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Study of $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ decays in the framework of Resonance Chiral Theory

J. L. Gutiérrez Santiago*

[a] *Departamento de Física, Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional,*
Apdo. Postal 14-740, 07000 Ciudad de México, México.

E-mail: jlgutierrez@fis.cinvestav.mx

In this study, the $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ ($\ell = e,\ \mu$) processes are analyzed. This are $O(\alpha^2)$-suppressed with respect to the dominant di-pion tau decay channel. Both the inner-bremsstrahlung and the structure- (and model-)dependent contributions are considered. In the $\ell = e$ case, structure-dependent effects are $O(1\%)$ in the decay rate, yielding a clean prediction of its branching ratio, 2.3×10^{-5}, measurable with BaBar or Belle(-II) data. For $\ell = \mu$, both contributions have similar magnitude and we get a branching fraction of $(1.6 \pm 0.3) \times 10^{-7}$, reachable by the end of Belle-II operation. These decays allow to study the dynamics of strong interactions with simultaneous weak and electromagnetic probes; their knowledge will contribute to reducing backgrounds in lepton flavor/number violating searches.

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*Speaker

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1. Introduction

This work is based in an article that we have published recently (for details see [1]). This article deals whith the semileptonic five-body decay $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ with the lepton pair ($\ell = e$ or μ) produced via a virtual photon. The corresponding radiative case $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$ has been analyzed using the Resonance Chiral Theory (RChT) [2–4] and vector meson dominance [5, 6] approaches to describe vector and axial-vector form factors involved in the $W^- \to \pi^- \pi^0 \gamma$ vertex. This weak vertex involves also the interplay with strong and electromagnetic interactions.

This process is useful because it may pollute searches for processes involving lepton flavor violation in the charged sector or lepton number violation. The decay $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$, with π^- misidentified as muon and undetected π^0, may be an important background that can mimic the signal in searches for lepton flavour violation (LFV) processes of the form $\tau^- \to \ell^{(\prime)-} \ell^+ \ell^-$. Currently the branching ratios of these LFV decay modes have upper bounds of $O(10^{-8})$ [7], while their SM predictions are unmeasurably small [8, 9]. These decays (for $\ell = \mu$) can also be misidentified as lepton number violating processes of the type $\tau^- \to \nu_\tau \mu^- \mu^- \pi^+$ [10]. To avoid these decays polluting new physics searches, it will be most useful to include them in the Monte Carlo Generator TAUOLA [11].

In addition, it could serve to verify the radiative corrections used in the contribution of the hadronic vacuum polarization (HVP) entering the anomalous magnetic moment of the muon (a_μ) obtained using hadronic τ decays data [4, 6, 12–19]. This analysis can be helpful in reducing the error of $a_\mu^{HVP,LO}$ using tau data. It is found that the $\ell = \mu$ case is promising in this respect, as the model-dependent contributions are of the same size of the IB part. Conversely, its low branching ratio, $\sim 10^{-7}$ will challenge the Belle-II analysis [20]. For the $\ell = e$ case the situation will be the opposite, with a $\sim 10^{-5}$ branching ratio (measurable already with BaBar or Belle data), that has little ($O(\%)$) model-dependence.

The IB part is model-independent [21], while the structure-dependent part is not. The vector and axial-vector form factors entering the latter can be computed using the RChT framework. Including operators contributing -upon integrating resonances out- to the $O(p^4)$ Chiral Perturbation Theory (ChPT) [22] low-energy couplings all free parameters are related to the pion decay constant (after applying short-distance QCD constraints), which results in controlled ($O(20\%)$) errors for our prediction in the $\ell = \mu$ mode. In the $\ell = e$ case, structure-dependent effects are as small as (uncomputed) one-loop QED corrections, which set the size of our corresponding uncertainty.

2. Model-independent contribution of the $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ amplitude

The $\tau^- \to \nu_\tau \pi^- \pi^0$ decay is the dominant channel among tau decays. The precise measurement of the di-pion mass spectrum allows to extract the weak pion form factor, which can be compared to the electromagnetic pion form factor measured in electron-positron collisions via the Conserved Vector Current (CVC) hypothesis. This same spectrum is useful to extract, in a clean way, information on the tower of vector resonance parameters that are produced in the hadronization of the isovector current. The radiative di-pion tau lepton decay, $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$, provides additional information on the hadronization of the weak current and it is necessary to account for the di-pion observables at the few percent level. Meanwhile, the lepton-pair production induced by the virtual photon in the radiative decay, namely $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma^* (\to \ell^+ \ell^-)$, serves to further scrutinize the hadronization of the weak current in an extended kinematical domain.

The leading terms of the amplitude for lepton-pair production in the di-pion tau lepton decay depend only upon the form factors and electromagnetic properties of the particles involved in the non-radiative amplitude and are fixed from the gauge invariance requirement. The contributions to this part are given in Fig. 1.

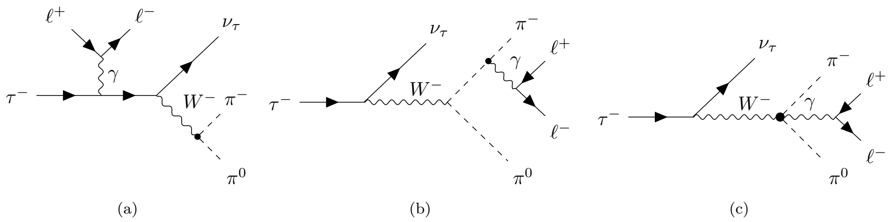

Figure 1: Feynman diagrams contributing to the $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ decay: (a) Inner Bremsstrahlung (IB) off the tau lepton; (b) IB off the π^- meson, (c) contact term.

The matrix element for the decay $\tau^-(P) \to \nu_\tau(q) \pi^-(p_-) \pi^0(p_0) \ell^+(p_{\ell^+}) \ell^-(p_{\ell^-})$ has a similar structure to the radiative amplitude [14]:

$$\begin{aligned} \mathcal{M}(\ell\ell) \quad &= \quad \frac{e^2}{k^2} G_F V_{ud}^* l^\mu \Bigg\{ \frac{(p_- - p_0)_\nu}{k^2 - 2P \cdot k} f_+(t) \overline{u}_\nu(q) \gamma^\nu (1 - \gamma_5) \left(\not{P} - \not{k} + M_\tau \right) \gamma_\mu u_\tau(P) \\ &+ \quad \left(V_{\mu\nu} - A_{\mu\nu} \right) L^\nu \Bigg\}, \end{aligned} \tag{1}$$

Here, G_F is the Fermi coupling constant, V_{ud} the CKM quark mixing matrix element, e is the magnitude of the electron charge. In addition, $L^\nu = \overline{u}_\nu(q) \gamma^\nu (1 - \gamma_5) u_\tau(P)$ and $l^\mu = \overline{u}(p_{\ell^-}) \gamma^\mu v(p_{\ell^+})$ are the weak and electromagnetic leptonic currents, respectively. Owing to Dirac's equation and $k = p_{\ell^+} + p_{\ell^-}$, we have $l^\mu k_\mu = 0$. The variable t is defined as the invariant mass of the two-pion system, and, for virtual photons, $t' \equiv (P - q)^2 = t + 2(p_- + p_0) \cdot k + k^2$.

The first term in Eq. (1) is the photon emission off the tau lepton. It is described in terms of the charged pion vector form factor, defined as $\langle \pi^- \pi | \overline{d} \gamma^\mu u | 0 \rangle = \sqrt{2} f_+(t) \, (p_- - p_0)^\mu$, In the numerical analysis we will use $f_+(x)$ as derived in Refs. [23, 24]. The second term contains the (structure-dependent) vector $V_{\mu\nu}$ and axial-vector $A_{\mu\nu}$ components. They describe the hadronization of the weak current involving an additional photon: $W^-(P-q) \to \pi^-(p_-) \pi^0(p_0) \gamma^*(k)$. As in the radiative case, the vector contribution can be splitted into two terms: $V_{\mu\nu} = V_{\mu\nu}^{LO} + \widehat{V}_{\mu\nu}$. The leading order LO (model-independent) contribution is fixed from the diagrams in Fig. 1 and the gauge-invariance requirement. One gets:

$$\begin{aligned} V_{\mu\nu}^{LO} \quad &= \quad f_+(t') \frac{2p_{-\mu} + k_\mu}{2p_- \cdot k + k^2} \left(p_- + k - p_0 \right)_\nu - f_+(t') g_{\mu\nu} \\ &+ \quad \left(\frac{f_+(t') - f_+(t)}{2(p_0 + p_-) \cdot k + k^2} \right) \left(2 \left(p_- + p_0 \right)_\mu + k_\mu \right) \left(p_- - p_0 \right)_\nu . \end{aligned} \tag{2}$$

This model-independent amplitude, also known as the inner bremsstrahlung term in this letter, is the leading contribution at low photon momenta given the off-shell propagators of charged particles, and the enhancement factor provided by the photon pole propagator. This feature makes model-independent contributions more important for observables in e^+e^-, which has a lower threshold than for muon-pair production. On the other hand, model or structure-dependent contributions, described by $\widehat{V}_{\mu\nu} - A_{\mu\nu}$ terms, start at $O(k)$ and may become important for large photon momenta. The effects of model-dependent contributions become more visible in the $\mu^+\mu^-$ pair production, allowing to explore the rich dynamics of strong interactions in the intermediate energy regime.

Since axial-vector contributions $A_{\mu\nu}$ are not present in the non-radiative amplitude, they start at $O(k)$ in the photon energy expansion. Thus, they are model-dependent and must be manifestly gauge-invariant: $k^\mu A_{\mu\nu} = 0$.

3. Structure-dependent contributions

The structure-dependent piece, $\widehat{V}_{\mu\nu}$ and $A_{\mu\nu}$ are computed in the framework of RChT. They involve a virtual photon ($k^2 = (p_{\ell^+} + p_{\ell^-})^2$). The hadronic vertex $W^*(W) \to \pi^-(p_-)\pi^0(p_0)\gamma^*(k)$ with two virtual gauge bosons (characterized by $W^2 = t' = (p_- + p_0 + k)^2 \neq m_W^2$, $k^2 = (p_{\ell^+} + p_{\ell^-})^2 \neq 0$) can be parameterized in terms of two different sets of form factors, vector and axial-vector. In addition to (t', k^2), the form factors can depend upon two independent kinematical variables which can be taken as (t, s_-) or (t, s_0), where $s_{-,0} \equiv (p_{-,0} + k)^2$, or equivalently $p_- \cdot k$ or $p_0 \cdot k$. Once we chose (t, t', k^2) as relevant variables, either $p_- \cdot k$ or $p_0 \cdot k$ can be chosen as the remaining kinematical scalar to describe the hadronic vertex.

3.1 Structure-dependent vector contributions

The most general form of the structure-dependent part of the vector contributions to the hadronic vertex can be built out by imposing gauge-invariance. It remains:

$$
\begin{aligned}
V^{\mu\nu} \quad = \quad & v_1 \left(p_- \cdot k g^{\mu\nu} - p_-^\mu k^\nu\right) + v_2 \left(p_0 \cdot k g^{\mu\nu} - p_0^\mu k^\nu\right) + v_3 \left(p_0 \cdot k p_-^\mu - (p_- \cdot k p_0^\mu\right) p_-^\nu \\
+ \quad & v_4 \left(p_0 \cdot k p_-^\mu - p_- \cdot k p_0^\mu\right) (p_- + p_0 + k)^\nu + v_5 \left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) \\
+ \quad & v_6 \left(k^2 p_-^\mu - p_- \cdot k k^\mu\right) p_0^\nu + v_7 \left(p_0 \cdot k k^\mu - k^2 p_0^\mu\right) p_-^\nu - f_+(t') g^{\mu\nu} \\
+ \quad & \frac{f_+(t') - f_+(t)}{2\,(p_0 + p_-) \cdot k + k^2} \left(2(p_0 + p_-) + k^\mu\right)^\mu (p_0 - p_-)^\nu \\
+ \quad & \frac{f_+(t')}{2k \cdot p_- + k^2} \left(2p_-^\mu + k^\mu\right) (p_- + k - p_0)^\nu \ . \qquad (3)
\end{aligned}
$$

The Lorentz-invariant form factors $v_{1,\cdots,7}$ encode the information about the dynamics of the strong, weak and electromagnetic interactions involved in the $W^*\pi^-\pi^0\gamma^*$ vertex. In the case of a real photon, $v_{5,6,7}$ do not contribute to the amplitude given that $k^2 = 0$ and $\epsilon \cdot k = 0$ and one recovers the results of Ref. [14] for the radiative amplitude, as it should be. The terms proportional to k^μ are kept because explicit calculations of the hadronic vertex give rise to such structures and in order to exhibit explicitly gauge invariance.

PoS(CHARM2020)035

The different Feynman diagrams appearing in Figure 2 are computed within the RChT framework [2, 3], which ensures the low-energy behaviour of ChPT [22] and includes resonances as dynamical degrees of freedom upon their approximate $U(3)$ flavor symmetry are computed. Besides the kinetic terms for the resonances, the interaction Lagrangian is given by [2]

$$\mathcal{L}_{2V} = \frac{F_V}{2\sqrt{2}} Tr\left(\widetilde{V}_{\mu\nu} f_+^{\mu\nu}\right) + i\frac{G_V}{\sqrt{2}} Tr\left(\widetilde{V}_{\mu\nu} u^\mu u^\nu\right), \qquad \mathcal{L}_{2A} = \frac{F_A}{2\sqrt{2}} Tr\left(\widetilde{A}_{\mu\nu} f_-^{\mu\nu}\right),$$

where Tr stands for a trace in flavor space. Resonance fields are represented by the antisymmetric tensors [25, 26] $\widetilde{V}_{\mu\nu}$ and $\widetilde{A}_{\mu\nu}$, the coupling to the weak charged $V-A$ current proceeds through the $f_\pm^{\mu\nu}$ tensors, and the u^μ tensors couple the resonances to either the vector part of the W boson or (derivatively) to pion fields. The coupling constants of resonances F_V, G_V and F_A can be fixed from short-distance constraints in terms of $f = F_\pi \sim 92$ MeV. Short-distance QCD constraints on the spin-one correlators [2, 3, 27] predict the former in terms of the latter as $F_V = \sqrt{2} f,\ G_V = f/\sqrt{2},\ F_A = f$.

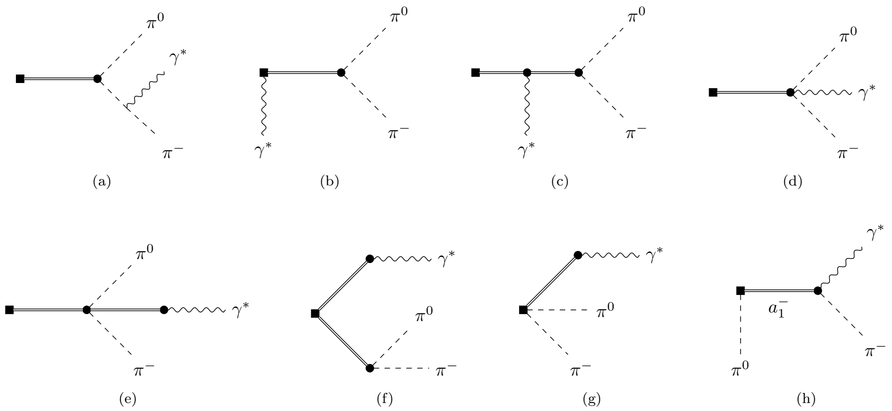

Figure 2: Contributions to $V^{\mu\nu}$ in RChT of the hadronic vertex $W^{*-} \to \pi^-\pi^0\gamma^*$ vertices. Those involving resonances are highlighted with a thick dot. Insertion of the weak charged current is represented by the square dot. Resonances (ρ unless specified) are represented with a double line and γ^* stands for $\gamma^* \to \ell^+\ell^-$.

An explicit evaluation of the seven form factors $v_{1,\dots,7}$ leads to:

$$\begin{aligned}
v_1 &= \frac{F_V G_V}{f^2}\left[2D_\rho^{-1}\left(k^2\right) + 2D_\rho^{-1}(t') + tD_\rho^{-1}(t)D_\rho^{-1}\left(k^2\right) + tD_\rho^{-1}(t)D_\rho^{-1}(t')\right] \\
&+ \frac{F_V^2}{2f^2}\left[-D_\rho^{-1}\left(k^2\right) - D_\rho^{-1}(t') + \left(t'-k^2\right)D_\rho^{-1}(t')\,D_\rho^{-1}\left(k^2\right)\right] \\
&+ \frac{F_A^2}{f^2 m_{a_1}^2}\left(m_{a_1}^2 - m_\pi^2 + \frac{t}{2}\right) D_{a_1}^{-1}\left[(p_- + k)^2\right], \\
v_2 &= \frac{F_V G_V t}{f^2}\left[-D_\rho^{-1}(t)D_\rho^{-1}\left(k^2\right) - D_\rho^{-1}(t)D_\rho^{-1}(t')\right] \\
&+ \frac{F_V^2}{2f^2}\left[-D_\rho^{-1}\left(k^2\right) - D_\rho^{-1}(t') - \left(t'-k^2\right)D_\rho^{-1}(t')\,D_\rho^{-1}\left(k^2\right)\right] \\
&+ \frac{F_A^2}{f^2 m_{a_1}^2}\left(m_{a_1}^2 - m_\pi^2 - k\cdot p_-\right) D_{a_1}^{-1}\left[(p_- + k)^2\right],
\end{aligned}$$

$$
\begin{aligned}
v_3 &= \frac{F_A^2}{f^2 m_{a_1}^2} D_{a_1}^{-1}\left[(p_- + k)^2\right], \\
v_4 &= \frac{F_V^2}{f^2} D_\rho^{-1}(t') D_\rho^{-1}\left(k^2\right) - \frac{2F_V G_V}{f^2} D_\rho^{-1}(t) D_\rho^{-1}(t'), \\
v_5 &= \frac{F_V^2}{2f^2}\left[-D_\rho^{-1}\left(k^2\right) - D_\rho^{-1}(t') - k\cdot(p_0 - p_-) D_\rho^{-1}(t') D_\rho^{-1}\left(k^2\right)\right] \\
&+ \frac{F_A^2}{f^2 m_{a_1}^2}\left(m_{a_1}^2 - m_\pi^2 + \frac{t}{2}\right) D_{a_1}^{-1}\left[(p_- + k)^2\right], \\
v_6 &= \frac{2F_V G_V}{f^2} D_\rho^{-1}(t) D_\rho^{-1}\left(k^2\right) + \frac{F_V^2}{f^2} D_\rho^{-1}(t') D_\rho^{-1}\left(k^2\right), \\
v_7 &= \frac{2F_V G_V}{f^2} D_\rho^{-1}(t) D_\rho^{-1}\left(k^2\right) + \frac{F_V^2}{f^2} D_\rho^{-1}(t') D_\rho^{-1}\left(k^2\right) + \frac{F_A^2}{f^2 m_{a_1}^2} D_{a_1}^{-1}\left[(p_- + k)^2\right], \quad (4)
\end{aligned}
$$

where the propagators are $D_\rho(s) = m_\rho^2 - s - i m_\rho \Gamma_\rho(s)$, $D_{a_1}(s) = m_{a_1}^2 - s - i m_{a_1}\Gamma_{a_1}(s)$. The off-shell widths of the $\rho(770)$ and $a_1(1260)$ mesons are obtained within RChT. The above expressions reduce to the corresponding equations in the case of real photons ($k^2 \to 0$) and owing to the conservation of electromagnetic current, $v_{5,6,7}$ do not contribute to the vector tensor in that case. The short-distance constraints for two-point Green functions (which include the set of relations $F_V = \sqrt{2}f$, $G_V = f/\sqrt{2}$, $F_A = f$) get modified when including three-point Green functions in both intrinsic parity sectors [4, 28–41]. The consistent set of relations in this more general case includes $F_V = \sqrt{3}f$ [42] that –through the appropriate asymptotic behaviour of the spin-one correlators– implies, $G_V = f/\sqrt{3}$ and $F_A = \sqrt{2}f$. As studied extensively in ref. [4] for the $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$ decays, shifting from

$$
F_V = \sqrt{2}f, \quad G_V = \frac{f}{\sqrt{2}}, \quad F_A = f, \qquad (5)
$$

to

$$
F_V = \sqrt{3}f, \quad G_V = \frac{f}{\sqrt{3}}, \quad F_A = \sqrt{2}f, \qquad (6)
$$

gives a rough estimate of the uncertainty in the calculation with the interaction Lagrangian (4) due to missing higher-order terms in the chiral expansion. The first relations have been taken as the reference ones but evaluate alternatively with the second set to assess the model-dependent error.

3.2 Structure-dependent axial-vector contributions

For the most general form of the axial-vector weak current contribution one gets [14, 39, 43] the same result as in the real photon case:

$$
\begin{aligned}
A^{\mu\nu} &= i a_1 \varepsilon^{\mu\nu\rho\sigma}(p_0 - p_-)_\rho k_\sigma + i a_2 W^\nu \epsilon^{\mu\lambda\rho\sigma} p_{-\lambda} p_{0\rho} k_\sigma \\
&+ i a_3 \varepsilon^{\mu\nu\rho\sigma} k_\rho W_\sigma + i a_4 (p_0 + k)^\nu \varepsilon^{\mu\lambda\rho\sigma} p_{-\lambda} p_{0\rho} k_\sigma, \qquad (7)
\end{aligned}
$$

where $W = P - q = p_- + p_0 + k$. As in the vector tensor case, the axial-vector form factors $a_{1\ldots4}$ are Lorentz invariant functions that depend upon two invariants (in addition to $t' = W^2$ and k^2). At $O(p^4)$ only a_1 and a_2, from the Wess-Zumino-Witten functional ([44, 45]), contribute and are obviously the same as in the real photon case at this order. They are given by [14]

$$a_1 = \left[8\pi^2 f^2\right]^{-1}, \quad a_2 = -\left[4\pi^2 f^2 \left(t' - m_\pi^2\right)\right]^{-1}. \tag{8}$$

As in the $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$ decays, it is expected that the corresponding contributions to the decay observables in lepton pair production to be negligibly small [4–6]. Therefore, it is no necessary to compute all remaining axial-vector contributions which will introduce in addition further (although small) uncertainties in computations.

4. Branching ratio and lepton-pair spectrum

The unpolarized squared amplitude of a five-body decay as $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$, depends on eight independent kinematical variables. Depending upon the specific observable we are interested in, it will become necessary to integrate some or all of them. It is convenient to use the set of invariant variables described in Ref. [46]. Specifically, we compute the invariant mass distribution of the lepton pair (k^2-distribution) and the corresponding branching fraction for the tau decays under consideration. The kinematical domain of the lepton pair distribution is the interval $\left[4m_\ell^2, (M_\tau - 2m_\pi)^2\right]$, being $\ell = e, \mu$.

We split the total decay observables into three terms: 1) the IB piece, 2) the VV (AA) model-dependent part, and, 3) the IB-V, IB-A and V-A pieces, which correspond to the interferences of IB, V and A contributions.

Table 1 shows the results of different contributions to the branching ratios of e^+e^- and $\mu^+\mu^-$ pair production (the numerical errors in the integration are shown within parentheses). The third (fifth) column of Table 1 shows the results obtained using the short-distance constraints on the couplings constants of resonances in Eq. (5) (Eq. (6)). The results exhibit the suppression expected since lepton-pair production is $O(\alpha^2)$ with respect to the dominant $\tau^- \to \pi^- \pi^0 \nu_\tau$ decay . Also, the $\mu^+\mu^-$ pair production is further suppressed with respect to e^+e^- production given that the later is largely dominated by model-independent contributions, which are enhanced and peaked at lower invariant mass values of the lepton-pair invariant mass due to the virtual photon propagator. The axial-vector contributions are suppressed in all cases.

The final predictions for the branching fractions are:

$$\mathrm{BR}(\tau^- \to \nu_\tau \pi^- \pi^0 e^+ e^-) = (2.27 \pm 0.03) \times 10^{-5}, \tag{9}$$

$$\mathrm{BR}(\tau^- \to \nu_\tau \pi^- \pi^0 \mu^+ \mu^-) = (1.55 \pm 0.25) \times 10^{-7}. \tag{10}$$

The associated errors cover the results shown in the different columns of Table 1

The normalized (to the total τ decay width) lepton-pair invariant mass distributions are shown in Figure 3 for electron-positron and in Figure 4 for $\mu^+\mu^-$ production. Both distributions are peaked very close to the corresponding threshold ($k^2_{\rm thr} = 4m_\ell^2$) for lepton pair production, with an enhanced peaking for e^+e^- production, due to the $1/k^4$ dependence of the squared amplitude. The second peak in the plots corresponds to the $\rho^0(770) - \gamma^*$ couplings dominance in the vector form factors. It is also clear that the model-dependent contributions are more visible in the $\mu^+\mu^-$ than in e^+e^- production, which is also related to the suppression of inner bremsstrahlung for large photon virtualities.

Contribution	$\ell^+\ell^- = e^+e^-$	$\ell^+\ell^- = e^+e^-$ using (6) for F_V, F_A and G_V	$\ell^+\ell^- = \mu^+\mu^-$	$\ell^+\ell^- = \mu^+\mu^-$ using (6) for F_V, F_A and G_V
IB	$2.213(11) \times 10^{-5}$		$5.961(3) \times 10^{-8}$	
VV	$6.745(36) \times 10^{-7}$	$9.571(48) \times 10^{-7}$	$5.462(4) \times 10^{-8}$	$9.429(7) \times 10^{-8}$
AA	$1.91(1) \times 10^{-8}$		$1.663(1) \times 10^{-9}$	
IB-V	$-3.83(18) \times 10^{-7}$	$-1.02(18)\times 10^{-7}$	$1.337(4) \times 10^{-8}$	$2.126(5)\times 10^{-8}$
IB-A	$9.1(4.5) \times 10^{-9}$		$2.85(3) \times 10^{-9}$	
V-A	$5.2(2.1) \times 10^{-9}$	$4.5(2.6)\times 10^{-9}$	$-1.73(3) \times 10^{-10}$	$-1.65(5) \times 10^{-10}$
Total	$2.245(13) \times 10^{-5}$	$2.302(13) \times 10^{-5}$	$1.319(2) \times 10^{-7}$	$1.795(2) \times 10^{-7}$

Table 1:
Contributions to the branching ratio of $\tau^- \to \nu_\tau\pi^-\pi^0\ell^+\ell^-$ decays. Columns three and five display the branching ratios obtained using Eq. (6) , while the second and four columns correspond to the use of relations (5).

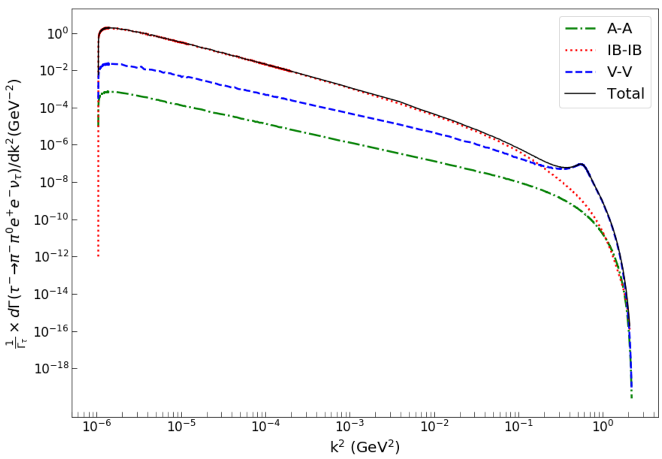

Figure 3: Contributions to the normalized invariant mass distribution for e^+e^- pair production (interferences are not displayed). A double logarithmic scale was used. The second peak is due to the $\rho(770)$ dominance of the virtual photon propagator.

5. Conclusions

The branching ratios and lepton-pair mass distributions of $\tau^- \to \nu_\tau\pi^-\pi^0\ell^+\ell^-$ ($\ell = e, \mu$) where calculated for the first time. As expected, these observables are of $O(\alpha^2)$ with respect to the corresponding dominant di-pion τ lepton decay. For the $\ell = e$ case, a clear inner bremsstrahlung (IB) dominance is observed due to the small $\ell^+\ell^-$ invariant mass (k^2) threshold values. On the other hand, for $\ell = \mu$, both contributions, structure-dependent and IB, are of the same order.

The structure-dependent contributions corresponding to the $W^- \to \pi^-\pi^0\gamma^*$ effective vertex, were calculated using the RChT framework. The structure-dependent vector form factors coincide

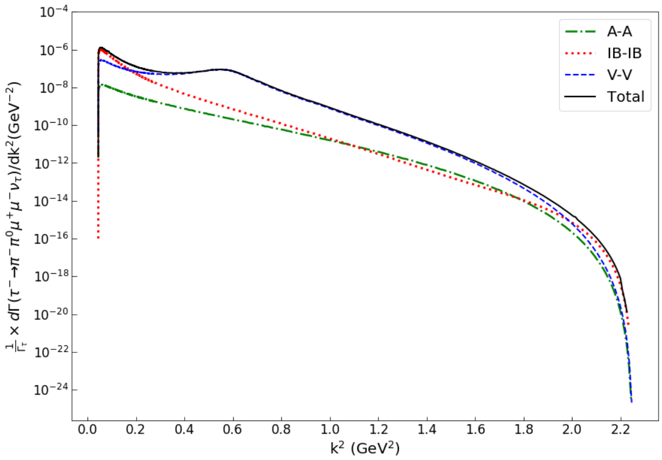

Figure 4: Contributions to the normalized invariant mass distribution for $\mu^+\mu^-$ pair production (inteferences are not displayed). The second peak is due to the $\rho(770)$ dominance of the virtual photon propagator.

(in the limit $k^2 \to 0$) with their counterparts computed in the case of the radiative $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$ decays. We expect axial-vector structure-dependent contributions to be negligible and we stick to their values provided by the Wess-Zumino-Witten anomalous terms.

Within this framework, the results are $\mathcal{BR}\left(\tau^- \to \nu_\tau \pi^- \pi^0 e^+ e^-\right) = 2.27(3) \times 10^{-5}$ (which is essentially free of hadronic uncertainties) and $\mathcal{BR}\left(\tau^- \to \nu_\tau \pi^- \pi^0 \mu^+ \mu^-\right) = 1.55(25) \times 10^{-7}$. The branching fraction for e^+e^- channel could be discovered already with BaBar or Belle data, while the $\mu^+\mu^-$ case will challenge the capabilities of Belle-II. The measurement of the $\mu^+\mu^-$ spectrum can be useful to test previous calculations of radiative corrections to di-pion tau lepton decays. Therefore, it has the potential of reducing the uncertainties on the dominant piece of the hadronic vacuum polarization part of a_μ using tau data.

Finally, the addition of the matrix elements derived in this work to the Monte Carlo generator TAUOLA [47] will be useful in improving background rejection for searches of three-prong lepton flavor or lepton number violating tau decays.

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