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Renormalisation Scale Setting in D-mixing

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A naive application of the heavy quark expansion (HQE) yields theory estimates for the decay rate of neutral D mesons that are four orders of magnitude below the experimental determination. It is well known that this huge suppression results from severe GIM cancellations. We find that this mismatch can be solved by individually choosing the renormalisation scale of the different internal quark contributions. For b and c hadron lifetimes, as well as for the decay rate difference of neutral B mesons the effect of our scale setting procedure lies within the previously quoted theory uncertainties, while we get enlarged theory uncertainties for the semileptonic CP asymmetries in the B system.

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1. Introduction

Charm has played a very important role in the structure of the SM, since its discovery as predicted by the GIM Mechanism [1]. Today we have a huge amount of data by LHCb [2], BESIII [3] and Belle II [4], however, the theoretical understanding needs to be improved in order to use them efficiently. A good example is the first discovery by the LHCb collaboration of CP violation in the charm system [5]. They announced an experimental measurement of $\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}$ which differs by 5.3σ from zero. It is currently not clear if this measurement of ΔA_{CP} requires BSM explanations [6, 7] (partly based on the calculation of Ref. [8]) or it can still be explained within the SM [9–12].
One of the biggest puzzles of charm physics is the mixing of D^0 meson. Earlier this year LHCb announced new measurements [13] of $x = \frac{\Delta M_D}{\Gamma_{D^0}}$ and $y = \frac{\Delta\Gamma_D}{2\Gamma_{D^0}}$ and by taking them into account the HFLAV [14] average reads now (in the case of allowing CP violation):

$$x = \frac{\Delta M_D}{\Gamma_{D^0}} = (0.409^{+0.048}_{-0.049})\% \ , y = \frac{\Delta\Gamma_D}{2\Gamma_{D^0}} = 0.615^{+0.056}_{-0.055}\% \, , \tag{1}$$

where ΔM_D is the mass difference between the two mass eigenstates of the D^0 meson and $\Delta\Gamma_D$ is the corresponding decay width difference. Expanding these in a series of small phase ϕ_{12}^D we get the following expressions [15]:

$$\begin{aligned} \Delta M_D &= 2|M_{12}^D| \cdot (1 + O((\phi_{12}^D)^2)) \, , \\ \Delta\Gamma_D &= 2|\Gamma_{12}^D| \cdot (1 + O((\phi_{12}^D)^2)) \, , \\ \phi_{12}^D &= \arg\left(-\frac{M_{12}^D}{\Gamma_{12}^D}\right) \, , \end{aligned} \tag{2}$$

where M_{12} and Γ_{12} are the non-diagonal elements of the mixing matrix of the $D^0 - \overline{D}^0$ system. Using these we can now define

$$x_{12} = \frac{2\,|M_{12}|}{\Gamma_{D^0}}, \ \ y_{12} = \frac{|\Gamma_{12}|}{\Gamma_{D^0}} \tag{3}$$

which unlike x and y they depend only on one of the non-diagonal elements.
Theoretical predictions of x and y though can cover a huge range of values, differing by several orders of magnitude, see e.g. [16, 17]. Future measurements will not only increase the precision of these quantities but also give stronger bounds or even a measurement of the CP violation in mixing [18], encoded in the phase ϕ_{12} which currently lies within $[-2.5°, 1.8°]$.

2. D-mixing in HQE

The heavy quark expansion (HQE) [19–25] (see Ref. [26] for a recent overview) describes the total decay rate of heavy hadrons and the decay rate difference of heavy neutral mesons as an expansion in inverse powers of the heavy quark mass. This theory is proven to work great for the B system (where the expansion parameter Λ/m_b is small) however it has been challenged if it

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could produce similar results in the charm system where the expansion parameter is increased by approximately a factor of 3.
In the table below you can see how good agreement there is between experiment and HQE in the B system [27–29]:

	HFLAV 2019	HQE 2019
$\frac{\tau(B_s)}{\tau(B_d)}$	0.994(4)	1.0007(25)
$\frac{\tau(B^+)}{\tau(B_d)}$	1.076(4)	$1.082^{+0.022}_{-0.026}$
$\frac{\tau(\Lambda_b)}{\tau(B_d)}$	0.969(6)	0.935(54)
$\Delta\Gamma_{B_s}$	0.091(13)ps^{-1}	0.090(5)ps^{-1}

In the charm system if we consider the lifetime ratio $\tau(D^+)/\tau(D^0)$ where NLO-QCD corrections to the dimension-six contribution [30] and values for the non-perturbative matrix elements of the 4-quark operators [31] are known, there is agreement between theory and experiment even with big theoretical uncertainties:

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|^{\text{PDG}} = 2.536(19)\,, \qquad \left.\frac{\tau(D^+)}{\tau(D^0)}\right|^{\text{HQE 2017}} = 2.7^{+0.7}_{-0.8}\,. \tag{4}$$

So what is going so wrong in the D-mixing?

In the present work as in the original paper [32] we will focus only on the calculation of Γ_{12} and not on M_{12}.Within the HQE framework Γ_{12} can be expanded as:

$$\Gamma_{12} = \left[\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right]\frac{\langle Q_6\rangle}{m_c^3} + \ldots \tag{5}$$

where the ellipsis stands for terms of higher order. The above expression can be shown diagrammatically in Fig. 1. The product of the $\Delta C = 1$ operators from the effective Hamiltonian ("full" theory) is matched into a series of local $\Delta C = 2$ operators Q_n of increasing dimension $n \geq 6$, with the short distance coefficients denoted by $\Gamma_{n-3}^{(i)}$. The expressions for $\Gamma_3^{(i)}$ can be simply obtained from the corresponding ones for B-mixing given in Refs. [33–38] while the matrix elements of the dimension-six operators have been determined in e.g. Refs. [31, 39]. The experimental result of the decay rate difference leads to the following bound

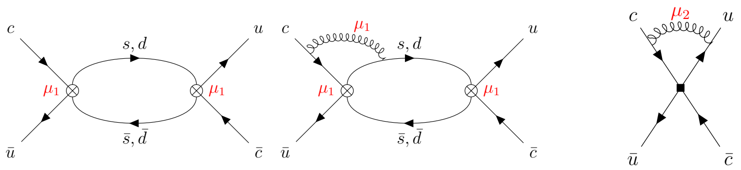
$$\Delta\Gamma_D^{\text{Exp}} \geq 0.027\,\text{ps}^{-1}, \tag{6}$$

at one standard deviation. Based on that we will focus on the following quantity[1]:

$$\Omega = \frac{2\,|\Gamma_{12}|^{\text{SM}}}{0.027\,\text{ps}^{-1}} \tag{7}$$

A value of Ω smaller than 1 indicates that we are unable to describe D-mixing within 1σ. A naive application of the HQE leads to $\Omega = 3.4 \cdot 10^{-5}$ at LO-QCD and $\Omega = 6.2 \cdot 10^{-5}$ at NLO-QCD. As we can see in both cases our prediction is around 5 orders of magnitude smaller than one. For the results stated above and the ones following we have used the PDG [40] values for all the masses as well as the strong coupling, CKM elements are from [41], the non perturbative matrix elements from [31] and finally the decay constant from [42].

[1]The results shown in our paper[15] are using a bound of 0.028 as the new average was not released yet. This however will not change the results in any significant way.



(a) (b)

Figure 1: (a) Diagrams describing the mixing of neutral D mesons via intermediate $s\bar{s}$, $s\bar{d}$, $d\bar{s}$ and $d\bar{d}$ states in the "full" theory at LO-QCD (left) and NLO-QCD (right). The crossed circles denote the insertion of $\Delta C = 1$ operators of the effective Hamiltonian describing the charm-quark decay. The dependence on the renormalisation scale μ_1 in the Wilson coefficients cancels against the μ_1 dependence of the QCD corrections. (b) Diagram describing mixing of neutral D mesons at NLO-QCD in the HQE. The full dot indicates the insertion of $\Delta C = 2$ operators. The dependence on the renormalisation scale μ_2 cancels between the QCD corrections to the diagram and the matrix elements of the corresponding operators.

3. GIM

The calculation of Γ_{12} can be expressed as:

$$\begin{aligned} \Gamma_{12} &= -\left(\lambda_s^2\,\Gamma_{12}^{ss} + 2\,\lambda_s\lambda_d\,\Gamma_{12}^{sd} + \lambda_d^2\,\Gamma_{12}^{dd}\right) \\ &= -\lambda_s^2\left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}\right) + 2\lambda_s\lambda_b\left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right) - \lambda_b^2\Gamma_{12}^{dd}. \end{aligned} \tag{8}$$

where $\lambda_q = V_{cq}V_{uq}^*$ and $\Gamma_{12}^{qq'}$ denotes the contribution from the diagrams with internal quark pair qq'. The peculiar feature of the second expression is that in terms of absolute size, the CKM dominant factor λ_s^2 multiplies the doubly GIM suppressed term, the CKM suppressed factor $\lambda_s\lambda_b$ multiplies the GIM suppressed term and the doubly CKM suppressed factor λ_b^2 multiplies a term with no GIM suppression. This results in all three terms of Eq.(8) having similar size:

$$\begin{aligned} \Gamma_{12} &= \left(2.08\cdot 10^{-7} - 1.34\cdot 10^{-11} I\right)\text{(1st term)} \\ &- \left(3.74\cdot 10^{-7} + 8.31\cdot 10^{-7} I\right)\text{(2nd term)} \\ &+ \left(2.22\cdot 10^{-8} - 2.5\cdot 10^{-8} I\right)\text{(3rd term)}. \end{aligned} \tag{9}$$

This is something observed only in D-mixing and not e.g. in B-mixing where the CKM dominant term multiplies the term with no GIM suppression.

It is also interesting to note that if we expand the terms of Eq.(8) in $z = m_s^2/m_c^2$ the GIM suppression seems to lifted by an order of this small parameter if we include NLO-QCD corrections [43], hinting that maybe if higher orders in QCD are calculated the GIM suppression can be less dominant.

To explain the big difference between HQE and experiment in D-mixing several solutions have been proposed: 1) Higher orders in HQE could be less affected by GIM suppression [44–46]. For this a full determination of dimension nine and twelve is necessary (first estimates of dimension

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nine can be found in [47]), 2) Quark hadron duality, as it has been shown in [48] that a duality violation of only 20% could suffice to reproduce the experimental value (see also [49] for a recent investigation), 3) The HQE is not applicable in the charm system and different methods like summing over exclusive decay channels should be used instead, see e.g. [50–52] and 4) Contributions from BSM physics could also enhance the decay difference, see e.g. [53–55].

4. Scale setting

In the calculation of Γ_{12} two renormalisation scales are arising, see Fig. (1). The scale μ_1 at which the $\Delta C = 1$ Wilson coefficients of the effective Hamiltonian and the QCD corrections of the 'full' theory are computed and the scale μ_2 which is introduced by the radiative corrections to the HQE diagrams and cancels with the scale dependence of the matrix elements of the $\Delta C = 2$ operators. We are not discussing any further the scale dependence on μ_2 as this cancellation is very effective. The reduction of the μ_1-dependence from LO-QCD to NLO-QCD in the D-system can be seen only if we consider the independent contributions to Γ_{12} i.e. $\Gamma_{12}^{ss}, \Gamma_{12}^{sd}$ and Γ_{12}^{dd} which can be understood as another effect of the GIM cancellations. This is depicted in Fig. (2). In a naive

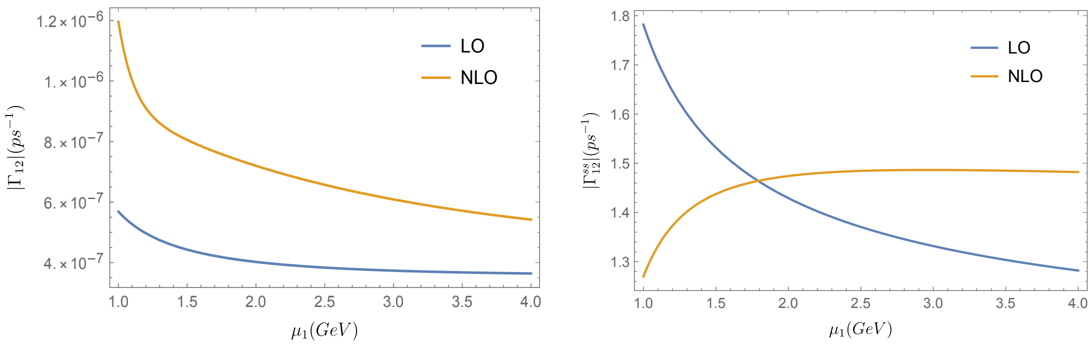

Figure 2: Comparison of the μ_1-dependence of $|\Gamma_{12}|$(left) and $|\Gamma_{12}^{ss}|$(right) at LO-QCD and NLO-QCD

calculation we set the scale $\mu_1 = m_c$ to minimise terms of the form $\alpha_s(\mu_1)\ln(\mu_1^2/m_c^2)$ and in order to estimate uncertainties due to higher orders we vary the scale from 1 GeV to $2m_c$.
Here we propose two alternative ways of treating the renormalisation scale. Both of them are based on the idea that different internal quark pairs contribute to different decay channels of the $D^0(\overline{D}^0)$ meson i.e. an $s\bar{s}$ to a K^+K^- final state and $s\bar{d}$ to a $K^-\pi^+$ final state. For each of these observables the choice of the renormalisation scale can be decided independently, even though traditionally they are chosen to be equal to m_c. For the remaining of the paper we introduce the parameters $\mu_1^{ss}, \mu_1^{sd}, \mu_1^{dd}$ which correspond to the scale used in the computation of $\Gamma_{12}^{ss}, \Gamma_{12}^{sd}$ and Γ_{12}^{dd} respectively.

The two alternative scenarios for the renormalisation scale are:

- $\mu_1^{ss} = \mu_1^{dd}$ and μ_1^{sd} are set to m_c but are varied independently between 1 GeV and $2m_c$. Here we set $\mu_1^{ss} = \mu_1^{dd}$ since K^+K^- and a $\pi^+\pi^-$ are not fully independent but could be related through re-scattering.

- We set the two scales to different values according to the available phase space. For ss we will use $\mu_1^{ss} = m_c - 2\epsilon$, for sd $\mu_1^{sd} = m_c - \epsilon$ and for dd $\mu_1^{dd} = m_c$ where ϵ is a parameter related to the kinematics of the decays.

If ϵ is not too large then both methods will give results inside the traditional uncertainties for $\Gamma_{12}^{ss}, \Gamma_{12}^{sd}$ and Γ_{12}^{dd} but clearly they will break the severe GIM suppression shown in Eq. (8). The first method gives a significantly enhanced range of values for Ω: $\Omega \in [4.6 \cdot 10^{-5}, 1.4]$ which covers the experimental value! Moreover by scanning independently the two available scale parameters out of the 121 values only 11 of them that correspond to identical scales give $\Omega < 0.001$ while 84 of them produce $\Omega > 0.1$.Throughout this paper we have used the $\overline{MS}$ renormalisation scheme and HQET results for the computation of the matrix elements, however, similar results are produced if we use the Pole scheme or lattice results. In all cases $\Omega > 1$ can be reached.

For the second method, we can estimate the parameter ϵ as the strange quark mass i.e. $\epsilon = m_s \approx 0.1\ GeV$ or comparing the energy release of $D^0 \to K^+K^-$, $M_{D^0} - 2M_{K^+} = 0.88$ GeV, with that of $D^0 \to \pi^+\pi^-$, $M_{D^0} - 2M_{\pi^+} = 1.59$ GeV we might expect that $\epsilon \approx 0.35$ GeV. As can be seen in Fig.(3) the experimental value can be achieved for $\epsilon \approx 0.2$ GeV.

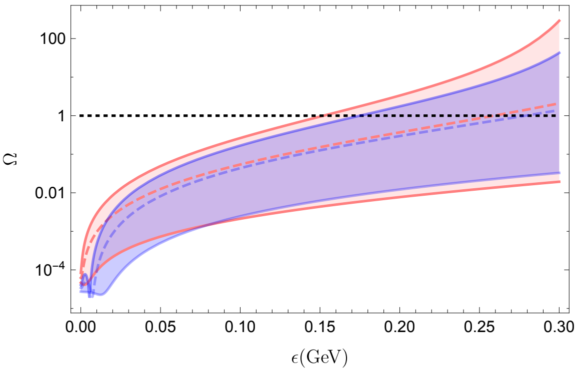

Figure 3: Comparison of the ϵ dependence of Ω at LO-QCD (blue) and NLO-QCD (pink) for different values of μ: the dashed line corresponds to $\mu = m_c$ while the two solid lines to $\mu = 1$ GeV and $\mu = 2m_c$.

Finally we should test this method with other HQE predictions and see how it affects them. For lifetime calculations (for both charm and bottom hadrons) as well as the decay rate difference in B_s-mixing, $\Delta\Gamma_s$ no GIM-like cancellations occur so this alternative scale setting will produce results that will be covered by the current theoretical uncertainties. However less pronounced GIM suppressions appear in the semi-leptonic CP asymmetries in B_s-mixing. In the SM we get:

$$\mathrm{Re}\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right)^{\mathrm{SM}} = -\frac{\Delta\Gamma_q}{\Delta M_q} = \begin{cases} -(49.9 \pm 6.7)\cdot 10^{-4} & q = s \\ -(49.7 \pm 6.8)\cdot 10^{-4} & q = d \end{cases},$$
$$\mathrm{Im}\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right)^{\mathrm{SM}} = a_{sl}^q = \begin{cases} (+2.2 \pm 0.2)\cdot 10^{-5} & q = s \\ (-5.0 \pm 0.4)\cdot 10^{-4} & q = d \end{cases}. \quad (10)$$

while by performing our ϵ analysis:

ϵ (GeV)	Γ_{12}^s/M_{12}^s	Γ_{12}^d/M_{12}^d
0.	$-0.00499 + 0.000022I$	$-0.00497 - 0.00050I$
0.2.	$-0.00494 + 0.000023I$	$-0.00492 - 0.00053I$
0.5.	$-0.00484 + 0.000026I$	$-0.00482 - 0.00059I$
1.0	$-0.00447 + 0.000037I$	$-0.00448 - 0.00084I$
1.5.	$-0.00287 + 0.000091I$	$-0.00309 - 0.0021I$

where the blue entries indicate values that lie within the known theoretical uncertainties. As we can see the real part stays within the uncertainties for values of ϵ up to 1 GeV while the imaginary part can be increased by almost 100%.

5. Conclusion

As we can see by just altering the traditional scale setting we can obtain values for y with a much larger range that includes the experimental result. The quantity y can be approximated as

$$y = (y^{sd} + y^{sd}) - (y^{ss} + y^{dd}) \tag{11}$$

if we take the approximation $\lambda_s \approx -\lambda_d$ in the first part of Eq. (8). Each of the two terms is larger than the experimental value for y and has an implicit uncertainty of at least 20%. By taking their numerical difference we end up with a value between $[10^{-4}, 10^{-5}]$. By taking such a result at its face value, we implicitly assume a precision of $10^{-4} \dots 10^{-5}$ in the individual terms which is of course unrealistic. We would like to point that this method still respects the GIM mechanism since for vanishing strange quark mass the parameter ϵ vanishes as well. For a further understanding however it is important to have a precise calculation of higher order corrections in the HQE as well as a complete computation of NNLO-QCD corrections of the leading term, see e.g. [56–58]. For a prediction of the CP violation in mixing the contribution from M_{12} is missing. This could be obtained with the help of dispersion relations, see e.g. [51, 59, 60].
Finally this procedure does not affect most other quantities like $\tau(D^+)/\tau(D^0)$, b hadron lifetimes and $\Delta\Gamma_s$ giving results within the current theoretical uncertainties, but it affects the semi-leptonic CP asymmetries, giving enhanced ranges:

$$a_{sl}^d \in [-9.2; -4.6] \cdot 10^{-4}, \qquad a_{sl}^s \in [2.0; 4.0] \cdot 10^{-5} \,. \tag{12}$$

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