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Beyond the Standard Model invisible particle searches in tau lepton decays

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Motivated by the evidence of physics beyond the Standard Model, the unprecedented amount of tau pairs to be collected by the Belle II experiment in the coming years, and inspired by previous proposals, we present an study of the kinematics of tau pair decays and propose a new method to search for lepton flavour violating processes in tau lepton decays to invisible beyond Standard Model particles, such as $\tau \to \ell\alpha$, where ℓ is either an electron or a muon, and α is a massive undetectable particle. Application of our method leads to an improvement by one order of magnitude the expected upper limit on the $\tau \to \ell\alpha$ production in 3x1 prong tau decays.

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Contents

1. INTRODUCTION

The success of the Standard Model of particle physics (SM) in explaining the wide range of experimental results is an outstanding achievement. Nevertheless, there are unanswered questions and some remaining tensions between prediction and experiment. In particular, observed phenomena such as the predominance of matter over antimatter in the universe, the neutrino masses, or dark matter, among others, suggest physics beyond the Standard Model (BSM). Hence the importance of searching for new physics. One of the available exploration strategies involves searching for SM suppressed lepton flavor violating (LFV) processes which observation would be the irrefutable evidence of BSM physics.

An ideal tool in the search for LFV processes is the tau lepton. Of key importance are LFV tau lepton decays to invisible BSM particles predicted in various SM extensions containing axionlike particles [1–4] or new Z' gauge bosons [5–7], and which allow to explain SM anomalies such as the a_μ discrepancy. One such decay is $\tau \to \ell\alpha$, where ℓ is either an electron or a muon, and α is a massive undetectable particle. This decay appears in several new physics models [3, 6–10] and are of interest not only due to the a_μ deviation, but also because very light particles could serve as dark matter candidates [11, 12], or to answer the proton radius puzzle [13].

Motivated by previous searches for dark matter or invisible heavy particles in $X\bar{X} \to (Y_a + N)(Y_b + \bar{N})$ processes [14–16], being Y_a and Y_b the only detectable products, we generalize the idea [17] to $X\bar{X} \to (\sum_{i=1}^{n} Y_{ai} + N_1)(\sum_{j=1}^{m} Y_{bj} + N_2)$ decays where Y represents visible particles and N undetectable particles. This generalization allows us to study $X\bar{X}$ pair decays with BSM processes in one decay, and SM processes with a missing particle in the complementary decay; consequently increasing the possibility of a non-SM particle production compared to requiring a double creation of the unknown particle as in previous studies.

Studying the generalized case of $X\bar{X}$ pair decay, we determine a kinematic constraint that relates the masses of the mother particle X and the undetectable particles N_1 and N_2. Then, using this relationship we propose new searching variables for BSM invisible particles in tau lepton

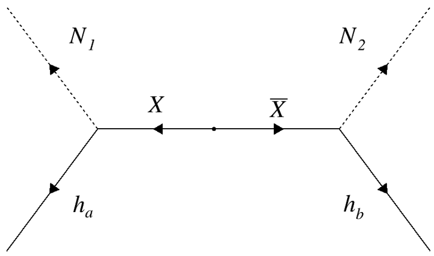

Figure 1: $X\bar{X}$ production topology in the center-of-mass frame. Each X decays to a detectable product h and an invisible particle N that escapes undetected.

decays from collisions with well defined initial state energy and momentum. Belle II provide an ideal environment with these characteristics.

Finally, we apply our findings to the search for LFV processes $\tau \to \ell\alpha$ in simulated data, emulating the Belle II experiment conditions.

2. A NEW METHOD

Let us consider the pair decay

$$X\bar{X} \to \Big(\sum_{i=1}^{n} Y_{ai} + N_1\Big)\Big(\sum_{j=1}^{m} Y_{bj} + N_2\Big) \tag{1}$$

at a well determined CMS energy $\sqrt{s}$. Here N_1 and N_2 are invisible particles that elude detection, and Y_{ai} and Y_{bj} are the i-th and j-th visible particles from the X and $\bar{X}$ decays, respectively. Using h_a and h_b to indicate $\sum_{i=0}^{n} Y_{ai}$ and $\sum_{i=0}^{m} Y_{bj}$. Then the $X\bar{X}$ pair decay is treated as illustrated in Fig. 1, with $p_a = (E_a, \mathbf{p}_a)$, $p_b = (E_b, \mathbf{p}_b)$, $p_1 = (E_1, \mathbf{p}_1)$, and $p_2 = (E_2, \mathbf{p}_2)$ being the four-momenta at CMS for h_a, h_b, N_1, and N_2, respectively. The kinematic equations for the process are

$$q^\mu = p_a^\mu + p_b^\mu + p_1^\mu + p_2^\mu, \quad \mu = 0, 1, 2, 3, \tag{2}$$

$$p_1^2 = m_1^2, \tag{3}$$

$$p_2^2 = m_2^2, \tag{4}$$

$$(p_a + p_1)^2 = (p_b + p_2)^2 = m_X^2, \tag{5}$$

where m_1, m_2, and m_X are the masses of N_1, N_2, and X, respectively. By defining the normalized variables $\mu_i = m_i/\sqrt{s}$, $z_i = E_i/\sqrt{s}$, $\mathbf{a} = \mathbf{p}_a/\sqrt{s}$, $\mathbf{b} = \mathbf{p}_b/\sqrt{s}$, $\mathbf{k}_1 = \mathbf{p}_1/\sqrt{s}$, $\mathbf{k}_2 = \mathbf{p}_2/\sqrt{s}$, from Eq. 2 we have $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{a} + \mathbf{b} = \mathbf{0}$ and $z_1 + z_2 + z_a + z_b = 1$. Then we can rewrite Eqs. 3–5 as

$$|\mathbf{k}_1|^2 + \mu_1^2 = z_1^2, \tag{6}$$

$$|\mathbf{k}_1 + \mathbf{a} + \mathbf{b}|^2 + \mu_2^2 = (1 - z_a - z_b - z_1)^2, \tag{7}$$

$$|\mathbf{k}_1 + \mathbf{a}|^2 + \mu_X^2 = \frac{1}{4}, \tag{8}$$

where we use $z_X = 1/2$. From Eq. 6 we have

$$\mathbf{k}_1 \cdot \mathbf{k}_1 = k_1^2 = \left(\frac{1}{2} - z_a\right)^2 - \mu_1^2, \tag{9}$$

and from Eq. 7 and Eq. 8 we obtain

$$\mathbf{a} \cdot \mathbf{k}_1 = A, \tag{10}$$

and

$$\mathbf{b} \cdot \mathbf{k}_1 = B, \tag{11}$$

where

$$A = \frac{1}{2}\left(z_a - z_a^2 - \mu_X^2 + \mu_1^2 - |\mathbf{a}|^2\right), \tag{12}$$

$$B = \frac{1}{2}\left(z_b^2 - z_b + \mu_X^2 - \mu_2^2 - |\mathbf{b}|^2\right) - \mathbf{a} \cdot \mathbf{b}, \tag{13}$$

In addition, $\mathbf{k}_1$, $\mathbf{a}$, and $\mathbf{b}$, must comply with

$$\begin{aligned} |\mathbf{k}_1 \times \mathbf{a} \times \mathbf{b}|^2 &= |(\mathbf{b} \cdot \mathbf{k}_1)\mathbf{a} - (\mathbf{a} \cdot \mathbf{k}_1)\mathbf{b}|^2, \\ &= |\mathbf{k}_1|^2 |\mathbf{a} \times \mathbf{b}|^2 \sin^2\theta, \\ &\leq |\mathbf{k}_1|^2 |\mathbf{a} \times \mathbf{b}|^2, \end{aligned} \tag{14}$$

and by using Eqs. 9–11, this transforms to

$$|B\mathbf{a} - A\mathbf{b}|^2 - \left[\left(\frac{1}{2} - z_a\right)^2 - \mu_1^2\right]|\mathbf{a} \times \mathbf{b}|^2 \leq 0, \tag{15}$$

From Eqs. 12 and 13, it is straightforward to show that

$$B\mathbf{a} - A\mathbf{b} = \frac{1}{2}\left((\mu_X^2 - \mu_2^2)\mathbf{a} + (\mu_X^2 - \mu_1^2)\mathbf{b} + \mathbf{H}\right), \tag{16}$$

where

$$\mathbf{H} \equiv \left(z_b^2 - z_b - |\mathbf{b}|^2 - 2\mathbf{a} \cdot \mathbf{b}\right)\mathbf{a} + \left(z_a^2 - z_a + |\mathbf{a}|^2\right)\mathbf{b}. \tag{17}$$

Then Eq. 15 transforms to

$$\begin{aligned} A_1(\mu_X^2 - \mu_1^2)^2 + A_2(\mu_X^2 - \mu_2^2)^2 + \\ A_3(\mu_X^2 - \mu_1^2)(\mu_X^2 - \mu_2^2) + \\ B_1(\mu_X^2 - \mu_1^2) + B_2(\mu_X^2 - \mu_2^2) + \\ C_1\mu_1^2 + D_1 \leq 0, \end{aligned} \tag{18}$$

where

$$A_1 = |\mathbf{b}|^2, \tag{19}$$

$$A_2 = |\mathbf{a}|^2, \tag{20}$$

$$A_3 = 2(\mathbf{a} \cdot \mathbf{b}), \tag{21}$$

$$B_1 = 2(\mathbf{b} \cdot \mathbf{H}), \tag{22}$$

$$B_2 = 2(\mathbf{a} \cdot \mathbf{H}), \tag{23}$$

$$C_1 = 4|\mathbf{a} \times \mathbf{b}|^2, \tag{24}$$

$$D_1 = \mathbf{H} \cdot \mathbf{H} - 4|\mathbf{a} \times \mathbf{b}|^2\left(\frac{1}{2} - z_a\right)^2. \tag{25}$$

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Equation 18 is our main result and contains the available kinematics information of the $X\bar{X} \to (h_a + N_1)(h_b + N_2)$ decay.

3. USE CASE: SEARCH FOR $\tau \to \ell\alpha$ DECAYS

The most recent search for $\tau \to \ell\alpha$ decays was performed by the ARGUS Collaboration [18] in $\tau \to \ell + invisible$ data, ℓ being an electron or a muon. The main challenge in the $\tau \to \ell\alpha$ search is to separate these signal decays from the same-signature SM process $\tau \to \ell\bar{\nu}_\ell\nu_\tau$. ARGUS used the fact that the signal is a two-body decay, in contrast to the three-body SM process, therefore, in the tau rest frame the lepton momentum is a constant value. This feature would allow us to separate the two processes. Unfortunately, each tau decay involves a missing particle, making impossible a full reconstruction of the tau. To solve this problem, ARGUS developed a method now knows as pseudo-rest-frame [18]. From now on, we will refer to this method as the ARGUS method.

By using Eq. 18, we can construct other methods to search for $\tau \to \ell\alpha$ decays. Let us consider the process $(\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \to e^-\alpha)$. For the decays studied in Section 2, this is a particular case where $\mu_1 = \mu_\alpha$, $\mu_2 = \mu_{\nu_\tau}$ and $\mu_X = \mu_\tau$. Assuming $\mu_{\nu_\tau} = 0$, Eq. 18 reduces to

$$A_0(\mu_\alpha^2)^2 + B_0\mu_\alpha^2 + C_0 \le 0, \tag{26}$$

where

$$\begin{aligned} A_0 &= A_1, & (27)\\ B_0 &= -B_1 + C_1 - (2A_1 + A_3)\mu_\tau^2, & (28)\\ C_0 &= (A_1 + A_2 + A_3)\mu_\tau^4 + (B_1 + B_2)\mu_\tau^2 + D_1. & (29)\end{aligned}$$

Then, Eq. 26 translates to

$$M_{min}^2 \le m_\alpha^2 \le M_{max}^2, \tag{30}$$

where

$$\begin{aligned} M_{min}^2 &= (\sqrt{s})^2\Big(\frac{-B_0 - \sqrt{B_0^2 - 4A_0C_0}}{2A_0}\Big), & (31)\\ M_{max}^2 &= (\sqrt{s})^2\Big(\frac{-B_0 + \sqrt{B_0^2 - 4A_0C_0}}{2A_0}\Big). & (32)\end{aligned}$$

According to Eq. 30, the distribution of the square value of these new variables, M_{min} and M_{max}, must show endpoints at the value of m_α^2. We can use these endpoints to untangle $\tau \to \ell\alpha$ decays from the SM processes and measure the mass of the α particle in case of observation. Also, if these new variables are not highly correlated, they could be combined in a two-dimensional distribution to increase the statistical discrimination power of the method. In what follows, we will refer to these as the M_{min}, M_{max}, and 2D methods, respectively.

3.1 Simulated data selection

In order to study the kinematic bounds in Eq. 30 at the energies of the Belle II experiment, we simulate $e^+e^- \to \tau^+\tau^-$ and $e^+e^- \to q\bar{q}$ processes at $\sqrt{s} = 10.58$ GeV using Pythia8 [19] implemented in ROOT 6.20 [20]. To account for $\tau \to e\alpha$ decays, we added to Pythia8 a new stable α spin-0 particle, which decay is simulated using a phase-space model. We estimate the number of simulated events for $\tau^+\tau^-$ and $q\bar{q}$ decaying to SM processes from the cross-sections reported by Belle II [21]. For stable, charged particles with transverse momentum p_T, the momentum precision σ in the Belle II experiment [22], varies from $\sigma/p_T \approx 5\%$ for very low p_T particles, to 0.3% for $p_T \geq 0.5$ GeV. To have more realistic simulated data, we apply Gaussian smearing to the momentum components of the final state particles for an average precision of $\sigma_{p_T}/p_T = 1\%$.

In order to select 3x1 prong tau decays, we require per event four charged particles in the final state, and no more than one photon with an energy higher than 0.05 GeV; the latter to suppress decays with neutral pions decaying to photons in the kinematic regime of the photon reconstruction in the Belle II detector. Tau pair decays are produced back-to-back in the CMS and their decay produce jet-like events, with two cones of collimated particles around the thrust axis $\mathbf{n}_T$, defined as the vector that maximizes the thrust magnitude T [23, 24]:

$$T = \frac{\sum_i |\mathbf{p}_i \cdot \hat{\mathbf{n}}_{\mathrm{T}}|}{\sum |\mathbf{p}_i|}, \tag{33}$$

where $\mathbf{p}_i$ is the momentum of the i-th particle in the CMS. To enhance the selection of $(\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \to e^- + anything)$, 3-prong candidates are reconstructed in combinations of three pions on the same side of a plane perpendicular to the thrust axis, while the 1-prong candidate requires one electron on the opposite side.

Figure 2 and Fig. 3 show the distributions for M^2_{min} and M^2_{max} in the simulated data before momentum smearing. The number of $\tau \to e\alpha$ decays for $m_\alpha = 1$ GeV has been set equal to the number of SM background events as an example. In both variables, the signal distribution shows clear endpoints, and a peaking structure at m^2_α, and the background extends all over the kinematic allowed region without significantly peaking at any point. These qualitative features of the distributions are similar for all values of m_α. These striking differences between signal and background data distributions will allow us to disentangle one from each other. In the two-dimensional distribution of (M^2_{min}, M^2_{max}), we do not observe a direct correlation for the signal events. However, background events appear to be correlated.

3.2 Production measurement

The production of a BSM decay is usually measured relative to a similar SM process; this to cancel out similar systematic effects. The $\tau \to e + invisible$ data is dominated by the irreducible decays $\tau \to e\bar{\nu}_e\nu_\tau$, and this is used as the normalization process in the $\tau \to \ell\alpha$ production measurement. Then, to estimate this relative production we need to identify three components in the $\tau \to e + anything$ data: the $\tau \to e\alpha$ decays; the SM process $\tau \to e\bar{\nu}_e\nu_\tau$; and anything else is

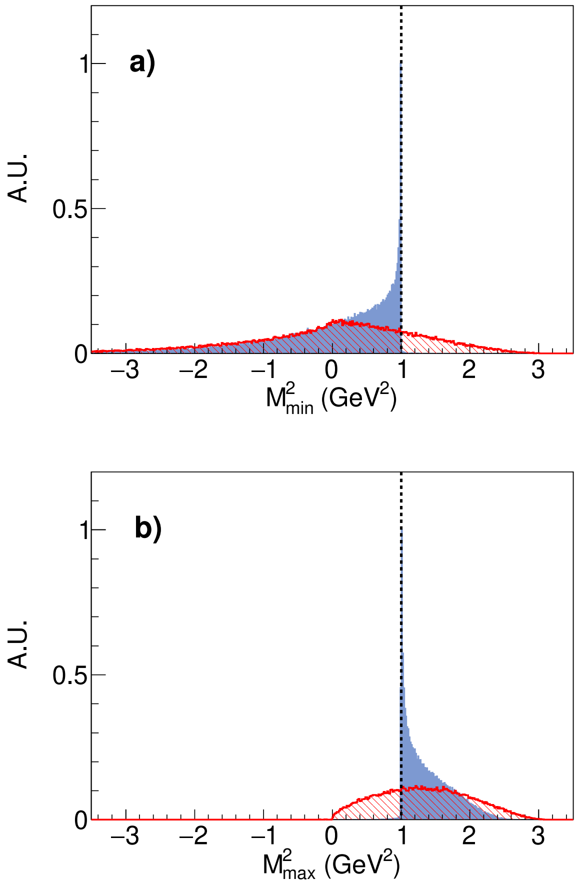

Figure 2: Distribution of (a) M^2_{min} and (b) M^2_{max} in simulated data. The solid region represents the signal process $(\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \to e^-\alpha)$, for $m_\alpha = 1.0$ GeV, indicated by the black dashed line. The red dashed region shows backgrounds from $\tau^+\tau^-$ into $(\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \to e^- + anything)$ and $q\bar{q}$ pairs similarly reconstructed. For illustration, the signal production is set to an equal number of background events.

considered as background. For this, the data should follow a probability distribution given by

$$f(x) = \frac{N_\alpha S_\alpha(\mathbf{x}) + N_\nu S_\nu(\mathbf{x}) + N_b B(\mathbf{x})}{N_\alpha + N_\nu + N_b}, \\ = \frac{N_\nu \mu \frac{\epsilon_\alpha}{\epsilon_\nu} S_\alpha(\mathbf{x}) + N_\nu S_\nu(\mathbf{x}) + N_b B(\mathbf{x})}{N_\nu \mu \frac{\epsilon_\alpha}{\epsilon_\nu} + N_\nu + N_b}, \quad (34)$$

where N_α, N_ν, and N_b are number of $\tau \to e\alpha$ decays, the number of $\tau \to e\bar{\nu}_e\nu_\tau$ decays, and the number of background events, respectively. These components are described by the probability density functions $S_\alpha(\mathbf{x})$, $S_\nu(\mathbf{x})$ and $B(\mathbf{x})$. Here $\epsilon_\alpha/\epsilon_\nu$ is the relative observation efficiency of the first two components, and μ is the relative branching ratio

$$\mu = \frac{Br(\tau \to e\alpha)}{Br(\tau \to e\bar{\nu}_e\nu_\tau)}. \quad (35)$$

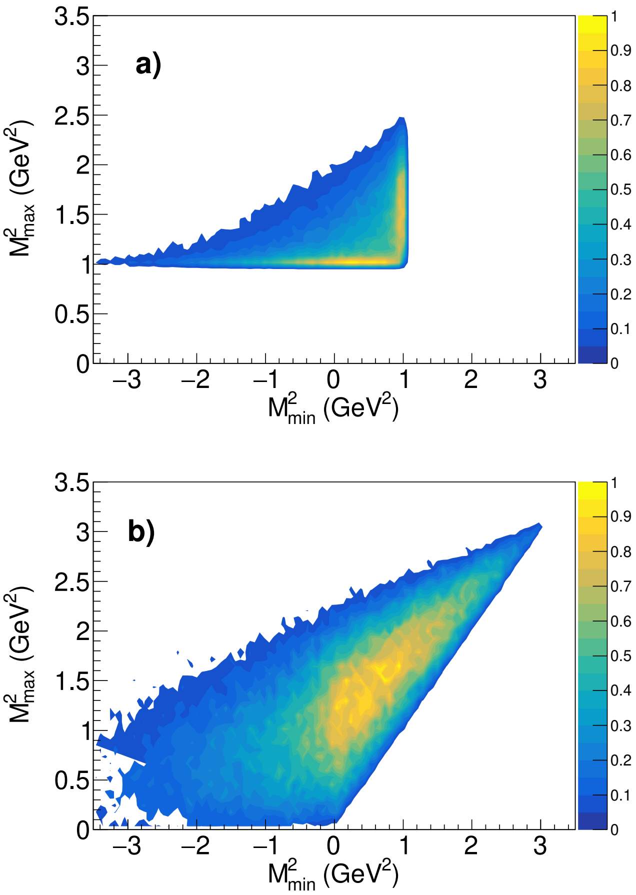

Figure 3: Normalized distribution of M^2_{min} vs M^2_{max} in simulated data for a) signal process $(\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \to e^-\alpha)$ with $m_\alpha = 1.0$ GeV, and b) SM background processes $(\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \to e^- + anything)$ and $q\bar{q}$ pairs. For illustration, the signal production is set to an equal number of background events.

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This is the parameter of interest in which a non-zero value indicates the presence of a signal in data. Then the measurement of the $\tau \to e\alpha$ production reduces to estimate the value of μ. For the search of tiny signals, it is better to formulate the μ determination in terms of a hypothesis test to exclude a possible signal at the desired confidence level (CL).

To test the performance of the M_{min}, M_{max}, and 2D methods in the determination of μ, we use simulated data samples of $\tau \to e + anything$ composed of SM-only processes that follow the selection criteria described in Section 3.1. Then by using the model in Eq. 34, 95% C.L. upper limits on μ are estimated with an asymptotic CLs technique [25] implemented in RooStats [26]. For the data modeling, the probability density distributions $S_\alpha(\mathbf{x})$, $S_\nu(\mathbf{x})$ and $B(\mathbf{x})$, are extracted as templates from independently simulated data samples, where the relative efficiency is found to be $\epsilon_\alpha/\epsilon_\nu = 1.17$ for $m_\alpha = 1.0$ GeV. Similar values of relative efficiency are obtained with different values of m_α. Four methods are studied for the upper limit estimate,

1. The ARGUS method, using the normalized electron energy in the pseudo-rest-frame, x =

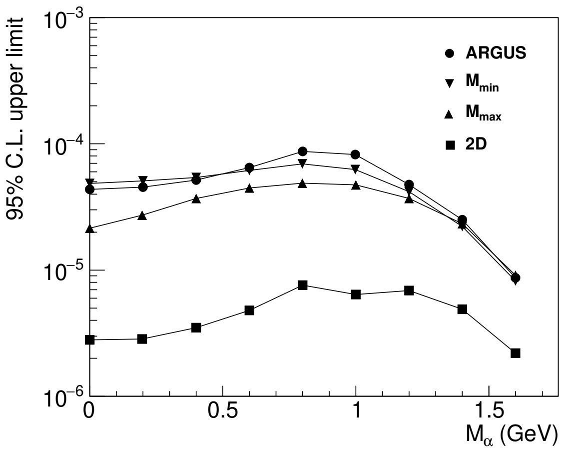

Figure 4: 95% C.L. upper limits for the relative branching ratio $Br(\tau \to e\alpha)/Br(\tau \to e\bar{\nu}_e\nu_\tau)$ for an integrated luminosity of 50 ab^{-1} for tau pairs in 3x1 prong decays.

$2E_e/m_\tau$, as the discriminating variable.

2. The M_{min} method, using M^2_{min} as the discriminating variable.

3. The M_{max} method, using M^2_{max} as the discriminating variable.

4. The 2D method. A combination of M^2_{min} and M^2_{max} in a two-dimensional density distribution.

Figure 4 summarizes the results on the upper limit estimate for masses of the α particle between 0 and 1.6 GeV for an integrated luminosity of 50 ab^{-1}; the data Belle II expects to collect during the next decade. The ARGUS and the M_{min} methods present similar performance for the upper limit estimate. However, for lower m_α values, we obtain better upper limits when using the M_{max} variable than with these two methods. This improvement is not negligible at all; for $m_\alpha = 0$, the upper limit in the M_{max} method is half the one achieved with the ARGUS technique. If we use a simple scaling of $1/\sqrt{N}$ for the limit estimate as data increase, this translates to four times more data in the ARGUS or the M_{min} method to perform as good as the M_{max} variable. However, the 2D method produces a better upper limit than the other three methods alone, improving the expected upper limit by one order of magnitude compared to the ARGUS technique. For the $m_\alpha = 0$, the ARGUS method upper limit is 15 times larger than the upper limit in the 2D method. Using simple data scaling means the ARGUS technique requires 225 times more data to perform as the 2D method.

For $m_\alpha = 0$, Fig. 5 shows for the ARGUS and the 2D method the upper limit on the relative branching ratio as a function of the integrated luminosity. We note that to reach an upper limit below 10^{-4}, 1 ab $^{-1}$ of data is necessary for the 2D method. However, we would require an order of magnitude more statistics for the ARGUS technique to reach this precision. With the proposed 2D method, the Belle II experiment could reach the level of the upper limit in the ARGUS technique for the full data sample, but with only a fraction of the data, which could be collected during the first years of operation.

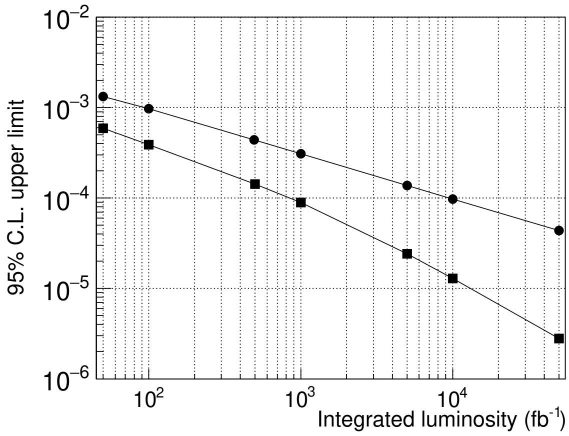

Figure 5: 95% C.L. upper limits for the relative branching fraction $Br(\tau \to e\alpha)/Br(\tau \to e\bar{\nu}_e\nu_\tau)$ as a function of the integrated luminosity for tau pairs in 3x1 prong decays. Black circle (squared) points are for ARGUS (2D) method. The upper limit are for $m_\alpha = 0$.

4. CONCLUSIONS

We studied the kinematics of pair decays as illustrated in Fig. 1 for well known center-of-mass energy. This allowed us to determine a kinematic constraint that relates the masses of the mother particle X and the undetectable particles N_1 and N_2. Then, applying Eq. 18 to the search for the LFV decay $Br(\tau \to e\alpha)$ we proposed two one-dimensional searching variables, and a 2D method to measure the production of this BSM process in tau pair decays at electron-positron colliders. We also showed that under the Belle II experiment conditions, the expect upper limit on $Br(\tau \to e\alpha)$ could be improved by one order of magnitude by using the 2D method respect to the one obtained by either of the ARGUS, M_{min} or M_{max} methods alone [27].

The results presented here were published in [17].

Recently new ideas has been proposed in the same direction [28].

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