## A study of $J / \psi$ decays into baryon－antibaryon pairs

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We perform an analysis of $J / \psi$ decays into octet baryons using the QCD factorisation framework． All decay amplitudes are calculated within the effective field theory approach．The subleading amplitude，which describes the decay of longitudinally polarised charmonia，is obtained for the first time using higher twist three－quark distribution amplitudes．A qualitative analysis of the experimental data is performed．It is found that the polarisation parameter $\alpha_{B}$ can be described with an accuracy $10-30 \%$ ，which indicates that the pQCD contribution provides the dominant part for this observable．

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## 1. Introduction

Decays of $J / \psi$ into baryon-antibaryon pairs have been studied for a very long time, see e.g. Ref.[1, 2]. The latest measurements of BESII and BESIII collaborations provide very accurate data for decay widths and polarisation parameters $\alpha_{B}$, which describes the angular behaviour of the cross section

$$
\begin{equation*}
\frac{d N}{d \cos \theta}=A\left(1+\alpha_{B} \cos ^{2} \theta\right) \tag{1}
\end{equation*}
$$

where $\theta$ is the angle between the baryon or antibaryon direction and the lepton beam, $A$ is an overall normalisation. The value of $\alpha_{B}$ is related to the ratio of the decay amplitudes $\mathcal{G}_{M}^{B} / \mathcal{G}_{E}^{B}$. These amplitudes $\mathcal{G}_{M}^{B}$ and $\mathcal{G}_{E}^{B}$ describe the decay of transversely or longitudinally polarised $J / \psi$, respectively. The contribution of the amplitude $\mathcal{G}_{E}^{B}$ in the cross section is suppressed by the power $m_{B}^{2} / M_{\psi}^{2}$ and therefore subleading ( $m_{B}$ is baryon mass). In the naive limit $m_{c} \rightarrow \infty$ the value of $\alpha_{B}$ is given by $\alpha_{B} \rightarrow 1+O\left(m_{B}^{2} / m_{c}^{2}\right)$ [3]. In Tab. 1 we summarise the existing data for the octet baryons. From this table one can see that the naive estimate at $m_{c} \rightarrow \infty$ implies large corrections. A more

Table 1: The experimental data for decays $J / \psi \rightarrow B \bar{B}$. The values of the branching ratios are taken from PDG [4]. The data for $\alpha_{B}$ are from Refs. [5-9].

|  | $p$ | $n$ | $\Lambda$ | $\Sigma^{0}$ | $\Sigma^{+}$ | $\Xi^{+}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Br}[J / \psi \rightarrow B \bar{B}] \times 10^{3}$ | $2.12(3)$ | $2.1(2)$ | $1.89(9)$ | $1.17(3)$ | $1.5(3)$ | $0.97(8)$ |
| $\alpha_{B}$ | $0.59(1)$ | $0.50(4)$ | $0.47(3)$ | $-0.45(2)$ | $-0.51(2)$ | $0.58(4)$ |
| $\alpha_{B}^{\text {th }}$, Eq. | (2) | 0.46 | 0.46 | 0.32 | 0.26 | 0.25 |

realistic estimate for $\alpha_{B}$ is suggested in Ref. [10], where the it is assumed that $\gamma_{B}=\mathcal{G}_{M}^{B} / \mathcal{G}_{E}^{B} \simeq 1$ and the mass ratio $m_{B}^{2} / M_{\psi}^{2}$ is taken to be finite. Then one finds

$$
\begin{equation*}
\alpha_{B}=\frac{1-4 m_{B}^{2}\left|\gamma_{B}\right|^{2} / M_{\psi}^{2}}{1+4 m_{B}^{2}\left|\gamma_{B}\right|^{2} / M_{\psi}^{2}} \simeq \frac{1-4 m_{B}^{2} / M_{\psi}^{2}}{1+4 m_{B}^{2} / M_{\psi}^{2}} \tag{2}
\end{equation*}
$$

This gives reliable estimates for nucleon, but yields less accurate values for other baryons, see Tab. 1. One can conclude that the data indicate about the sufficiently large effect of the subleading amplitude $\mathcal{G}_{E}^{B}$, which is sensitive to the higher Fock wave functions of the nucleon. Various attempts to estimate $\alpha_{N}$ have been considered in Refs. [11, 12] using a different phenomenological models. However the amplitude $\mathcal{G}_{E}^{B}$ have not been computed on a systematical way as it is must follow from the effective field theory (EFT) framework. For the first time such approach was recently used in Ref. [13] for the nucleon and later this analysis was extended to the baryon octet states in Ref. [14]. Below we review the results obtained in these works.

## 2. Decay amplitudes within the factorisation framework

The QCD description is given by the non-relativistic expansion (NRQCD) and by the collinear factorisation for the outgoing hadrons. The hard subprocess is associated with the $c \bar{c}$-annihilation into three hard gluons, which further create the light quarks-antiquarks pairs. The final state of the perturbative partonic subprocess is described by the collinear quarks and antiquarks, which provide
the long-distance overlap with the hadronic states. In the EFT approach such non-perturbative contributions are described by the light-cone collinear matrix elements. The operators in these matrix elements are constructed from the collinear quark and gluon fields. In this work we only consider the 3-quark operators. The matrix elements are parametrised in terms of scalar functions, which are known as light-cone distribution amplitudes (DAs). These non-perturbative functions depends on the momentum fractions carried by quarks in the bound states. The non-perturbative dynamics associated with the initial charmonia is described by the matrix element in NRQCD. Corresponding constant can be associated with the charmonium wave function at the origin. The expressions for the decay amplitudes are given by the collinear convolution integrals with respect to the quark momentum fractions. The integrands are given by the product of the perturbative kernels and non-perturbative DAs. The result for the amplitude $\mathcal{G}_{M}^{B}$ have been obtained long time ago, see e.g. Ref. [15]. The expression for the second subleading amplitude $\mathcal{G}_{E}^{B}$ have been obtained recently in Refs. [13, 14]. This result can be written as

$$
\begin{equation*}
\mathcal{G}_{M / E}^{B}=\frac{f_{\psi}}{m_{c}^{2}} \frac{f_{B}^{2}}{m_{c}^{4}}\left(\pi \alpha_{s}\right)^{3} \frac{10}{81} J_{E}^{B} \tag{3}
\end{equation*}
$$

where the collinear integral reads

$$
\begin{align*}
J_{E}^{B}= & \frac{c_{B}}{f_{B}^{2}} \int D y_{i} \frac{1}{y_{1} y_{2} y_{3}} \int D x_{i} \frac{1}{x_{1} x_{2} x_{3}} \frac{1}{D_{1} D_{2} D_{3}} \\
& \times\left\{\left(\mathcal{A}_{1}-\mathcal{V}_{1}\right)^{B}\left(x_{1}, x_{2}, x_{3}\right)\left(A_{1}+V_{1}\right)^{B}\left(y_{1}, y_{2}, y_{3}\right) x_{1}\left(x_{2}\left(y_{2}-y_{3}\right)-\bar{y}_{1} y_{2}\right)\right. \\
& +\left(\mathcal{A}_{1}+\mathcal{V}_{1}\right)^{B}\left(x_{1}, x_{2}, x_{3}\right)\left(A_{1}-V_{1}\right)^{B}\left(y_{1}, y_{2}, y_{3}\right) x_{2}\left(x_{2}-y_{2}\right)\left(y_{1}-y_{3}\right) \\
& \left.+\left(\mathcal{T}_{21}-\mathcal{T}_{41}\right)^{B}\left(x_{1}, x_{2}, x_{3}\right) T_{1}^{B}\left(y_{1}, y_{2}, y_{3}\right) 2 x_{3}\left(x_{2}\left(y_{1}-y_{2}\right)+y_{2} \bar{y}_{3}\right)\right\},  \tag{4}\\
& D x_{i}=d x_{1} d x_{2} d x_{3} \delta\left(x_{1}+x_{2}+x_{3}-1\right), \quad D_{i}=x_{i} \bar{y}_{i}+y_{i} \bar{x}_{i}, \quad \bar{y}_{i}=1-y_{i} . \tag{5}
\end{align*}
$$

The coupling $f_{\psi}$ in Eq.(3) describe the NRQCD matrix element. The coupling $f_{B}$ describes the normalisation of the baryon twist-3 DA. The constant $c_{B}$ in Eq.(5) is defined as

$$
c_{B}=\left\{\begin{array}{l}
1, B \neq \Lambda  \tag{6}\\
2, B=\Lambda
\end{array}\right.
$$

The integrand in Eq.(5) includes twist- 3 baryon DAs $\left\{V_{1}, A_{1}, T_{1}\right\}$ and twist-4 baryon DAs $\left\{\mathcal{V}_{1}, \mathcal{A}_{1}, \mathcal{T}_{21}-\right.$ $\left.\mathcal{T}_{41}\right\}$. All these DAs have the following structure

$$
\begin{equation*}
\operatorname{DA}\left(x_{1}, x_{2}, x_{3}\right)=x_{1} x_{2} x_{3} \times \sum_{k_{i} \geq 0} C_{k_{1} k_{2} k_{3}} x_{1}^{k_{1}} x_{2}^{k_{2}} x_{3}^{k_{3}} \tag{7}
\end{equation*}
$$

This allows one to see that the integral in Eq.(5) is well defined.
The moments of the twist-3 and twist-4 DA have been estimated in Refs. [15-17] ( light-cone sum rules) and computed on the lattice, see Ref. [21]. The twist-4 DAs also have contributions with the twist-3 moments, which are often referred as Wandzura-Wilczek (WW) contributions. This can be schematically written as

$$
\begin{align*}
& \mathcal{V}_{1}\left(x_{1}, x_{2}, x_{3}\right)=\mathcal{V}_{1}^{(3)}\left(x_{1}, x_{2}, x_{3}\right)+\overline{\mathcal{V}}_{1}\left(x_{1}, x_{2}, x_{3}\right)  \tag{8}\\
& \mathcal{A}_{1}\left(x_{1}, x_{2}, x_{3}\right)=\mathcal{A}_{1}^{(3)}\left(x_{1}, x_{2}, x_{3}\right)+\overline{\mathcal{A}}_{1}\left(x_{1}, x_{2}, x_{3}\right) \tag{9}
\end{align*}
$$

The similarly structure also holds for the chiral-odd combination $\mathcal{T}_{21}-\mathcal{T}_{41}$. The parts with superscript "(3)" completely depend on the moments of the 3-quark operators, which have geometrical twist3. The functions $\overline{\mathcal{V}}_{1}$ and $\overline{\mathcal{A}}_{1}$ in Eqs.(9) denote the genuine twist- 4 contributions. A detailed consideration of the nucleon twist-4 DAs and their WW-structure can be found in Refs. [18-20].

## 3. Phenomenological results and discussion

In our numerical estimations we use the following models of baryon DAs. For the nucleon DAs we take the model set ABO 1 from Ref. [17] and the coupling $f_{N}$ from the sum rule estimate in Ref. [15]. For other baryons we use simpler models with fewer parameters. The numerical values for the different parameters are given in Tab. 2. For the twist-3 DAs we use the results from Ref. [21]. However the normalisation constants $f_{B}$ have been modified in order to improve the description. The twist-4 parameters $\lambda_{1}^{B}$ and $\lambda_{\perp}$ are also taken from the Ref. [21]. The other parameters $\eta_{10}^{B}, \eta_{11}^{B}, \zeta_{10}$ and $\zeta_{11}$ have not yet been studied in the literature (except the nucleon case). Therefore for $\eta_{10}^{B}$ and $\zeta_{10}^{B}$ we take the same values as for the nucleon. The numerical values of the $\eta_{11}^{B}$ and $\zeta_{11}$ are selected so as to better describe the data.

Table 2: The parameters, which define the twist-3 and twist-4 models of the baryon DAs (upper and bottom tables, respectively). All values are given at scale $\mu^{2}=4 \mathrm{GeV}^{2}$.

| $B$ | $f_{B}, \mathrm{GeV}^{2}$ | $\phi_{10}$ | $\phi_{11}$ | $\phi_{20}$ | $\phi_{21}$ | $\phi_{22}$ | $f_{\perp}^{B}, \mathrm{GeV}^{2}$ | $\pi_{10}^{B}$ | $\pi_{11}^{B}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $N$ | $4.80 \times 10^{-3}$ | 0.047 | 0.047 | 0.069 | -0.024 | 0.15 | - | - | - |
| $\Lambda$ | $5.5 \times 10^{-3}$ | 0.125 | 0.050 | 0 | 0 | 0 | - | 0.044 | - |
| $\Sigma$ | $4.5 \times 10^{-3}$ | 0.017 | 0.037 | 0 | 0 | 0 | $5.14 \times 10^{-3}$ | - | -0.017 |
| $\Xi$ | $5.1 \times 10^{-3}$ | 0.057 | -0.0023 | 0 | 0 | 0 | $5.29 \times 10^{-3}$ | - | 0.063 |


| $B$ | $\lambda_{1}^{B}, \mathrm{GeV}^{2}$ | $\eta_{10}^{B}$ | $\eta_{11}^{B}$ | $\lambda_{\perp}^{B}, \mathrm{GeV}^{2}$ | $\zeta_{10}^{B}$ | $\zeta_{11}^{B}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $N$ | $-30 \times 10^{-3}$ | -0.037 | 0.127 | - | - | 0.127 |
| $\Lambda$ | $-42 \times 10^{-3}$ | -0.037 | 0.127 | $-52 \times 10^{-3}$ | -0.037 | - |
| $\Sigma$ | $-46 \times 10^{-3}$ | -0.037 | 0.23 | - | - | 0.23 |
| $\Xi$ | $-49 \times 10^{-3}$ | -0.037 | 0.11 | - | - | 0.11 |

Numerical results for the branching ratios and for the $\gamma_{B}=\left|\mathcal{G}_{M}^{B} / \mathcal{G}_{E}^{B}\right|$ are shown in Tab. 3. The obtained results show that the branching fractions for all baryons can be reasonably described for the relatively low normalisation scale $\mu^{2} \simeq 1.5 \mathrm{GeV}^{2}$ only. The large sensitivity to the renormalisation scale is due to the value of $\alpha_{s}$. On the other hand the value of $\gamma_{B}$ is quite stable because many uncertainties cancel in the ratio. The obtained values $\gamma_{B}$ describe the experimental data within the $(10-30) \%$ accuracy, which is quite reasonable taking into account different theoretical uncertainties. This indicates that the factorisable contribution provides sufficiently large numerical effect for this observables. In the current qualitative analysis we do not consider the effect from the electromagnetic amplitude $\left(J / \psi \rightarrow \gamma^{*} \rightarrow B \bar{B}\right)$. The interference with the electromagnetic amplitudes definitely provides a reasonable numerical impact. The new data for baryon time-like electromagnetic form factors allows one to perform the estimates of these effects, this work is in progress.

Table 3: The results of the numerical calculation in comparison with the experimental data. The obtained values are shown for the the scale interval $2 m_{c}^{2}<\mu^{2}<1.5 \mathrm{GeV}^{2}$. Instead of $\alpha_{B}$ we show the results for $\gamma_{B}=\left|\mathcal{G}_{M}^{B} / \mathcal{G}_{E}^{B}\right|$, see Eq.(2).

| $B$ | $B r_{\exp } \times 10^{3}$ | $B r \times 10^{3}$ | $\gamma_{B}^{\exp }$ | $\gamma_{B}$ |
| :---: | :---: | :---: | :---: | :---: |
| $p$ | $2.12(3)$ | $0.47-1.43$ | $0.83(2)$ | $0.66-0.68$ |
| $n$ | $2.09(2)$ |  | $0.95(6)$ |  |
| $\Lambda$ | $1.89(9)$ | $0.45-1.32$ | $0.83(4)$ | $0.69-0.69$ |
| $\Sigma^{0}$ | $1.17(3)$ | $0.41-1.14$ | $2.11(5)$ | $1.68-1.77$ |
| $\Sigma^{+}$ | $1.5(3)$ |  | $2.27(5)$ |  |
| $\Xi^{+}$ | $0.97(8)$ | $0.26-0.74$ | $0.61(5)$ | $0.59-0.59$ |

The experimental data indicate that the value of $\gamma_{\Sigma}$ is about factor $2-3$ larger comparing with other octet baryons. This leads to the interesting observation: the power suppressed contribution in expression for width is strongly enhanced and provides the very large numerical effect for the branching ratio. The dynamic origin of this effect is not clear. In order to describe this effect within the considered framework it is necessary to assume sufficiently large $S U(3)$-breaking corrections. This implies that the twist-4 parameters $\eta_{11}^{\Sigma}$ and $\zeta_{11}^{\Sigma}$, are about factor 2 larger comparing with other ones. It remains unclear whether one can explain this enhancement of $\gamma_{\Sigma}$ by some intrinsic properties of the baryon wave functions or perhaps this effect is may also be related with the hadron dynamics at large distances such as final state interactions.

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## References

[1] N. Brambilla et al. [Quarkonium Working Group], hep-ph/0412158.
[2] N. Brambilla et al., Eur. Phys. J. C 71 (2011) 1534 doi:10.1140/epjc/s10052-010-1534-9 [arXiv:1010.5827 [hep-ph]].
[3] S. J. Brodsky and G. P. Lepage, Phys. Rev. D 24 (1981) 2848. doi:10.1103/PhysRevD. 24.2848
[4] P.A. Zyla et al. [Particle Data Group], PTEP 2020 (2020) no.8, 083C01 doi:10.1093/ptep/ptaa104
[5] M. Ablikim et al. [BES], Phys. Rev. D 78 (2008), 092005 doi:10.1103/PhysRevD.78.092005 [arXiv:0810.1896 [hep-ex]].
[6] M. Ablikim et al. [BESIII], Phys. Rev. D 86 (2012), 032014 doi:10.1103/PhysRevD. 86.032014 [arXiv: 1205.1036 [hep-ex]].
[7] M. Ablikim et al. [BESIII], Phys. Rev. D 93 (2016) no.7, 072003 doi:10.1103/PhysRevD.93.072003 [arXiv:1602.06754 [hep-ex]].
[8] M. Ablikim et al. [BESIII], Phys. Rev. D 95 (2017) no.5, 052003 doi:10.1103/PhysRevD.95.052003 [arXiv:1701.07191 [hep-ex]].
[9] M. Ablikim et al. [BESIII], Phys. Rev. Lett. 125 (2020) no.5, 052004 doi:10.1103/PhysRevLett.125.052004 [arXiv:2004.07701 [hep-ex]].
[10] M. Claudson, S. L. Glashow and M. B. Wise, Phys. Rev. D 25 (1982) 1345. doi:10.1103/PhysRevD.25.1345
[11] C. Carimalo, Int. J. Mod. Phys. A 2 (1987) 249. doi:10.1142/S0217751X87000107
[12] F. Murgia and M. Melis, Phys. Rev. D 51 (1995) 3487 doi:10.1103/PhysRevD. 51.3487 [hepph/9412205].
[13] N. Kivel, Eur. Phys. J. A 56 (2020) no.2, 64 [erratum: Eur. Phys. J. A 57 (2021) no.9, 271] doi:10.1140/epja/s10050-021-00575-9 [arXiv:1910.02850 [hep-ph]].
[14] N. Kivel, [arXiv:2109.05847 [hep-ph]].
[15] V. L. Chernyak, A. A. Ogloblin and I. R. Zhitnitsky, Z. Phys. C 42 (1989) 583 [Yad. Fiz. 48 (1988) 1398] [Sov. J. Nucl. Phys. 48 (1988) 889]. doi:10.1007/BF01557664
[16] V. Braun, R. J. Fries, N. Mahnke and E. Stein, Nucl. Phys. B 589 (2000) 381 Erratum: [Nucl. Phys. B 607 (2001) 433] doi:10.1016/S0550-3213(00)00516-2, 10.1016/S0550-3213(01)00254-1 [hep-ph/0007279].
[17] I. V. Anikin, V. M. Braun and N. Offen, Phys. Rev. D 88 (2013) 114021 doi:10.1103/PhysRevD.88.114021 [arXiv:1310.1375 [hep-ph]].
[18] V. M. Braun, A. N. Manashov and J. Rohrwild, Nucl. Phys. B 807 (2009) 89 doi:10.1016/j.nuclphysb.2008.08.012 [arXiv:0806.2531 [hep-ph]].
[19] P. Wein and A. Schäfer, JHEP 05 (2015), 073 doi:10.1007/JHEP05(2015)073 [arXiv:1501.07218 [hep-ph]].
[20] I. V. Anikin and A. N. Manashov, Phys. Rev. D 93 (2016) no.3, 034024 doi:10.1103/PhysRevD.93.034024 [arXiv:1512.07141 [hep-ph]].
[21] G. S. Bali et al. [RQCD Collaboration], Eur. Phys. J. A 55 (2019) no.7, 116 doi:10.1140/epja/i2019-12803-6 [arXiv:1903.12590 [hep-lat]].


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