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S-shell Λ and $\Lambda\Lambda$ hypernuclei based on chiral effective field theory

Hoai Le,[a,*] **Johann Haidenbauer,**[a] **Ulf-G. Meißner**[b,a,c] **and Andreas Nogga**[a]

[a] *IAS-4, IKP-3 and JHCP, Forschungszentrum Jülich, D-52428 Jülich, Germany*
[b] *HISKP and BCTP, Universität Bonn, D-53115 Bonn, Germany*
[c] *Tbilisi State University, 0186 Tbilisi, Georgia*
E-mail: h.le@fz-juelich.de, j.haidenbauer@fz-juelich.de, meissner@hiskp.uni-bonn.de, a.nogga@fz-juelich.de

We present results for Λ and $\Lambda\Lambda$ hypernuclei with $A = 4-7$ using the Jacobi no-core shell model in combination with baryon-baryon interactions derived within the framework of chiral effective field theory. First, we discuss the predictions of two almost phase-equivalent next-to-leading order YN interactions, NLO13 and NLO19, for the ${}^{4}_{\Lambda}$He, ${}^{5}_{\Lambda}$He, and ${}^{7}_{\Lambda}$Li systems. We then report on a calculation of s-shell $\Lambda\Lambda$ hypernuclei based on chiral YY potentials at LO and NLO. The prediction of NLO for ${}^{6}_{\Lambda\Lambda}$He are consistent with experiment. Both interactions also yield a bound state for ${}^{5}_{\Lambda\Lambda}$He, whereas the ${}^{4}_{\Lambda\Lambda}$H system is predicted to be unbound.

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*Speaker

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Introduction

Over many decades strangeness $S = -1$ and $S = -2$ hypernuclei have attracted particular attention of nuclear physicists. Since nucleons and hyperons (Λ, Σ, Ξ) form an SU(3) flavor octet, the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions are related to that in the nucleon-nucleon (NN) system by the underlying (though broken) $SU(3)$ flavor symmetry of QCD, and hypernuclei offer a perfect ground for exploring the dynamics from a different perspective. Furthermore, it is well known that hyperons play an important role for the size and mass of neutron stars as reflected in the so-called hyperon puzzle (see, e.g., Ref. [1] and references therein). Thus, the knowledge of the underlying baryon-baryon (BB) interactions is a prerequisite for understanding complex structures, i.e neutron stars and supernovae.

Due to the wealth of experimental data, the NN interaction is rather well understood. In particular, it has been successfully described within the framework of chiral effective field theory (EFT) [2]. In this approach, the long-range part of the interaction (due to exchange of pseudoscalar mesons) is fixed by chiral symmetry. The short-distance part is not resolved and effectively described by contact terms whose strengths, encoded in low-energy constants (LECs), need to be determined by a fit to data [2]. Recently, chiral EFT has also been applied to the strangeness $S = -1$ [3, 4] and $S = -2$ [5–7] sectors, so far up to next-to-leading order (NLO) in the chiral expansion. Unfortunately, the scarcity of YN and YY data does not allow for a unique determination of the pertinent LECs, even when $SU(3)_f$ symmetry is exploited. Indeed, by pursuing different strategies for fixing those LECs, two realizations of the YN interaction, NLO13 [3] and NLO19 [4], have been established that describe all the available YN data with comparable quality (χ^2). But, as discussed in [4], NLO19 involves a somewhat weaker $\Lambda N - \Sigma N$ transition potential and the two potentials predict slightly different results for ${}^4_\Lambda$He and for nuclear matter. Below we report results of a study of $A = 5, 7$ hypernuclei based on those YN potentials, using the Jacobi no-core shell model (J-NCSM) [8]. It allows us to explore the sensitivity of the Λ separation energies of those systems to the details of the aforementioned YN interactions, and to shed light on possible effects of chiral YNN three-body forces (3BFs).

Due to the even more scarce empirical information on the interaction in the YY and ΞN systems, the status of the strangeness $S = -2$ sector is even less satisfactory. Given technical difficulties in performing pertinent scattering experiments, this situation is not expected to change significantly in the near future. Complementary information could, however, be obtained from studying $\Lambda\Lambda$ and Ξ hypernuclei, especially of light systems for which microscopic calculations can be performed [9, 10]. Here, we present first predictions for $A = 4-6$ $\Lambda\Lambda$ hypernuclei within the *ab initio* J-NCSM [9], based on chiral LO and NLO YY interactions [5–7]. By comparing the theoretical predictions of ${}^{\;6}_{\Lambda\Lambda}$He with experiment, useful information to further constrain the underlying two-body interaction could be obtained.

The J-NCSM is based on an expansion of the many-body wave function in a harmonic oscillator (HO) basis that depends on relative Jacobi coordinates. HO states are, however, not very well suited for calculations using interactions that induce strong correlations like nuclear and hypernuclear interactions, hence, the similarity renormalization group (SRG) evolution [11] is applied to NN, YN and YY potentials in order to soften the interactions [8]. Generally, the SRG evolution will not only modify the two-body interactions but also induce three- and higher-body forces. These SRG-induced forces are omitted in the current study and their effect is estimated by studying the dependence of the binding energies on the SRG flow parameter λ.

Effects of the NLO YN interactions on B_Λ

In this section, we discuss the impact of the two practically phase-equivalent NLO13 [3] and NLO19 [4] YN potentials on the Λ-separation energies B_Λ. Results for ${}^4_\Lambda$He$(0^+, 1^+)$ and ${}^5_\Lambda$He are displayed in panels (a) -(c) of Fig. 1, respectively, and those for the $A = 7$ system can be found in [8]. The SMS N^{4}LO+(450) potential [12] with $\lambda_{NN} = 1.6$ fm^{-1} is used to describe the NN interaction. The YN potentials with cutoffs $\Lambda_Y = 500$-650 MeV [3, 4] are SRG-evolved to the same range of SRG flow parameter, namely $0.8 \leq \lambda_{YN} \leq 3.0$ fm^{-1}. Overall, the dependence of B_Λ on the chiral regulator Λ_Y is somewhat stronger for NLO19 than for NLO13.

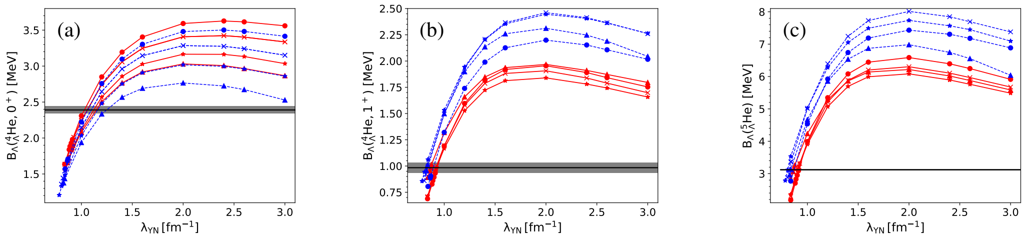

Figure 1: B_Λ of (a) $^4_\Lambda$He(0^+), (b) $^4_\Lambda$He(1^+), (c) $^5_\Lambda$He as a function of λ_{YN}. Black lines with grey bands represent the experimental B_Λ and the uncertainties, respectively. Calculations are based on the NN SMS N^{4}LO+(450) with λ_{NN} = 1.6 fm^{-1}, in combination with the NLO13 (red solid lines) and NLO19 (dashed blue lines) YN potentials for four regulators, Λ_Y = 500 (triangles), 550 (stars), 600 (crosses) and 650 (circles) MeV.

This is because in the NLO19 realization one has less freedom to absorb regulator artifacts into the LECs [4]. Also, there are large differences between B_Λ obtained with NLO13 and NLO19, which apparently exceed the Λ_Y-dependence. Furthermore, for all states except $^4_\Lambda$He(0^+), one observes a clear tendency toward larger B_Λ values predicted by NLO19 than those calculated with NLO13. Evidently, the interaction with a weaker Λ-Σ conversion potential generally leads to larger Λ-separation energies. That trend is, however, not clearly observed for the ground state of $^4_\Lambda$He as can be seen in panel (a). As discussed in [4, 8], the strong sensitivity of B_Λ to the underlying interaction can qualitatively be understood in terms of the different weights with which the spin-singlet and triplet ΛN interactions effectively contribute to specific *s*-shell Λ hypernuclei. The pronounced variations of B_Λ predicted by NLO13 and NLO19 are a striking evidence for possible contributions of 3BFs to the separation energies. Such discrepancies are expected to be largely removed once proper chiral YNN forces are included [13].

Correlation of Λ separation energies

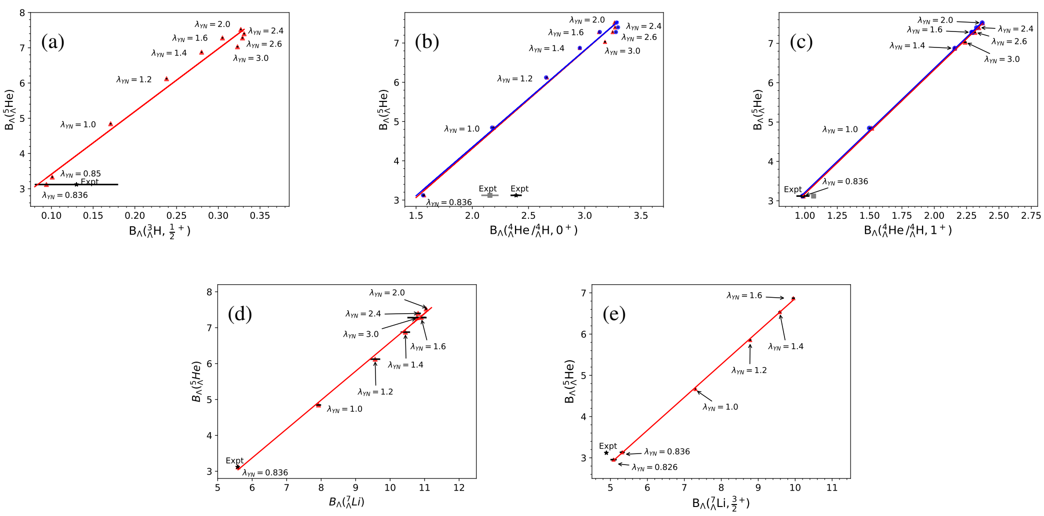

Figure 2: Correlations of Λ-separation energies between $^5_\Lambda$He and (a) $^3_\Lambda$H, (b) the 0^+ state of $^4_\Lambda$He (red) and $^4_\Lambda$H (blue), (c) the 1^+ state of $^4_\Lambda$He (red) and $^4_\Lambda$H (blue), (d) $^7_\Lambda$Li($1/2^+$, 0) and (f) $^7_\Lambda$Li($3/2^+$, 0), for a wide range of flow parameters λ_{YN}. The experimental B_Λ are taken from [14]. The Idaho-N^{3}LO(500) SRG-evolved to λ_{NN} = 1.6 fm^{-1} and NLO19(600) was used for the NN and YN interaction, respectively.

As seen in Fig. 1, the SRG YN evolution has a strong impact on the Λ separation energies B_Λ, leading to an overall variation of $\Delta B_\Lambda \approx 2$ and 5 MeV in $^4_\Lambda$He and $^5_\Lambda$He, respectively. This makes it rather difficult to compare the obtained separation energies with experiment. However, the striking similarity between the SRG dependence of B_Λ for $A = 3-7$ systems clearly hints at some intriguing correlations between the separation energies of these systems. Such correlations between $B_\Lambda(^5_\Lambda\mathrm{He})$ and B_Λ of $A = 3, 4$ and 7 hypernuclei are illustrated in Fig. 2. Here $^5_\Lambda$He has been chosen as a benchmark system since the binding energy of this hypernucleus is precisely known and, in addition, our results for $B(^5_\Lambda\mathrm{He})$ are also well-converged. $B_\Lambda(^3_\Lambda\mathrm{H})$ is obtained solving Faddeev equations in momentum space since the NCSM calculations for loosely bound systems converge very slowly. The results shown in Fig. 2 are based on the NN interaction Idaho-N^{3}LO(500) with $\lambda_{NN} = 1.6$ fm^{-1} and the NLO19(600) YN interaction, but we stress that similar trends are observed for SMS N^{4}LO+(450) and in combination with other YN interactions, see also [8]. Note that the results for the isospin mirrors $^4_\Lambda$He and $^4_\Lambda$H shown in panels (b)-(c) are almost identical because there is no charge-symmetry breaking (CSB) in the employed YN potential. Each symbol in panels (a)-(e) represents the numerical B_Λ of the corresponding two systems computed at the same λ_{YN}. The straight lines are obtained from a linear fit to the results, resembling the Tjon lines between the binding energies of ^{4}He and ^{3}He [15]. Clearly, for all systems considered, there is a nearly perfect linear correlation for flow parameters up to $\lambda_{YN} = 2.0$ fm^{-1} and a slight deviation from the straight line as λ_{YN} further increases. The latter can be attributed to the possible contribution of the SRG-induced 3BF [8]. Interestingly, the correlation line goes through the experimental B_Λ of the $^3_\Lambda$H, $^4_\Lambda$He(1^+), $^5_\Lambda$He, and $^7_\Lambda$Li$(1/2^+)$ hypernuclei at $\lambda_{YN} = 0.836$ fm^{-1}, whereas $^4_\Lambda$He(0^+) is slightly underbound. This special value of λ_{YN}, referred to as the magic λ^m_{YN}, will obviously depend on the employed YN interactions as well as their regulators. Furthermore, at this λ^m_{YN}, our results for the hypertrition, $B_\Lambda(^3_\Lambda\mathrm{H}) = 0.92$ MeV, and for the spin doublet of $^4_\Lambda$He, $B_\Lambda(0^+(1^+)) = 1.57(0.97)$ MeV, are surprisingly close to the predictions of the non-evolved bare YN interaction obtained within exact Faddeev-Yakubovsky calculations, $B_\Lambda = 0.112$, 1.61 and 1.18 MeV, respectively. Again, the slight deviations between the two results are consistent with the size of 3BFs expected from the power counting of chiral EFT [4]. The striking linear correlations between B_Λ of different hypernuclei suggest that, probably, one could parameterize the missing SRG-induced 3BFs by only one adjustable parameter. If this is the case, one is able to minimize the effect of the omitted 3BFs by tuning the SRG-YN flow parameter to the magic value λ^m_{YN} for which a particular hypernucleus, say $^5_\Lambda$He, is properly described. That λ^m_{YN} can then serve as a good starting point for hypernuclear calculations that require a SRG-YN evolution. This, in turn, may provide a good opportunity to study hypernuclear structure as well as the YN forces in a less expensive but realistic approach. A possible application of this finding has been discussed in [16].

$\Lambda\Lambda$ hypernuclei: $^{\;\;6}_{\Lambda\Lambda}$He, $^{\;\;5}_{\Lambda\Lambda}$He and $^{\;\;4}_{\Lambda\Lambda}H(1^+, 0)$

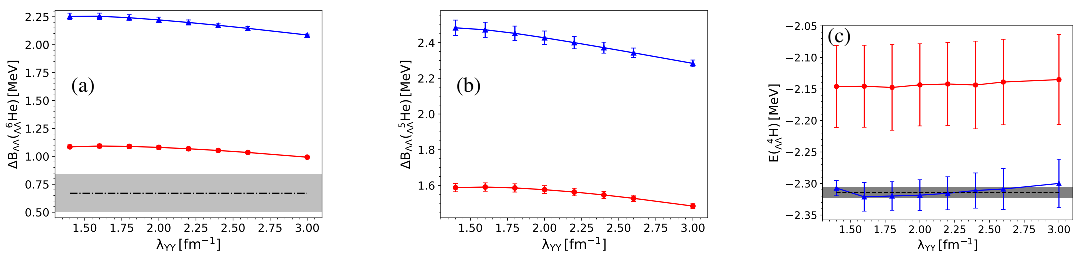

Figure 3: $\Lambda\Lambda$ excess energies, $\Delta B_{\Lambda\Lambda}(^{\;\;6}_{\Lambda\Lambda}\mathrm{He})$ (a), $\Delta B_{\Lambda\Lambda}(^{\;\;5}_{\Lambda\Lambda}\mathrm{He})$ (b), and binding energy $E_{\Lambda\Lambda}(^{\;\;4}_{\Lambda\Lambda}\mathrm{H}(1^+, 0))$ as function of the flow parameter λ_{YY}. Calculations are based on the NN SMS N^{4}LO+(450) with $\lambda_{NN} = 1.6$ fm^{-1}, the YN NLO19(650) with $\lambda_{YN} = 0.868$ fm^{-1} and in combination with the LO(600) (blue triangles) and NLO(600) (red circles) YY potentials. Dashed line with shaded area is the experimental $\Delta B_{\Lambda\Lambda}(^{\;\;6}_{\Lambda\Lambda}\mathrm{He})$ (a) [17] and the computed $E(^3_\Lambda\mathrm{H})$ (c).

The predictions of the LO [5] and NLO [7] YY potentials for $A = 4-6$ $\Lambda\Lambda$ hypernuclei are displayed in panels (a)-(c) of Fig. 3. The N^{4}LO+(450) with $\lambda_{NN} = 1.6$ fm^{-1} and NLO19(650) with $\lambda_{YN} = 0.868$ fm^{-1} are employed for the NN and YN interactions, respectively. This combination of NN and YN potentials successfully predicts the empirical B_Λ for $^3_\Lambda$H, $^4_\Lambda$He(1^+) and $^5_\Lambda$He, and underbinds $^4_\Lambda$He(0^+) only slightly

[8]. Overall, the LO YY interaction predicts too much attraction in ${}^{\;6}_{\Lambda\Lambda}$He, overbinding the system by about 1.5 MeV (see panel (a)). On the other hand, the ΛΛ excess energy $\Delta B_{\Lambda\Lambda}$ predicted by the NLO potential, $\Delta B_{\Lambda\Lambda} \approx 1.1$ MeV, is only slightly larger than the empirical value of $\Delta B_{\Lambda\Lambda} = 0.67 \pm 0.17$ MeV [17]. As can be seen in panel (b), both potentials clearly yield a bound state in ${}^{\;5}_{\Lambda\Lambda}$He. Also here the LO prediction, $\Delta B_{\Lambda\Lambda}({}^{\;5}_{\Lambda\Lambda}\text{He}) \approx 2.5$ MeV, visibly exceeds the NLO result of about 1.6 MeV. Note that the probability of finding a Ξ hyperon in the ground-state wave function of ${}^{\;5}_{\Lambda\Lambda}$He is much bigger than that for ${}^{\;6}_{\Lambda\Lambda}$He [9], which is consistent with the expected suppression of conversion processes like ΛΛ − ΞN in the latter hypernucleus and in p-shell hypernuclei. It is furthermore remarkable that both $\Delta B_{\Lambda\Lambda}({}^{\;5}_{\Lambda\Lambda}\text{He})$ and $\Delta B_{\Lambda\Lambda}({}^{\;6}_{\Lambda\Lambda}\text{He})$ exhibit a rather weak dependence on the YY flow parameter λ_{YY}, with an overall variation of only about 100 keV. It is at least one order of magnitude smaller than the variation of $B_\Lambda({}^5_\Lambda\text{He})$ with respect to λ_{YN}, as discussed in the previous section. Finally, we show in panel (c) the binding energies of ${}^{\;4}_{\Lambda\Lambda}\text{H}(1^+,0)$ computed with the LO (blue triangles) and NLO (read circles) potentials together with the computed hypertriton binding energy (dashed line with shaded area). Clearly, the NLO YY interaction does not support a bound ${}^{\;4}_{\Lambda\Lambda}\text{H}(1^+,0)$, and also the results for LO likely hint at a particle-unstable system with respect to ${}^3_\Lambda$H. Specifically, given the considerable overbinding of ${}^{\;6}_{\Lambda\Lambda}$He by the LO interaction, one can safely assume that this potential also overestimates the actual attraction in ${}^{\;4}_{\Lambda\Lambda}$H.

Conclusion

In this work, we reported predictions of two practically phase-equivalent chiral YN potentials (NLO13, NLO19) for $A = 4 - 7$ Λ hypernuclei. The large differences in the Λ separation energies based on the two potentials can be attributed to (missing) contributions of chiral YNN forces. We also observed an almost perfect linear correlations between B_Λ of $A = 3 - 7$ systems for a wide range of λ_{YN} values. Furthermore, at the magic flow parameter λ^m_{YN} that reproduces the experimental value of $B_\Lambda({}^5_\Lambda\text{He})$, the results for ${}^3_\Lambda$H and for the two states $(0^+, 1^+)$ in ${}^4_\Lambda$He are in agreement with the values obtained with the non-evolved bare YN interactions, while $B_\Lambda({}^7_\Lambda\text{Li}(1/2^+, 3/2^+))$ are close to the experiment. This suggests that λ^m_{YN} could be a good starting point for hypernuclear calculations that require a SRG-YN evolution. Furthermore, we considered ΛΛ hypernuclei using LO and NLO YY interactions from chiral EFT. The results for ${}^{\;6}_{\Lambda\Lambda}$He computed with NLO are in a reasonable agreement with experiment. Both LO and NLO also clearly support a bound ${}^{\;5}_{\Lambda\Lambda}$He hypernucleus, whereas the ${}^{\;4}_{\Lambda\Lambda}$H system is predicted to be unstable with respect to ${}^3_\Lambda\text{H} + \Lambda$ threshold.

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