bar{K}N}^{2,\mathrm{SIDD}}$ and $V_{\bar{K}N}^{\mathrm{Chiral}}$ and smaller, when the one-pole $V_{\bar{K}N}^{1,\mathrm{SIDD}}$ was used.

Asknowledgmens

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