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Proton Decay Amplitudes with Physical Chirally-Symmetric Quarks

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Proton decay is a major prediction of Grand-Unified Theories (GUT) and its observation would indicate baryon number violation that is required for baryogenesis. Many decades of searching for proton decay have constrained its rate and ruled out some of the simplest GUT models. Apart from the baryon number-violating interactions, this rate also depends on transition amplitudes between the proton and mesons or leptons produced in the decay, which are matrix elements of three-quark operators. We report nonperturbative calculation of these matrix elements for the most studied two-body decay channels into a meson and antilepton done on a lattice with physical light and strange quark masses and lattice spacings $a \approx 0.14$ and 0.20 fm. We perform nonperturbative renormalization and excited state analysis to control associated systematic effects. Our results largely agree with previous lattice calculations done with heavier quark masses and thus remove ambiguity in ruling out some simple GUT theories due to quark mass dependence of hadron structure.

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Introduction Proton decay is a $|\Delta B| = 1$ baryon number-violating process that has been predicted by Grand Unification Theories (GUT)[1–3] but has not been observed so far. Discovery of proton decay may potentially fulfil one of the three prerequisites to explain the Baryon asymmetry in the Universe [4][1], and also demand extension of the Standard Model to accomodate baryon number violation [6], potentially involving supersymmetry [7, 8].

Some of the most important proton decay channels are into an (anti)lepton and one or more mesons. At the lowest order, interactions leading to such decay are local operators [9, 10],

$$\mathcal{L}_{\text{eff}} = \sum_I C_I O_I + \text{h.c.}, \qquad O_I = \epsilon^{abc}(\bar{q}^{aC} P_{\chi_I} q^b)(\bar{\ell}^C P_{\chi'_I} q^c), \tag{1}$$

where chirality projectors $P_{\chi^{(\prime)}=R,L} = \frac{1\pm\gamma_5}{2}$ and the Wilson coefficients C_I depend on the specifics of an underlying unified theory. For small $m_{\bar{\ell}} \ll m_N$, the $p \to \Pi\bar{\ell}$ partial decay width is

$$\Gamma(p \to \Pi\bar{\ell}) = \frac{m_N}{32\pi}\Big[1 - \Big(\frac{m_\Pi}{m_N}\Big)^2\Big]^2 \Big|\sum_I C_I W^I_{\bar{\ell}}\Big|^2, \tag{2}$$

where $\Pi = \pi, K$ is a meson and $\bar{\ell} = e^+, \bar{\nu}, \mu^+$ is a lepton in the final state. The proton decay amplitudes $W_{\bar{\ell}} \approx W_0^I + O(m_\ell/m_N) \cdot W_1^I$ depend only on the quark component of the operators O_I (1). From dimensional analysis, $W_\ell^I \propto \Lambda^2_{\text{QCD}}$ and the proton decay rate is suppressed as $\Gamma \propto |c_I|^2(\Lambda_{\text{QCD}}/\Lambda_{(\text{GUT})})^4$, where c_I are dimensionless GUT couplings and $\Lambda_{(\text{GUT})}$ is the relevant scale. The decay form factors $W^I_{0,1}(Q^2)$ are defined as

$$\bar{v}^C_{\ell\alpha}(\vec{q})\langle\Pi(\vec{p})|O^{\chi\chi'}_\alpha(q)|N(\vec{k})\rangle = (\bar{v}^C_\ell(\vec{q})\, P_{\chi'}\, [W^O_0(Q^2) - \frac{i\not{q}}{m_N}W^O_1(Q^2)]u_N(\vec{k})) \tag{3}$$

and must be determined at the decay kinematical point $Q^2 = -(E_N - E_\Pi)^2 + (\vec{k} - \vec{p})^2 = -m^2_\ell$. They depend on nonberturbative quark dynamics and have to be evaluated in ab initio QCD calculations.

Table 1: Parameters of lattice ensembles with I-DSDR gauge and (zMobius) Domain Wall fermion actions.

$L_x^3 \times L_t$	a^{-1} [GeV]	β	am_π	am_K	$m_\pi L$	N_{cfg}	N_{exact}	N_{approx}
$24^3 \times 64$	1.023(2)	1.633	0.1378(7)	0.5004(25)	3.31	140	1	32
$32^3 \times 64$	1.378(5)	1.75	0.1008(5)	0.3543(6)	3.25	112	1	32

We perform our QCD calculation on a lattice using physical values of quarks with chirally-symmetric action (see Tab. 1 and Ref. [11] for details). For full description of the contents in this report, see Ref. [12]. To make such expensive calculations affordable we (1) use "zMobius" fermion action, (2) use pre-calculated multigrid eigenvectors [13] to accelerate propagator calculation, and (3) employ *all-mode-averaging* sampling. In the latter, we evaluate 32 approximate samples per configuration with quark propagators computed employing truncated Conjugate-Gradient (s quark) combined with deflation (u, d quarks), as well as one exact sample to correct for bias in the approximate samples.

[1] There are viable alternatives such as leptogenesis [5].

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Hadron spectrum First step of the analysis is to extract the meson (pion and kaon) and proton masses and energies as well as normalization of their operators from their two-point functions

$$C^{\Pi\Pi}(\vec{k},t) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \langle J_\Pi(x)\, J_\Pi^\dagger(0)\rangle, \tag{4}$$

$$C_+^{N\bar{N}} = \mathrm{Tr}\Big[\frac{1+\gamma_4}{2} C^{N\bar{N}}\Big], \qquad C_{\alpha\beta}^{N\bar{N}}(\vec{k},t) = \sum_{\vec{x}} e^{-i\vec{k}\vec{x}} \langle N_\alpha(x)\, \bar{N}_\beta(0)\rangle, \tag{5}$$

where $J_\Pi = \bar{d}\gamma_5 u$, $\bar{s}\gamma_5 u$, and $\bar{s}\gamma_5 d$ for π^+, K^+, and K^0, respectively, and $N = (u^T C\gamma_5 d)u$ for the proton. In Figure 1, we show their effective energies and 2-state fit results compared to the continuum dispersion relations extrapolated from their masses determined on the lattice.

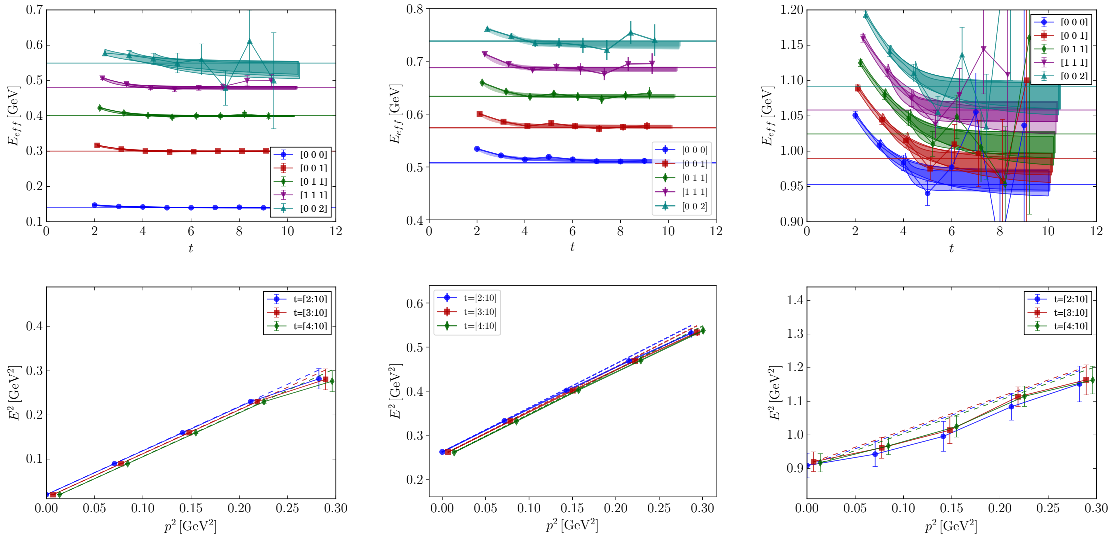

Figure 1: Two-state fits to pion (left), kaon (center), and nucleon (right) two-point correlation functions shown as effective energies (top) and ground-state dispersion relations (bottom) on the 24ID ensemble. The continuum dispersion relations $E^2(p) = E^2(0) + p^2$ for ground-state energies are shown with horizontal lines in the top panels and with dashed lines in the bottom panels.

Matrix elements are extracted from the three-point functions

$$C_{\alpha\beta}^{\Pi O N}(\vec{p},\vec{q};t_2,t_1) = \sum_{\vec{y},\vec{z}} e^{-i\vec{p}\vec{y}-i\vec{q}\vec{z}+i\vec{k}\vec{x}} \langle J_\Pi(\vec{y}, x_4+t_2)\, O_\alpha^{\chi\chi'}(\vec{z}, x_4+t_1)\, \bar{N}_\beta(x)\rangle, \tag{6}$$

where the spin indices α, β are the lepton and proton polarization and the 4-momentum transfer $q = (k-p)$ is the lepton "recoil" that must satisfy the decay kinematics $q^2 = m_\ell^2$. We use the two spin projections $C_{\mathcal{P}}^{\Pi O N} = \mathrm{Tr}[\mathcal{P} C^{\Pi O N}]$ with $\mathcal{P} = \frac{1}{2}(1+\gamma_4)$ and $\frac{1}{2}(1+\gamma_4)(\vec{\gamma}\cdot q)$, which yield linearly independent combinations of the proton decay form factors $W_{0,1}$ [12]. Ground-state matrix elements are obtained from the correlation functions (6) using 2-state fits as well, with state energies and overlaps determined in the spectrum analysis. We compare that to the alternative "ratio" method

$$R_{\mathcal{P}}^{O}(\vec{p},\vec{q};t_2,t_1) = \frac{\sqrt{Z_\Pi(\vec{p}) Z_N(\vec{k})}\, C_{\mathcal{P}}^{\Pi O\bar{N}}(\vec{p},\vec{q};t_2,t_1)}{C^{\Pi\Pi}(\vec{p},t_2-t_1)\, C_+^{N\bar{N}}(\vec{k},t_1)}, \tag{7}$$

where $R|_{t_1 \approx t_2/2}$ is constructed to converge to the ground-state matrix element at large t_2. The agreement between the fits, the ratios, and the ground-state results in the pion channel shown in Fig. 2 indicates that excited-state effects are negligible. Due to discrete values of momenta in the finite volume, we calculate the matrix elements at three kinematical points and perform linear interpolation in Q^2 (see Fig. 3, left), and then perform a^2 extrapolation to the continuum limit.

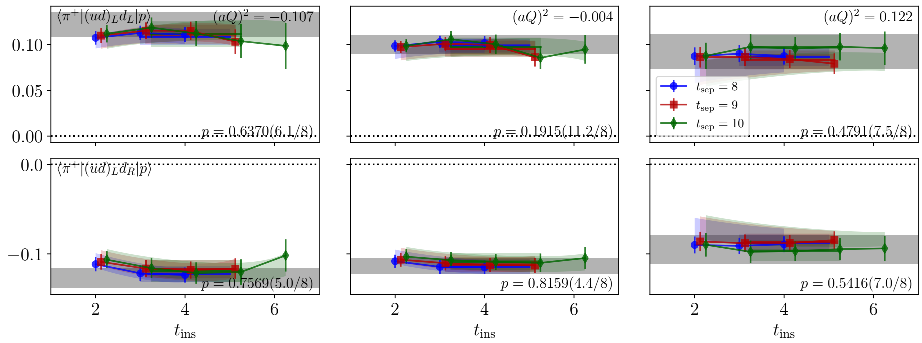

Figure 2: Two-state fits of renormalized $N \to \pi$ correlation functions for form factor W_0 at the three kinematic points on the 24ID lattices with $a \approx 0.20$ fm. The data points show the ratios (7), the color bands show 2-state fits, and the grey bands show the ground-state fit results. All data are normalized in the final $\overline{\rm MS}$ scheme. The fit quality (p-value) is estimated using the Hoteling distribution.

Discussion Our final results are presented in Fig. 3 (right). We find reasonable agreement with earlier calculations that employed heavier quark masses[17], quenched [15] or chiral symmetry-breaking fermion action [16]. We also compare our results to estimates based on tree-level chiral perturbation theory using lattice nucleon-to-vacuum decay constants, and find that the latter yield larger values than the direct calculation. We do not observe any suppression as suggested in Ref. [18], confirming earlier limits on some the GUT models. Although conservatively estimated uncertainties in our results sufficiently small [12], they can be further improved by performing calculations at finer lattice spacings, larger volume, and with more statistics.

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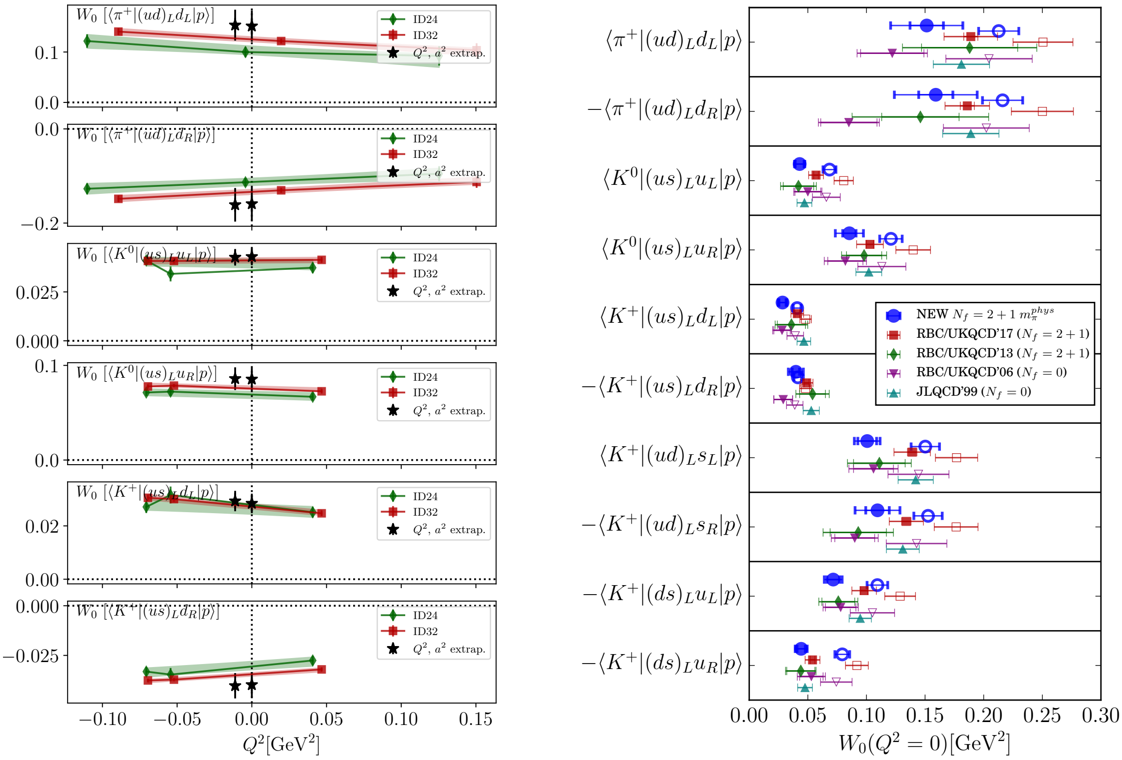

Figure 3: (Left) Momentum interpolation and continuum extrapolation of form factor W_0 for select six channels. (Right) Comparison of our results ("NEW") for the proton decay amplitudes $W_0(0)$ computed directly (filled symbols) and indirectly (open symbols) to previous determinations [14–16]. All results are renormalized to the $\overline{\mathrm{MS}}(2\,\mathrm{GeV})$ scheme.

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