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Mixed QCD×EW corrections for Drell-Yan processes

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In this talk we cover the current status of the calculation of mixed QCD×EW corrections to the Drell-Yan process at the LHC. After briefly reviewing ongoing efforts in the computation of the fully off-shell case, we will focus on the scenario where an electroweak vector boson is resonant. In this respect, complete $O(\alpha\alpha_s)$ predictions for the production of on-shell Z and W bosons at the LHC recently became available in literature. As a case study, we present results for the inclusive and fiducial cross sections as well as kinematic distributions for the production of a Z boson. We discuss the impact of these corrections and describe their salient features.

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1. Introduction

The Drell-Yan (DY) process is at the centre of the current and future programme of the Large Hadron Collider (LHC), both for Standard Model precision measurements and beyond Standard Model searches. From the production of a dilepton system the W boson mass M_W is expected to be extracted with a sensitivity below 10 MeV and the electroweak mixing angle could be measured with LEP precision. Given the required accuracy, theoretical predictions which include NLO and NNLO QCD as well as NLO electroweak (EW) corrections must be taken into account. Mixed QCD×EW effects are thus among the most prominent sources of unknown theoretical uncertainty for the current precision goal.

The complete calculation of the mixed QCD×EW corrections for a fully off-shell dilepton system involves several technical complexities. Most notably, the computation of the two-loop amplitudes and the regularisation of soft and colliner singularities. As for the former, $O(\alpha\alpha_s)$ corrections entail two-loop diagrams with up to four external legs with massive internal and external particles, thus depending on several and disparate energy scales. The relevant master integrals have been recently computed in Refs. [1–3], but the full scattering amplitudes are not available yet. As far as the subtraction of infra-red singularities is concerned, QCD×EW corrections to Drell-Yan are formally of NNLO type, further complicated by the fact that any charged particle involved interacts with photons. Thus not only external partons radiate photons, but also virtual W bosons, which can eventually become resonant. In Ref. [4] NLO EW corrections to a pair of massive leptons were computed using the q_T slicing method, offering a potential extension to $O(\alpha\alpha_s)$. The same method was used in Ref. [5] for the calculation of mixed QCD×QED corrections to the production of an off-shell Z boson decaying to a pair of neutrinos[1].

In the lack of a complete fixed-order calculation, a first realistic estimate of the impact of mixed corrections was assessed in Refs. [7–9] in the context of the `POWHEG BOX` Parton Shower (PS) generator, combining NLO QCD+PS and NLO EW+QED PS predictions. A detailed phenomenological analysis pointed out that the impact of mixed factorisable $O(\alpha\alpha_s)$ corrections can induce a shift on the extraction of the W mass M_W as of $O(-16\,\text{MeV})$.

2. Mixed QCD×EW corrections to the Drell-Yan process in the resonance region

Precision measurements of the W boson mass and of the electroweak mixing angle are dominated by the production of resonant W and Z bosons respectively. In this region of the phase space, non-factorisable corrections which connect initial and final state are suppressed by powers of Γ_V/M_V, see Refs. [10, 11]. This permits to separate the production and decay stages of the resonant boson, reducing the technical complexity of the calculation considerably. In Ref. [11] factorisable corrections of QCD type to the production stage and EW type to the decay stage were presented for both Z and W production. As for the latter, it was argued that such corrections can lead up to $O(14\,\text{MeV})$ shift on the W mass extraction.

Recently, complete NNLO QCD×EW corrections to the inclusive cross section for Z boson production were computed in Ref. [12] and at the fully differential level in Ref. [13]. The calculation

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[1] We point out that shortly before the publication of this proceeding the calculation of the $O(n_f\alpha\alpha_s)$ corrections to off-shell W and Z bosons production were presented in Ref. [6]

of the QCD×EW corrections to on-shell W boson prodution at the fully differential level was presented in Ref. [14]. Here the authors give a thourough description of the subtraction scheme adopted and show results for the two-loop $O(\alpha\alpha_s)$ $qq' \to W$ form factor.
In the following we focus on the production of on-shell Z bosons[2].

2.1 Fully inclusive cross section through $O(\alpha\alpha_s)$ for the production of an on-shell Z boson

In this section we summarise the recent computation of mixed QCD×EW corrections to the inclusive cross section for the production of an on-shell Z boson of Ref. [12].

The total cross section σ_{tot} can be expressed as a perturbative series

$$\sigma_{\text{tot}} = \int \mathrm{d}\sigma, \quad \text{where} \quad \mathrm{d}\sigma = \mathrm{d}\sigma^{\text{LO}} + \sum_{i,j} \frac{\alpha_s^i}{2\pi}\frac{\alpha^j}{2\pi}\delta\sigma^{(i,j)} = \mathrm{d}\sigma^{\text{LO}} + \sum_{i,j} \mathrm{d}\sigma^{(i,j)} \,. \tag{1}$$

The calculation of the complete set of contributions to $\sigma^{(1,1)}$ has been carried out in a fully analytic way. This required the evaluation of the two-loop $O(\alpha\alpha_s)$ $q\bar{q} \to Z$ form-factor as well as one-loop and tree-level amplitudes integrated over the phase space of one and two unresolved partons respectively. Phase space integrals are dealt with using the reverse unitarity approach of Ref. [15], thus all contributions are effectively of two-loop kind and can be treated with modern techniques for multiloop calculations. The final results are then expressed in terms of Goncharov Polylogarithms, made an exception for a subset of double-real corrections where the phase-space integration gives rise to three elliptic integrals which have been evaluated as series expansions.

The EW renormalisation has been performed in the on-shell scheme and the authors consider two alternatives for the EW input parameters: the $\alpha(0)$-scheme and the G_μ-scheme. In the context of hadron colliders the G_μ scheme is to be preferred, because it reabsorbs large logarithms stemming from light-quark mass effects. However, a comparison between these two extreme scheme choices can be used as a conservative estimate of missing EW higher-order effects and to investigate the convergence of the perturbative series (1).

In order to highlight the impact of individual higher-order QCD and EW corrections, we consider the perturbative expansions

$$\begin{aligned} A_1 &= \sigma^{\text{LO}} + \sigma^{(1,0)} + \sigma^{(2,0)} \,, \\ B_2 &= \sigma^{\text{LO}} + \sigma^{(1,0)} + \sigma^{(2,0)} + \sigma^{(0,1)} \,, \\ B_3 &= \sigma^{\text{LO}} + \sigma^{(1,0)} + \sigma^{(2,0)} + \sigma^{(0,1)} + \sigma^{(1,1)} . \end{aligned} \tag{2}$$

Both A_1 and $B_{1,2}$ are computed in the four-flavour scheme using the `NNPDF31_nnlo_as_0118_nf_4` and `NNPDF31_nnlo_as_0118_luxqed_nf_4` PDF sets of Ref. [16] respectively[3]. In Table 1 we report results for the inclusive cross section in proton-proton collisions at 13 TeV.

The NNLO QCD result A_1 is LO from the point of view of the EW theory, thus has a input-scheme uncertainty of 3.53%. This is reduced to 0.88% by the inclusion of the NLO EW correction in B_2, and further ameliorated to 0.23% by the mixed QCD×EW contribution. The latter effect is mostly due to the stabilisation of the input-scheme uncertainty of large $O(\alpha_s)$ corrections. Finally

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[2] We refer the interested in the W boson case to Ref. [14]

[3] For the numerical values of the complete set of input parameters see Ref. [12].

	G_μ [nb]	$\alpha(0)$ [nb]	$\delta_{G_\mu-\alpha(0)}$ [%]
A_1	55.787	53.884	3.53
B_2	55.501	55.015	0.88
B_3	55.469	55.340	0.23

Table 1: Inclusive cross sections for Z boson production as defined in (2), both in the G_μ and $\alpha(0)$ input schemes. Factorisation and renormalisation scales are both set to M_Z.

we note that the best QCD×EW prediction B_3 reduces the NNLO QCD result A_1 by -0.57% in the G_μ scheme. For a thorough assessment of the theory uncertainty at this precision level, the N3LO QCD corrections will be necessary as well.

2.2 Differential results for on-shell Z boson production at the LHC

An independent calculation of the mixed QCD×EW corrections to on-shell Z boson production has been recently presented in Ref. [13]. The authors include the decay of the Z boson to massless charged leptons and the computation is fully differential with respect to resolved final-state partons.

From the technical point of view, the two-loop $O(\alpha\alpha_s)\ q\bar{q} \to Z$ form factor has been computed and checked against the result available in Ref. [17], as well as the $O(\alpha\alpha_s)$ charge renormalisation which has been checked against Refs. [11, 18]. One-loop EW corrections to $q\bar{q} \to g\,Z$ and related partonic channels have been obtained with OpenLoops2 [19]. All the other relevant amplitudes have been computed analytically or extracted from the literature where available. For the purpose of a fully differential calculation, soft and collinear divergences stemming from pure QCD×QED corrections need to be regulated. This problem had been already addressed and solved in Ref. [20], adapting the soft-collinear subtraction scheme developed in Ref. [21] for NNLO QCD computations.

In order to better analyse the impact of higher-order corrections, let us consider ratios of cross sections and kinematic distributions defined as

$$\mathrm{d}\Delta^{(i,j)} = \frac{\mathrm{d}\sigma^{(i,j)}}{\mathrm{d}\sigma^{\mathrm{LO}} + \mathrm{d}\sigma^{(1,0)}}\,, \tag{3}$$

where $\mathrm{d}\sigma^{(i,j)}$ has been introduced in (1) and $\mathrm{d}\sigma^{(1,0)}$ is the NLO QCD correction. In the case of the neutral-current Drell-Yan process, QED and purely weak contributions can be separated in a gauge invariant way, and in the following we will comment on the impact of these two subsets.

In Table 2 we report results for the relative corrections (3) to the integrated cross section computed in the G_μ scheme for the 13 TeV LHC for two choices of the renormalisation and factorisation scales $\mu_R = \mu_F = M_Z(M_Z/2)$ [4]. Corrections to the Z production phase by themselves are reported as well. The authors employ standard kinematic selection cuts requiring

$$p_{\mathrm{T},\ell_{1(2)}} > 24(16)\ \mathrm{GeV}, \quad |y_\ell| < 2.4, \quad R_{e\gamma} = \sqrt{(y_e - y_\gamma)^2 + (\varphi_e - \varphi_\gamma)^2} < 0.1, \tag{4}$$

where $\ell_{1(2)}$ denotes the harder(softer) lepton.

QED effects to the Z boson decay are known to be strongly sensitive to the cuts on the final-state leptons and this can be seen in Table 2, where QCD×QED corrections flip sign and increase by

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[4] For the numerical values of all the input parameters and the choice of PDF set see Ref. [13].

	Inclusive		Cuts		Cuts (production)	
	$\mu = M_Z/2$	$\mu = M_Z$	$\mu = M_Z/2$	$\mu = M_Z$	$\mu = M_Z/2$	$\mu = M_Z$
$\Delta^{(0,1)}_{\mathrm{QED}}$	$+2.3 \times 10^{-3}$	$+3.1 \times 10^{-3}$	-5.3×10^{-3}	-5.5×10^{-3}	$+2.2 \times 10^{-3}$	$+3.0 \times 10^{-3}$
$\Delta^{(0,1)}_{\mathrm{weak}}$	-5.5×10^{-3}	-6.2×10^{-3}	-5.0×10^{-3}	-5.8×10^{-3}	-5.0×10^{-3}	-5.8×10^{-3}
$\Delta^{(0,1)}$	-3.2×10^{-3}	-3.1×10^{-3}	-1.0×10^{-2}	-1.1×10^{-2}	-2.8×10^{-3}	-2.9×10^{-3}
$\Delta^{(2,0)}$	$+1.3 \times 10^{-2}$	-6.3×10^{-3}	$+5.8 \times 10^{-3}$	-1.2×10^{-2}	$+5.8 \times 10^{-3}$	-1.2×10^{-2}
$\Delta^{(1,1)}_{\mathrm{QED}}$	$+5.5 \times 10^{-4}$	$+2.9 \times 10^{-4}$	-5.9×10^{-3}	-5.2×10^{-3}	$+1.4 \times 10^{-4}$	-1.5×10^{-4}
$\Delta^{(1,1)}_{\mathrm{weak}}$	-1.6×10^{-3}	-9.2×10^{-4}	-2.1×10^{-3}	-1.3×10^{-3}	-2.1×10^{-3}	-1.3×10^{-3}
$\Delta^{(1,1)}$	-1.1×10^{-3}	-6.4×10^{-4}	-8.0×10^{-3}	-6.5×10^{-3}	-2.0×10^{-3}	-1.5×10^{-3}

Table 2: Corrections (3) to the inclusive and fiducial cross sections, the latter defined by the setup (4).

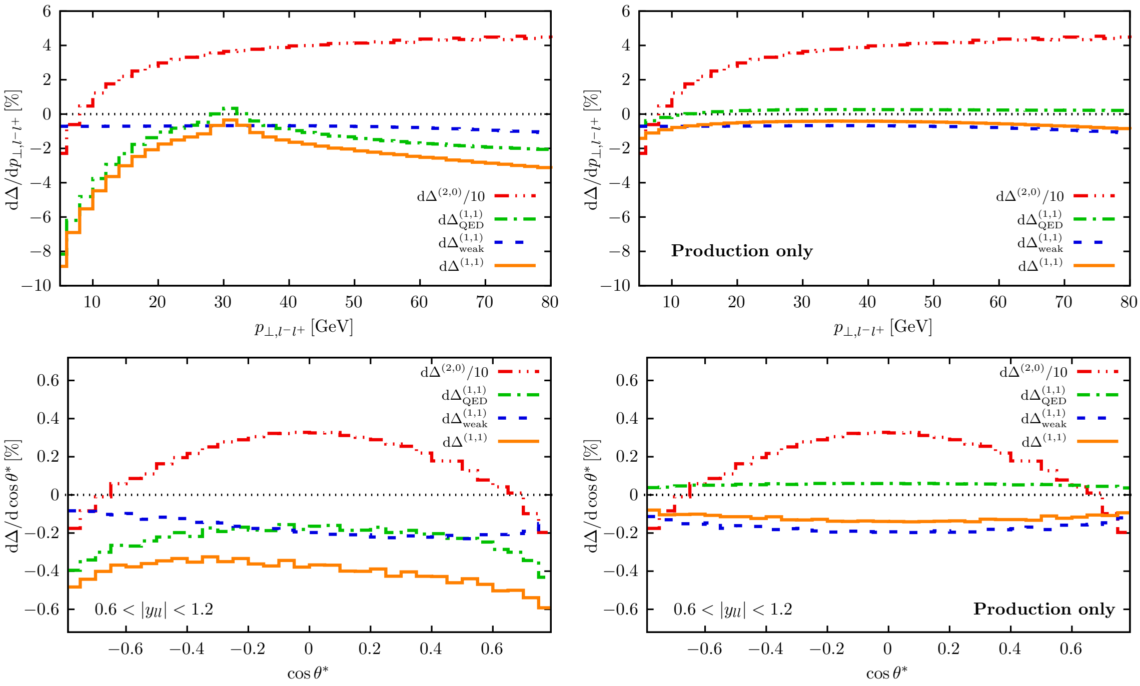

Figure 1: QCD×EW corrections to the dilepton transverse momentum distribution and cosine of the Collins-Soper θ^* angle at the 13 TeV LHC for $\mu_R = \mu_F = M_Z/2$. Left pane includes corrections to both production and decay whereas right pane includes corrections to the production stage only.

almost an order of magnitude when cuts are imposed. However, the results show that the mixed QCD×weak corrections dominate over the QCD×QED ones both at the inclusive level and when the Z boson decay is considered in LO approximation. Further, depending on the choice of the factorisation scale, mixed QCD×EW corrections compete with the NNLO QCD ones[5].

In Figure 1 we present selected kinematic distributions, in particular the dilepton transverse

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[5] This effect should not be interpreted as the mixed corrections being unnaturally large, rather the NNLO QCD ones being accidentally small.

momentum distribution and the cosine of the Collins-Soper θ^* angle. As for the integrated cross sections, we note that when corrections to production and decay are considered, QED effects play an important role, but when only corrections to the production stage are taken into account, weak corrections actually dominate. Therefore both of them have to be included for a proper description of $O(\alpha\alpha_s)$ effects. We refer the reader to Ref. [13] for a detailed discussion on these observables and further relevant ones.

3. Summary and outlook

The Drell-Yan process is a paradigm of precision physics at hadron colliders. For the current precision goal and for the upcoming high-luminosity phase of the LHC, NNLO QCD×EW corrections need to be taken into account. From the side of theoretical predictions we are witnessing fast and constant progress. New results for the production of on-shell Z and W bosons became recently available, showing that $O(\alpha\alpha_s)$ corrections are small, usually at the permill level. However, their impact varies with the observable of interest and can depend strongly on the selection cuts imposed. Therefore, complete QCD×EW corrections have to be considered if the accuracy target is below 1% level.

Away from the resonance, $O(\alpha\alpha_s)$ corrections are crucial for studies in the high-invariant mass region. A complete fixed-order calculation will also shed light on the validity of the most common approximations adopted so far in the literature.

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