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Studying chiral imbalance using Chiral Perturbation Theory.

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We analize the most general low-energy effective Lagrangian including local parity violating terms parametrized by an axial chemical potential μ_5. This result is obtained following the external source method, up to $O(p^4)$ order in the chiral expansion for two light flavours. We show that the $O(p^4)$ Lagrangian includes new terms proportional to μ_5^2 and new low-energy constants. Finally, the μ_5 and temperature dependences of several observables related to the vacuum energy density are studied.

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1. Introduction

A convenient way to parametrize a parity-breaking source or chiral imbalance is by means of a constant axial chemical potential μ_5 to be added to the QCD action over a given finite space-time region. The chiral charge may remain approximately conserved during the fireball evolution in a typical heavy-ion collision, giving rise in the light quark sector to a chemical potential term which is equivalent to consider an axial source $a_\mu^0 = \mu_5 \delta_{\mu 0}$ in the QCD generating functional.

We provide in [1] a model-independent approach, constructing the most general effective Lagrangian for the lightest degrees of freedom in the presence of the μ_5 source. To keep a consistent chiral power counting in a generic momentum scale p, μ_5 should be a $O(p)$ quantity. Preliminary ideas along this line have been proposed in [2]. In this contribution we will review the main phenomenological consequences in terms of observables such as the energy density, the pion decay constant, the pion mass, the quark condensate and the topological susceptibilities.

2. Effective Lagrangian

The construction of the most general, model-independent, effective Lagrangian can be carried out within the framework of the external source method [3]. To do that, the so called "spurion" fields $Q_{R,L}(x)$ are introduced, where in our case $Q_L = -Q_R = (\mu_5/F)\mathbb{1}$ and F is the pion decay constant in the chiral limit. There are additional terms in the effective Lagrangian depending on $Q_{R,L}$. Moreover, the operator $\mathrm{tr}(U^\dagger d_\mu U)$ has to be considered, unlike in standard ChPT where that operator vanishes. To $O(p^2)$, the only modification to the chiral Lagrangian is a constant term:

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{tr}\left[\partial_\mu U^\dagger \partial^\mu U + \chi^\dagger U + \chi U^\dagger\right] + 2\mu_5^2 F^2 \left(1 - Z + \kappa_0\right), \quad (1)$$

where $\chi = 2B_0\mathcal{M}$ and $\mathcal{M}$ is the quark mass matrix, $-F^2 B_0$ is the tree-level quark condensate, U is the field containing the meson fields and Z is defined in [4].

At $O(p^4)$ we have new terms constructed out of the Q operators and $\mathrm{tr}(U^\dagger d_\mu U)$. The possible contributions (including Q operators) are of the form: $ddQQ, \chi QQ, QQQQ$, and the explicit μ_5 corrections are:

$$\mathcal{L}_4(\mu_5) = \mathcal{L}_4^0(\mu_5 = 0) + \kappa_1\mu_5^2\mathrm{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \kappa_2\mu_5^2\mathrm{tr}\left(\partial_0 U^\dagger \partial^0 U\right) + \kappa_3\mu_5^2\mathrm{tr}\left(\chi^\dagger U + \chi U^\dagger\right) + \kappa_4\mu_5^2, \quad (2)$$

where these κ_i constants above can be compared with the electromagnetic LEC k_i given in [4] by taking from the general Lagrangian $Q_R = Q_L = Q$ with Q the electromagnetic charge matrix.

3. Physical consequences

The external field breaks manifest Lorentz invariance and the spatial and time components of the pion decay constant are different [5]. The two main physical consequences of that are the pion velocity and pion mass. Requiring that the pion velocity remains smaller than the speed of light for any μ_5, the constrain $\kappa_2 < 0$ is obtained. On the other hand, in order to the square pion mass remains positive, $\kappa_1 - \kappa_3 < 0$. However, a decreasing pion mass for low values of μ_5 does not necessarily imply a tachyonic mode so this last condition may be too restrictive. We plot in Figure 1 the dependence of v_π and M_π^2 with μ_5 expected within natural values for the LECs.

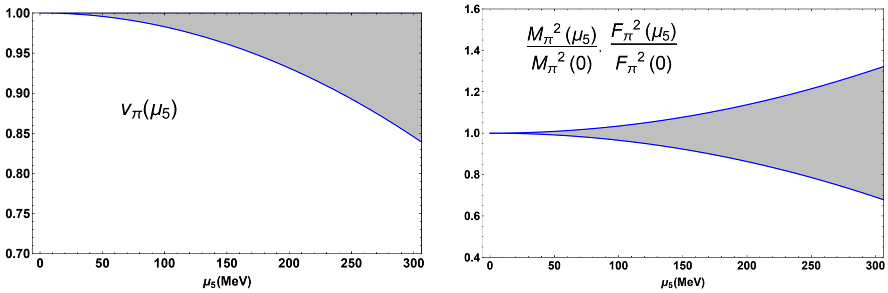

Figure 1: μ_5 dependence of pion velocity, pion mass and pion decay constant, to leading order in ChPT. The grey bands correspond to the uncertainties of the LEC within natural values $1/(16\pi^2)$ which is their expected size from loop corrections.

The vacuum energy density at order $O(p^2)$ is obtained from the constant part of the $\mathcal{L}_2$ Lagrangian. The $O(p^4)$ includes the contribution from the kinetic part of $\mathcal{L}_2$ and the field-independent contributions coming from $\mathcal{L}_4$. Finally, to $O(p^6)$ the contributions are the two-loop closed diagram with four-pion vertices coming from $\mathcal{L}_2$, the $O(\pi^2)$ part of $\mathcal{L}_4$ and the field-independent contributions coming from $\mathcal{L}_6$.

The main features of chiral symmetry restoration can be read from the quark condensate and the scalar susceptibility which are derived from the vacuum energy density. The first μ_5 correction to the quark condensate is temperature independent and its behavior is determined by the sign of κ_3, it increases if $\kappa_3 > 0$ and it decreases if $\kappa_3 < 0$. Lattice results clearly show growing condensate and T_c with μ_5 [6]. For the ratio $\langle\bar{q}q\rangle_l^{\text{NNLO}}(T,\mu_5)/\langle\bar{q}q\rangle_l^{\text{NNLO}}(0,\mu_5)$ the κ_i dependence reduces to the combinations $\kappa_a = 2\kappa_1 - \kappa_2$ and $\kappa_b = \kappa_1 + \kappa_2 - \kappa_3$ (although in the chiral limit it depends only on κ_a). In order to provide more quantitative conclusions we plot the μ_5 dependence of the critical temperature normalized by its value at $\mu_5 = 0$. The dependence of this quantity is just quadratic in μ_5 and is given in [1]. This curve for the physical pion mass lies very close to the chiral limit one and the lattice points [7] clearly fall into the uncertainty given by the natural values range of κ_a and κ_b. We compared a fit with two and three points (left side of Figure 2) in the chiral limit with a fit in the massive case fixing the κ_a parameter. These fits show that the chiral limit approach with just one parameter κ_a is a robust approximation.

The dependence of the topological susceptibility, χ_{top}, with low and moderate μ_5 is controlled by the κ_3 constant. The fit of that observable to the lattice data, which we can see in the right side of Figure 2, shows that the results for κ_3 are compatible with zero and the error bands are much narrower than the natural values for this constant. The lattice points used for the fit are those in [8].

Another quantity studied in the lattice is the chiral charge density, $\rho_5(\mu_5)$. We perform a fit of $\rho_5(\mu_5)$ to the lowest values of μ_5 provided in [8]. The simple linear dependence fits very well the lowest μ_5 lattice points. The prediction for κ_0 is consistent with the fit allowing the other parameters to be free. An interesting result is that the analysis of the energy density does not favor a $\mu_5 \neq 0$ minimum for the free energy for moderate values of μ_5.

Finally, the thermodynamic pressure (P) and the speed of sound (c_s) have also been calculated. The μ_5 corrections to P are parametrized by κ_2 and κ_b but in the chiral limit, which corresponds to the ultrarelativistic free pion gas, only the κ_2 term survives. On the other hand, c_s depends only on κ_b and as can be seen in [1], when we include the μ_5 corrections, c_s^2 remains below $1/3$. The uncertainty band for κ_b actually narrows as T increases.

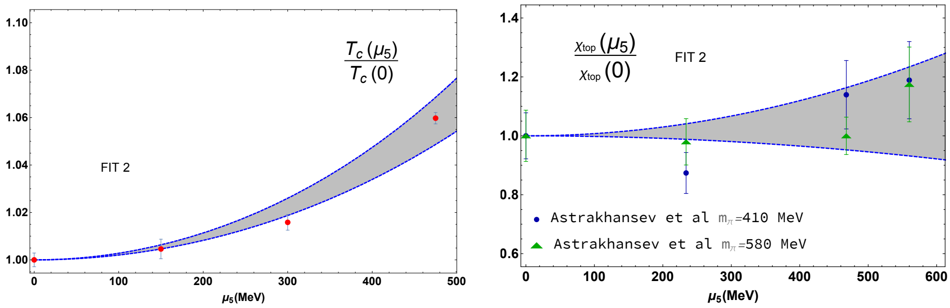

Figure 2: Left: Fit of the critical temperature, $T_c(\mu_5)/T_c(0)$, for three lattice points in the chiral limit. We have used the three points with lower μ_5 (and $\mu_5 \neq 0$) obtained in [7]. Right: Fit of lattice data to $\chi_{top}(\mu_5)/\chi_{top}(0)$ where the number of lower μ_5 points with $\mu_5 \neq 0$ considered is 3 (m_π = 410 MeV)+3 (m_π = 580 MeV).

4. Conclusions

In this work we have analyzed the effective chiral Lagrangian for nonzero chiral imbalance up to fourth order and from that we have studied several observables and the dependence of their μ_5 corrections with the κ_i constants. Besides this we have compared our results with existing lattice data.

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