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A combined description of $D^+ \to K^-\pi^+\pi^+$ and $D_s \to \pi^- K^+ K^+$ decays

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In this work, we perform a combined study of $D^+ \to K^-\pi^+\pi^+$ and $D_s \to \pi^- K^+ K^+$ decays using the naive factorization approach. The formalism allows for a description of both decays in terms of the well-known vector- and scalar- $K\pi$ form factors, together with a form factor appearing in semileptonic $D^+ \to K^-\pi^+\ell^+\nu$ decays. We propose a useful—yet simple—parametrization to describe the latter that incorporates unitarity and analyticity constraints. As a result, we find a satisfactory description for the P-wave part in $D^+ \to K^-\pi^+\pi^+$ decays, dominated by the $\bar{K}^{*0}(892)$ resonance, while the full description requires minor adjustments for the S-wave. The final description allows to predict the $D_s \to \pi^- K^+ K^+$ decay, that is in nice agreement with data and provides an excellent consistency check of our study.

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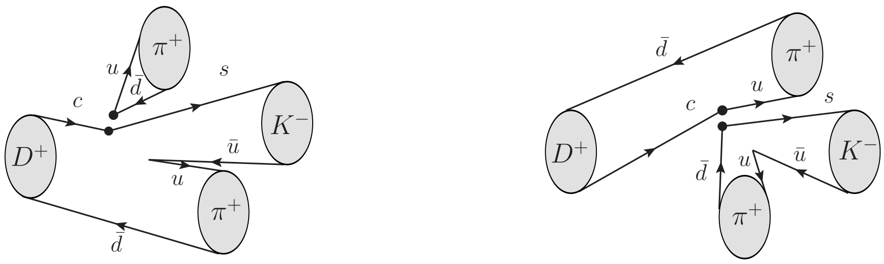

Figure 1: The different contributions to $D^+ \to K^-\pi^+\pi^+$ decay in the naive factorization approach.

1. Introduction

Describing three-body weak meson decays represents a challenging task, given the complex structures arising in the Dalitz plot analysis due to final state meson interactions. As such, these processes can help improving our understanding of meson interactions at low and intermediate energies, providing valuable information in hadronic physics. At the same time, there are often competing (weak) topologies contributing to the same decay, that requires a certain amount of modelling to understand the underlying physics. In the present work [1], we revisit the $D^+ \to K^-\pi^+\pi^+$ decay studied in Ref. [2] using naive factorization. As a novelty, we take advantage of the recent data from $D^+ \to K^-\pi^+\ell^+\nu$ decays, that is analyzed by means of a simple model incorporating essential analytic and unitarity constraints. As a byproduct, we obtain a competitive description for $D_s \to \pi^- K^+ K^+$ decays, that provides a valuable consistency check of our framework.

2. Naive factorization prediction for $D^+ \to K^-\pi^+\pi^+$ decay

Following Ref. [2], the relevant operators from the weak effective Lagrangian are

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* [C_1(\mu) O_1 + C_2(\mu) O_2] + \text{h.c.}, \qquad O_{1(2)} = 4[\bar{s}_L^i \gamma^\mu c_L^{i(j)}][\bar{u}_L^j \gamma^\mu d_L^{j(i)}], \quad (1)$$

that, in the naive factorization framework, produce the kind of contributions outlined in Fig. 1. The resulting matrix element reads

$$i\mathcal{M} = -i\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \Big[a_1 \langle K^-\pi_1^+ | \bar{s}\gamma^\mu (1-\gamma^5) c | D^+ \rangle \langle \pi_2^+ | \bar{u} \gamma_\mu (1-\gamma^5) d | 0 \rangle + a_2 \langle K^-\pi_1^+ | \bar{s}\gamma^\mu (1-\gamma^5) d | 0 \rangle \langle \pi_2^+ | \bar{u} \gamma_\mu (1-\gamma^5) c | D^+ \rangle \Big] + (\pi_1^+ \leftrightarrow \pi_2^+), \quad (2)$$

where π_2^+ is a bachelor pion and $a_1 = C_1 + C_2/N_c = 1.2(1)$, and $a_2 = C_2 + C_1/N_c = -0.5(1)$ [3]. With the value of the Wilson coefficients at hand, the problem ultimately reduces to the description of the four relevant matrix elements above. As noted in Ref. [2], the matrix elements accompanying a_2 can be related via isospin symmetry to those in $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ and $D^0 \to \pi^-\ell^+\nu$ decays, respectively, for which we adopt the descriptions in Refs. [2, 4–6]. Concerning those accompanying a_1, the second matrix element is that arising in $\pi^+ \to \ell^+\nu_\ell$ decays. Accounting for this, the first term reduces to $-a_1 f_\pi \langle K^-\pi_1^+ | \partial_\mu \bar{s}\gamma^\mu (1-\gamma^5) c | D^+ \rangle$, which corresponding matrix element appears in semileptonic $D^+ \to K^-\pi^+\ell^+\nu$ decays. As we shall show in the following, the divergence appearing

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in $\langle K^-\pi_1^+ |\, \partial_\mu \bar{s}\gamma^\mu(1-\gamma^5)c\, |D^+\rangle$ selects a form factor with a marginal role in the semileptonic decays that, together with the lack of precise data at the time of Ref. [2], was the reason for devising a model fitted directly to $D^+ \to K^-\pi^+\pi^+$ decay [2]. In the following section, we carefully discuss the semileptonic decays to show how they still can be used to constrain the relevant matrix element. This is possible thanks to the recent data from BES III with unprecedent precision [7], that we analyze using a new model based on basic analytic and unitarity constraints that might be of general interest.

3. Semileptonic $D^+ \to K^-\pi^+\ell^+\nu$ decays

The amplitude for $D^+ \to \ell^+\nu K^-\pi^+$ decays reads

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{cs}^* \langle \pi^+ K^- |\, \bar{s}\gamma^\mu(1-\gamma^5)c\, |D^+\rangle\, [\bar{u}_\nu \gamma_\mu (1-\gamma^5) v_\ell], \tag{3}$$

that involves the relevant hadronic matrix element discussed in the previous section. The latter can be parametrized as ($p/\bar{p} = p_K \pm p_\pi$ and $q = p_\ell + p_\nu$)

$$\langle K^-\pi^+ |\, \bar{s}\gamma^\mu(1-\gamma^5)c\, |D^+\rangle = i\boldsymbol{w}_+ \left(p^\mu - q^\mu \frac{p\cdot q}{q^2} \right) + i\boldsymbol{w}_- \left(\bar{p}^\mu - q^\mu \frac{\bar{p}\cdot q}{q^2} \right) + \frac{i\tilde{\boldsymbol{r}}}{q^2} q^\mu - \boldsymbol{h}\epsilon^{\mu q p \bar{p}} . \tag{4}$$

The equations above imply that the form factor of interest, $\tilde{\boldsymbol{r}} = -\langle K^-\pi^+ |\, \partial_\mu \bar{s}\gamma^\mu(1-\gamma^5)c\, |D^+\rangle$, appears suppressed by m_ℓ in Eq. (3), and as such its effect is commonly neglected in experimental studies. Still, this can be extracted with few model assumptions [1]. In particular, the absence of massless particles coupling to the pseudoscalar ($\bar{s}i\gamma^5 c$) current demands the following relation to be satisfied

$$\lim_{q^2\to 0} [(p\cdot q)\boldsymbol{w}_+ + (\bar{p}\cdot q)\boldsymbol{w}_- - \tilde{\boldsymbol{r}}] = 0 \quad \Rightarrow \quad \lim_{q^2\to 0} [F_1(q^2, p^2, \bar{p}\cdot q) - F_4(q^2, p^2, \bar{p}\cdot q)] = 0, \tag{5}$$

where the last equality has been expressed in terms of the commonly used helicity form factors (F_i), with $F_4 = \tilde{\boldsymbol{r}}$. The relation above allows as such to construct a model where the F_4 form factor is expressed in terms of quantities appearing in $F_{i\neq 4}$ form factors, which can be accessed experimentally. In particular, the BES-III Coll. has provided the most precise study to date [7], while the experimental analysis employed therein makes use of simplified modes, where $K\pi$ interactions are described in terms of Breit-Wigners (with the S-wave incorporating a background term as well that, however, breaks analiticity). To ammend this, we put forward a better-motivated description built on the $K\pi$ phase-shifts. Note in particular that, according to Watson's theorem, this implies the well-known Omnés solution below higher inelasticities, in line with Refs. [5, 8–12]. As such, we abandon the use of Breit-Wigner distributions in favor of the following descriptions for the vector and scalar waves [1]

$$F_+^{D_{\ell 4}}(s) = \exp\left[\lambda_1 \frac{s}{m_\pi^2} + G_+(s) \right], \qquad G_+(s) = \frac{s^2}{\pi} \int_{s_{th}}^\infty d\eta \frac{\delta_1^{1/2}(\eta)}{\eta^2(\eta - s)}, \tag{6}$$

$$F_0^{D_{\ell 4}}(s) = \exp\left[\frac{s[\ln C_{D_{\ell 4}} + G_0(s)]}{\Delta_{K\pi}} \right], \quad G_0(s) = \frac{\Delta_{K\pi}(s - \Delta_{K\pi})}{\pi} \int_{s_{th}}^\infty d\eta \frac{\delta_0^{1/2}(\eta)}{\eta(\eta - \Delta_{K\pi})(\eta - s)} . \tag{7}$$

Note in particular that, while the phase-shifts above should be identical to those in $F_{0,+}^{K\pi}(s)$ form factors below inelasticities,[1] the subtraction constants, encapsulating the high-energy effects, might in general differ. As such, these are noted with $D_{\ell 4}$ indices and must be obtained from a fit to data (see below). Using this description, the $F_{1,4}$ form factors read (see Refs. [1, 13] for definitions)

$$F_1 = -F_+^{D_{\ell 4}}(p^2)\beta_{K\pi}\cos\theta_{K\pi}\frac{X^2\chi_C^{\text{eff}} + (q\cdot p)\chi_B^{\text{eff}}}{1 - q^2/m_{D_{s1}}^2} + \frac{2\chi_S^{\text{eff}}X}{1-q^2/m_{D_{s1}}^2}F_0^{D_{\ell 4}}(p^2)\,, \tag{8}$$

$$F_4 = -\chi_{\bar{K}^*}F_+^{D_{\ell 4}}(p^2)\frac{N(p^2)}{2}\frac{\chi_B^{\text{eff}} + \frac{m_D^2 - p^2}{2}\chi_C^{\text{eff}}}{1-q^2/m_{D_s}^2} + \frac{\chi_S^{\text{eff}}(m_D^2 - p^2)}{1-q^2/m_{D_s}^2}F_0^{D_{\ell 4}}(p^2)\,, \tag{9}$$

with parameters χ_X to be determined, together with the subtraction constants above, from a fit to data. Unfortunately, Ref. [7] does not provide the real data used in their analysis. Consequently, in order to obtain the parameters above, we resort to a fit to pseudodata taking as input the model from Ref. [7]. The obtained result is excellent, despite the $m_{K\pi}$-dependence of the scalar form factor substantially differs from the model in Ref. [7], that makes more compelling a possible (experimental) re-analysis of Ref. [7] employing the suggested parametrizations. Further details can be found in Ref. [1].

Another important outcome of the study is that, while $F_+^{D_{\ell 4}}(s)$ and $F_+^{K\pi}(s)$ form factors are quite similar, largely due to the $\bar{K}^{*0}(892)$ dominance, this is not the case for the scalar form factors that, despite sharing their phases, have different $m_{K\pi}$-dependence (e.g. different subtraction constants in our parametrization). With this parametrization at hand, we discuss the $D^+ \to K^-\pi^+\pi^+$ in the section below.

4. $D^+ \to K^-\pi^+\pi^+$ decays

In analogy to Ref. [2], we first check our prediction for the P-wave branching ratio, that should be the most reliable as it is mostly dominated by the narrow $\bar{K}^{*0}(892)$ resonance. In order to reproduce the central values quoted by PDG [14] (1.06%), we need a rescaling factor of the semileptonic matrix element of 1.24(21), with uncertainties dominated by the input for the Wilson coefficients $a_{1,2}$. This is, neglecting the inherent uncertainties from the naive factorization framework, the result is compatible with the naive factorization hypothesis at the 1σ level.

Next, we explore the role of the S-wave. We find that, in analogy to Ref. [2], a relative phase among the S- and P-waves of $(180-65)^\circ$ is required to correctly reproduce the interference patterns observed in the Dalitz-plot (see also Refs. [15, 16]). Furthermore, in order to reproduce the branching ratio, an additional rescaling factor of 1.55(30) is required for the $D_{\ell 4}$ S-wave; again, this is subject to large uncertainties, sourced by a_2 and the fit to semileptonic decays. With these inputs, the obtained prediction for the Dalitz plot and differential decay width is shown in Fig. 2, that matches the data reasonably well.

An interesting exercise to cross-check our approach and hypotheses is to confront our model against the doubly-Cabibbo suppressed $D_s \to \pi^- K^+ K^+$ decay, that is related to the one above via U-spin symmetry and that we discuss below.

[1] We take the phase-shift for the S-wave from Ref. [4], while that for the P-wave is taken from Ref. [5].

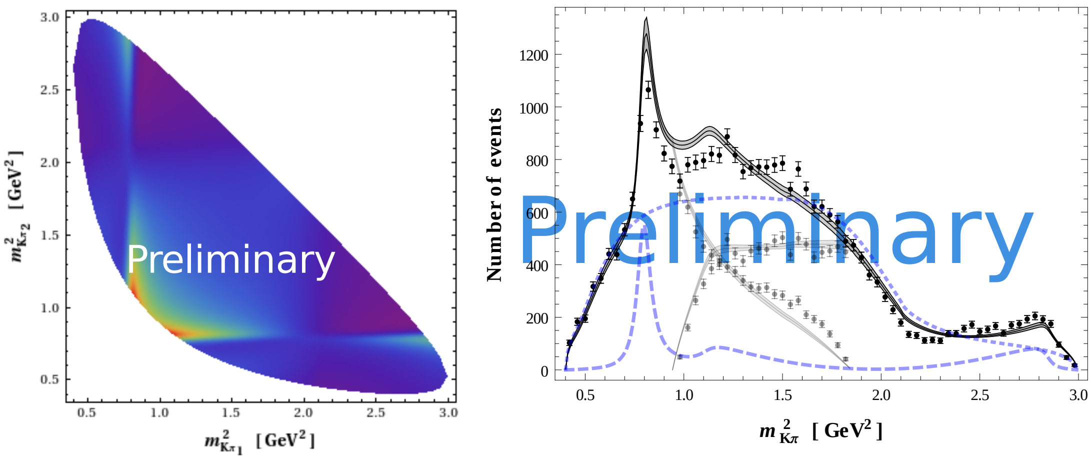

Figure 2: The Dalitz plot (left) and differential decay width (right) for $D^+ \to K^-\pi^+\pi^+$ decays.

5. $D_s \to \pi^- K^+ K^+$ decays

$D^+ \to K^-\pi^+\pi^+$ and $D_s \to \pi^- K^+ K^+$ decays are related via U-spin symmetry (e.g. upon $d \leftrightarrow s$ exchange) that, however, will in general receive important corrections from $SU(3)$-breaking. These can be nevertheless obtained in our framework using existing data. In particular, we find

$$i\mathcal{M} = -i\frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* \Big[a_1 \langle \pi^- K_1^+ | \bar{d}\gamma^\mu (1-\gamma^5) c | D_s \rangle \langle K_2^+ | \bar{u}\gamma_\mu (1-\gamma^5) s | 0 \rangle + a_2 \langle \pi^- K_1^+ | \bar{d}\gamma^\mu (1-\gamma^5) s | 0 \rangle \langle K_2^+ | \bar{u}\gamma_\mu (1-\gamma^5) c | D_s \rangle \Big] + (K_1^+ \leftrightarrow K_2^+)\,.$$

As such, the source of $SU(3)$-breaking appears in $f_\pi \to f_K$ as well as $D^0 \to \pi^- \ell^+ \nu \Rightarrow D_s \to K^0 \ell^+ \nu$ replacements, which are experimentally known [14, 17]. Finally, it is expected that charmed meson decays are rather insensitive to the spectator quark [18], that is also justified experimentally [17]. As such, we take the form factors in semileptonic D_s decays identical to those in the D^+ case. With these modifications, we obtain $1.44(13) \times 10^{-4}$ for the branching ratio, in good agreement with the PDG value $1.28(4) \times 10^{-4}$ [14]. The resulting Dalitz plot is shown in Fig. 3 together with the recent measurement from LHCb [19], showing an excellent agreement and further supporting our findings in the previous section. In particular, the relative phase among the S- and P-waves is crucial to describe the patterns in the Dalitz plot (find further details in Ref. [1].).

6. Summary and outlook

In this study [1], we have revisited the hadronic $D^+ \to K^-\pi^+\pi^+$ decay using the naive factorization approach. As compared to the previous study in Ref. [2], we have improved the hadronic matrix element related the semileptonic $D^+ \to K^-\pi^+\ell^+\nu$ decays, that is now constrained by experimental results. We have obtained a nice out-of-the-box result for the P-wave branching ratio and a reasonable description for the Dalitz plot and differential decay widths upon minor adjustments. Furthermore, we tested the naive factorization hypothesis using $D_s \to \pi^- K^+ K^+$ decays, where no further adjustment is possible. The outcome was in a remarkably good agreement with current data, thus strengthening our results.

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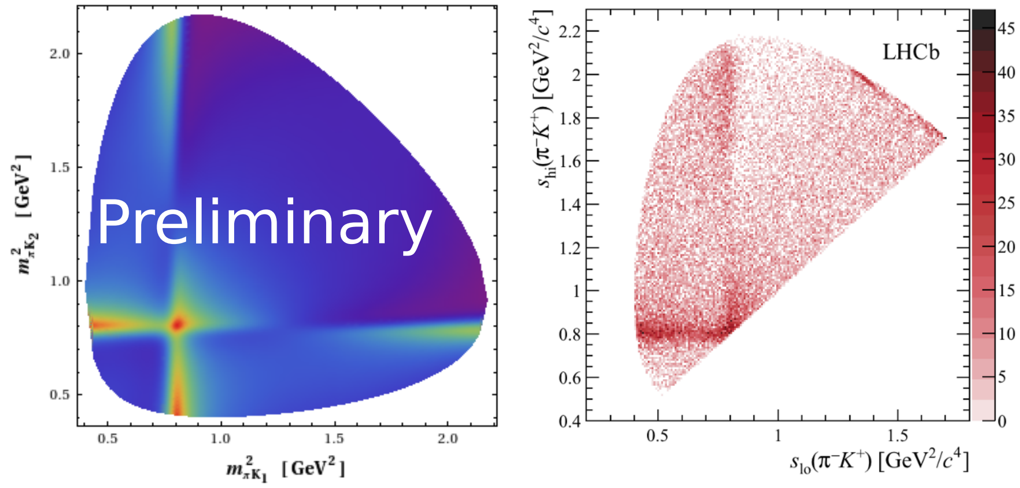

Figure 3: The predicted Dalitz plot from our models for $D^+ \to K^-\pi^+\pi^+$ decays (left) compared to LHCb results [19].

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