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# Decay of the scalar charmonium state $\chi_{c0}(IP)$ in the extended Linear Sigma Model

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We study the hadronic decays of the ground-state scalar charmonium $\chi_{c0}(IP)$, $J^{PC}=0^{++}$, in the framework of a $U(4)_r \times U(4)_l$ symmetric linear sigma model with (pseudo)scalar and (axial-)vector mesons. The model fairly succeeds to describe the masses of charmed mesons and the (OZI-dominant) strong decays of open charmed mesons. Here we compute the (OZI-suppressed) decays of charmonium state $\chi_{c0}(1P)$. We calculate also the decay widths of this state into the scalar-isoscalar resonances $f_0(1370)$, $f_0(1500)$ and a predominantly scalar glueball $f_0(1710)$. The results of measured decay widths are in good agreement with experiments. The unmeasured states and channels are interesting for the ongoing BESIII, Belle and BaBar experiments as well as the upcoming PANDA experiment at FAIR facility.

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## 1. Introduction

Charmonia are quantum systems composed of a charm quark ($c$) and an anti-charm quark ($\overline{c}$), which are among the simplest bound states of Quantum Chromodynamics (QCD), the strong fundamental interactions of quarks and gluons. Charmonia exhibit a spectrum of resonances and play the same important role for understanding hadronic dynamics as the hydrogen atom [1]. Since November 1974, the first discovery of the charmonium state which was for $(J/\psi)$ with quantum numbers $J^{PC} = 1^{--}$ at BNL [2] and at SLAC [3], the significant experimental progress has been achieved for charmonium spectroscopy. As an example of this, the hadronic and electromagnetic transitions between charmonium states and their decays have been measured with high precision with the BESIII spectrometer at the electron-positron collider at IHEP Beijing. Moreover, unconventional narrow charmonium-rich states have been recently discovered in an energy regime above the open-charm threshold by Belle [4] and BaBar [5], which potentially initiates a new area in charmonium spectroscopy. The upcoming PANDA experiment at the research facility FAIR will exploit the annihilation of cooled anti-protons with protons to perform charmonium spectroscopy with incredible precision. Moreover the theoretical process such as nonrelativistic QCD [6] and heavy-quark effective theory [7], potential models [8], lattice gauge theory [9], and light front quantization have shown the direct connection of charmonium properties with QCD. More details of the experimental and theoretical situation are given in Ref. [10]. Notice that QCD is reproducing successfully the physics phenomena at distances much shorter than the size of the nucleon, where perturbation theory can be used yielding results of high precision and predictive power. At larger distance scales, however, perturbation methods cannot be applied anymore, although spectacular phenomena- such as the generation of hadron masses and quark confinement. As shown in the Refs. [11, 12] effective field approaches of low-energy provided a very successful description of hadron phenomenology and hadronic reactions.

The extended Linear Sigma Model (eLSM) [13, 14] is an effective model, which has been studied successfully the vacuum phenomenology of (pseudo-)scalar and (axial-)vector mesons in the cases of $N_f = 2$ [15, 16, 17], $N_f = 3$ [18], and $N_f = 4$ [19, 20, 21]. The eLSM was also applied to study excited scalar mesons [22], hybrid mesons [23] and the decay modes of the pseudoscalar glueball [24] and of its first excited state [25]. The construction of the eLSM is based on a global chiral symmetry $U(N_f)_r \times U(N_f)_l$ as well as the classical dilation symmetry. In the vacuum, global chiral symmetry is broken spontaneously by a non-vanishing expectation value of the quark condensate ($< \overline{q}q > = < \overline{q}_r q_l + \overline{q}_l q_r > \neq 0$ [26] to $SU(N_f)_V$), explicitly by quantum effects (the $U(1)_A$ symmetry is broken to $Z(N_f)_A$ [27] as shown by 't Hooft [28]), and explicitly by non-vanishing quark masses. Furthermore, the dilation symmetry is broken explicitly and the local color symmetry is automatically fulfilled for colorless hadronic degree of freedom. For more completion, we expand our framework to study the vacuum properties of hidden charmed mesons like decay widths by eLSM.

In the present work we study the OZI-suppressed decays of the ground-state scalar charmonium $\chi_{c0}$ within the eLSM. We obtained that the scalar glueball could be produced through the decay of $\chi_{c0}$.

This work is organized as follows: in Sec. II we present $U(4)_r \times U(4)_l$ LSM. In Sec III we present the results of the two- and three-body decay widths of $\chi_{c0}$, and in Sec IV our conclusion.

Details of the calculations are relegated to the Appendices. Our units are $\hbar = c = 1$, the metric tensor is $g^{\mu\nu} = diag(+,-,-,-)$.

## 2. The $U(4)_r \times U(4)_l$ LSM interaction with glueballs

In this section, we present the $U(4)_L \times U(4)_R$ Linear sigma model with (pseudo)scalar and (axial-)vector mesons [19, 20], which contains a scalar glueball $G$ and a pseudoscalar glueball $\widetilde{G}$, and has a chiral symmetry, dilatation invariance, and invariance under the discrete symmetries $C$ and $P$, as follows

$$\begin{aligned}
\mathcal{L} = & \mathcal{L}_{dil} + \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[Tr(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& + \mathrm{Tr}\left\{\left[\left(\frac{G}{G_0}\right)^2\frac{m_1^2}{2} + \Delta\right]\left[(L^\mu)^2 + (R^\mu)^2\right]\right\} - \frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] - 2\,\mathrm{Tr}[\varepsilon\Phi^\dagger\Phi] \\
& + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] + c(det\Phi - det\Phi^\dagger)^2 - \delta c(det\Phi - det\Phi^\dagger)^2\mathrm{Tr}(\mathbb{P}_C\Phi^\dagger\mathbb{P}_C\Phi) + i\tilde{c}\tilde{G}\left(\det\Phi - \det\Phi^\dagger\right) \\
& + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)Tr[(L^\mu)^2 + (R^\mu)^2] + h_2\mathrm{Tr}[(\Phi R^\mu)^2 + (L^\mu\Phi)^2] + 2h_3\mathrm{Tr}(\Phi R_\mu\Phi^\dagger L^\mu) \\
& + i\frac{g_2}{2}\{\mathrm{Tr}(L_{\mu\nu}[L^\mu, L^\nu]) + \mathrm{Tr}(R_{\mu\nu}[R^\mu, R^\nu])\} + \ldots,
\end{aligned} \tag{2.1}$$

where $D^\mu\Phi \equiv \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu)$ is the covariant derivative; $L^{\mu\nu} \equiv \partial^\mu L^\nu - \partial^\nu L^\mu$, and $R^{\mu\nu} \equiv \partial^\mu R^\nu - \partial^\nu R^\mu$ are the left-handed and right-handed field strength tensors. While $\mathcal{L}_{dil}$ is the dilation Lagrangian which describes the scalar glueball $G \equiv |gg\rangle$ with quantum number $J^{PC} = 0^{++}$, and mimics the trace anomaly of the pure Yang-Mills sector of QCD [29, 18]:

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4\log\frac{G}{\Lambda} - \frac{G^4}{4}\right). \tag{2.2}$$

The energy scale of low-energy QCD is described by the dimensionful parameter $\Lambda$ which is identical to the minimum $G_0$ of the dilaton potential ($G_0 = \Lambda$). The scalar glueball mass $m_G$ has been evaluated by lattice QCD which gives a mass of about (1.5-1.7) GeV [30]. The assignment of $G$ is still uncertain. Recently, we confirmed the result of the study discussed in Ref. [31], the resonance $f_0(1710)$ is predominantly a scalar glueball.
The matrices $H$, $\Delta$ and $\varepsilon$ defined as

$$H = \frac{1}{2}diag(h_{0N}, h_{0N}, \sqrt{2}h_{0S}, \sqrt{2}h_{0C}),\ h_{0N} = const.,\ h_{0S} = const.,\ h_{0C} = const.,$$

$$\Delta = diag(\delta_{0N}, \delta_{0N}, \delta_{0S}, \delta_{0C}),\ \delta_{0N} = const,\ \delta_{0S} = const,,\ \delta_{0C} = const.,$$

$$\varepsilon = diag(\varepsilon_{0N}, \varepsilon_{0N}, \varepsilon_{0S}, \varepsilon_{0C}),\ \varepsilon_{0N} = const,\ \varepsilon_{0S} = const,,\ \varepsilon_{0C} = const.,$$

where $h_{0i} \sim m_i$ $\delta_{0i} \sim m_i^2$ and $\varepsilon_{0i} \sim m_i^2$ [20].

In our framework, we study charmonium state then $\Phi$ represents the $4\times 4$ (pseudo)scalar multiples whereas $L^\mu$ and $R^\mu$ represent the left- and right-handed (axial)vector multiplets [19] as follows:

$$\Phi=(S^a+iP^a)t^a=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N+a_0^0)+i(\eta_N+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+}+iK^+ & D_0^{*0}+iD^0 \\ a_0^- + i\pi^- & \frac{(\sigma_N-a_0^0)+i(\eta_N-\pi^0)}{\sqrt{2}} & K_0^{*0}+iK^0 & D_0^{*-}+iD^- \\ K_0^{*-}+iK^- & \overline{K}_0^{*0}+i\overline{K}^0 & \sigma_S+i\eta_S & D_{S0}^{*-}+iD_S^- \\ \overline{D}_0^{*0}+i\overline{D}^0 & D_0^{*+}+iD^+ & D_{S0}^{*+}+iD_S^+ & \chi_{C0}+i\eta_C \end{pmatrix}, \tag{2.3}$$

where $t^a$ are the generators of the group $U(N_f)$. The multiplet $\Phi$ transforms as $\Phi \to U_L \Phi U_R^\dagger$ under $U_L(4)\times U_R(4)$ chiral transformations, whereas $U_{L(R)}=e^{-i\theta^a_{L(R)}t^a}$ is an element of $U(3)_{R(L)}$, under parity which $\Phi(t,\overrightarrow{x}) \to \Phi^\dagger(t,-\overrightarrow{x})$, and under charge conjugate $\Phi \to \Phi^\dagger$. The determinant of $\Phi$ is invariant under $SU(4)_L\times SU(4)_R$, but not under $U(1)_A$ because $\det\Phi \to \det U_A \Phi U_A = e^{-i\theta_A^0\sqrt{2N_f}}\det\Phi \neq \det\Phi$.

Now we turn to present the left-handed and right-handed matrices containing the vector, $V^a$, and axial-vector, $A^a$, degrees of freedom [19]:

$$L^\mu=(V^a+iA^a)^\mu t^a=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}}+\frac{f_{1N}+a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+}+K_1^+ & D^{*0}+D_1^0 \\ \rho^- + a_1^- & \frac{\omega_N-\rho^0}{\sqrt{2}}+\frac{f_{1N}-a_1^0}{\sqrt{2}} & K^{*0}+K_1^0 & D^{*-}+D_1^- \\ K^{*-}+K_1^- & \overline{K}^{*0}+\overline{K}_1^0 & \omega_S+f_{1S} & D_S^{*-}+D_{S1}^- \\ \overline{D}^{*0}+\overline{D}_1^0 & D^{*+}+D_1^+ & D_S^{*+}+D_{S1}^+ & J/\psi+\chi_{C1} \end{pmatrix}^\mu, \tag{2.4}$$

$$R^\mu=(V^a-iA^a)^\mu t^a=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}}-\frac{f_{1N}+a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+}-K_1^+ & D^{*0}-D_1^0 \\ \rho^- - a_1^- & \frac{\omega_N-\rho^0}{\sqrt{2}}-\frac{f_{1N}-a_1^0}{\sqrt{2}} & K^{*0}-K_1^0 & D^{*-}-D_1^- \\ K^{*-}-K_1^- & \overline{K}^{*0}-\overline{K}_1^0 & \omega_S-f_{1S} & D_S^{*-}-D_{S1}^- \\ \overline{D}^{*0}-\overline{D}_1^0 & D^{*+}-D_1^+ & D_S^{*+}-D_{S1}^+ & J/\psi-\chi_{C1} \end{pmatrix}^\mu. \tag{2.5}$$

which transform as $L^\mu \to U_L L^\mu U_L^\dagger$ and $R^\mu \to U_R L^\mu U_R^\dagger$under chiral transformations. This transformation properties of $\Phi$, $L^\mu$, and $R^\mu$ have been used to build the chirally invariant Lagrangian (2.1).

All mesons in our model are assigned to the physical resonances in quark-antiquark states, only the (pseudo)scalar glueballs consist of gluons, as follows:

(i) In the pseudoscalar sector $P^a$: the fields $\vec{\pi}$ and $K$ correspond to the physical pion isotriplet and the kaon isodopublet, respectively [32]. The bare fields $\eta_N \equiv \left|\bar{u}u+\bar{d}d\right\rangle/\sqrt{2}$ and $\eta_S \equiv |\bar{s}s\rangle$ are the non-strange and strange mixing contributions of the physical states $\eta$ and $\eta'$ [32] with mixing angle $\varphi \simeq -44.6^\circ$ [18, 24]:

$$\eta=\eta_N\cos\varphi+\eta_S\sin\varphi,\ \eta'=-\eta_N\sin\varphi+\eta_S\cos\varphi\,. \tag{2.6}$$

In the pseudoscalar charm sector, we have the well-established $D$ resonance, the open strange-charmed state $D_s$, and charm-anticharm state $\eta_c$ which represent $\eta_c(1S)$.

(ii) In the scalar sector $S^a$: The isotriplet $\overrightarrow{a}_0$ and the kaonic $K_0^*$ states are the physical states $a_0(1450)$ and $K_0^*(1430)$, respectivley, (the details of this assignment are given in Ref. [18]). The scalar-isoscalar states $\sigma_N \equiv \left|\bar{u}u+\bar{d}d\right\rangle/\sqrt{2}$, $\sigma_S \equiv |\bar{s}s\rangle$, and the scalar glueball $G$ mix and generate the three resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ [31] as described in the following mixing matrix:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.94 & -0.17 & 0.29 \\ 0.21 & 0.97 & -0.12 \\ -0.26 & 0.18 & 0.95 \end{pmatrix} \begin{pmatrix} \sigma_N \\ \sigma_S \\ G \end{pmatrix} . \tag{2.7}$$

The open charmed sector $D_0^*$ is assigned to the resonance $D_0^*(2400)$ whereas the strange-charm sector $D_{S0}^*$ to the $D_{S0}^*(2317)$ and the charmonium sector $\chi_{C0}$ corresponds to the ground-state charm-anticharm resonance $\chi_{C0}$.

(iii) In the vector sector $V^a$: the isotriplet filed $\overrightarrow{\rho}$, the kaonic state $K^*$, and the isoscalar states $\omega_N$ and $\omega_S$ correspond to the $\rho(770)$, $K^*(892)$, $\omega$, and $\phi$ mesons, respectively. Notice that the mixing between strange and nonstrange isoscalars is small. The charm sectors $D^{*0,\bar{0},\pm}$, $D_S^{*\pm}$, and charmonium state $J/\psi$ correspond to the open-charm sectors $D^*(2007)^0$, $D^*(2010)^\pm$, $D_S^{*\pm}$ (with mass= $2112.3 \pm 0.5$ MeV), and $J/\psi(1S)$, respectively.

(iv) In the axial-vector sector $A^a$: the isotriplet field $a_1(1260)$, the kaonic state $K_1$, the isoscalar fields $f_{1,N}$ and $f_{1,S}$, the open-charm sector $D_1$, the strange-charmed doublet $D_{S1}^\pm$ are assigned to $a_1(1260)$, $K_1(1270)$,, or $K_1(1400)$ mesons, $f_1(1285)$, $f_1(1420)$, $D_1(2420)^{0,\pm}$, and $D_{S1}(2536)^\pm$, respectively. In the end the charm-anticharm state $\chi_{C,1}$ represent the ground-state charmonium resonance $\chi_{c,1}(1P)$. For more detail of strange-nonstrange fields assignment see Ref. [18] and for open and hidden charmed fields assignment see Ref. [20].

If $m_0^2 < 0$, the Lagrangian (2.1) undergoes spontaneous symmetry breaking. To implement this breaking we have to shift the scalar-isoscalar fields $G, \sigma_N$, $\sigma_S$, and $\chi_{C0}$ by their vacuum expectation values $G_0$, $\phi_N$, $\phi_S$, and $\phi_C$ [17, 20]

$$\begin{aligned} G \to G + G_0,\ \sigma_N \to \sigma_N + \phi_N \,, \\ \sigma_S \to \sigma_S + \phi_S \,,\ \chi_{C0} \to \chi_{C0} + \phi_C \,. \end{aligned} \tag{2.8}$$

All the parameters in the Lagrangian (2.1) have been fixed in the case of $N_f = 3$, see Ref.[18] for more details. The three additional parameter related to the charm sector ($\varepsilon_C$, $\delta_C$, and $\phi_C$), in the case of $N_f = 4$, have been determined in the Ref. [20]. The parameters $\lambda_1$ and $h_1$ are determined in Ref. [21], which are related to the OZI-suppressed decays of the charmonium state $\chi_{c0}$. The mixing between the hidden-charmed scalar meson $\chi_{c0}$ and the scalar glueball $G$ is neglected because it is small.

## 3. Results

In this section, we present the results of the decay widths of the scalar charmonium state $\chi_{C0}$. The two- and three-body decays of $\chi_{c0}$ into scalar and pseudoscalar mesons are summarized in the Table 1 and into scalar glueballs and scalar mesons are reported in Table II. In addition, the two- and three-body decays of $\chi_{c0}$ into (axial-)vector and (pseudo)scalar mesons are reported in Table III.

| Decay Channel | theoretical result [MeV] | Experimental result [MeV] |
|---|---|---|
| $\Gamma_{\chi_{c0}\to\bar{K}_0^{*0}K_0^{*0}}$ | 0.010±0.003 | $0.010\,^{+0.004}_{-0.003}$ |
| $\Gamma_{\chi_{c0}\to K^-K^+}$ | 0.059±0.008 | 0.062±0.005 |
| $\Gamma_{\chi_{c0}\to\pi\pi}$ | 0.090±0.011 | 0.087±0.006 |
| $\Gamma_{\chi_{c0}\to\bar{K}^{*0}K^{*0}}$ | 0.014±0.007 | 0.018±0.006 |
| $\Gamma_{\chi_{c0}\to\omega\omega}$ | 0.012±0.006 | 0.010±0.001 |
| $\Gamma_{\chi_{c0}\to\phi\phi}$ | 0.0035±0.0036 | 0.0081±0.0009 |
| $\Gamma_{\chi_{c0}\to\eta\eta}$ | 0.022±0.002 | 0.031±0.003 |
| $\Gamma_{\chi_{c0}\to\eta'\eta'}$ | 0.021±0.001 | 0.021±0.002 |
| $\Gamma_{\eta_c\to\eta\pi^-\pi^+}$ | 0.12±0.02 | 0.54±0.16 |
| $\Gamma_{\eta_c\to\eta'\pi\pi}$ | 0.081±0.019 | 1.30±0.54 |

**Table 1:** The decay widths of $\chi_{c0}$ into (pseudo)scalar mesons.

| Decay Channel | theoretical result [MeV] | Experimental result [MeV] |
|---|---|---|
| $\Gamma_{\chi_{c0}\to f_0(1370)f_0(1370)}$ | $5.10^{-3}$ | $<3.10^{-3}$ |
| $\Gamma_{\chi_{c0}\to f_0(1500)f_0(1500)}$ | $4.10^{-3}$ | $<5.10^{-4}$ |
| $\Gamma_{\chi_{c0}\to f_0(1370)f_0(1500)}$ | $2.10^{-6}$ | $<1.10^{-3}$ |
| $\Gamma_{\chi_{c0}\to f_0(1370)f_0(1710)}$ | $1.10^{-4}$ | 0.0069±0.004 |
| $\Gamma_{\chi_{c0}\to f_0(1500)f_0(1710)}$ | $2.10^{-5}$ | $<7.10^{-4}$ |
| $\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta}$ | $4.10^{-4}$ | - |
| $\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta}$ | $3.10^{-3}$ | - |
| $\Gamma_{\chi_{c0}\to f_0(1370)\eta'\eta'}$ | $27.10^{-4}$ | - |
| $\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta'}$ | $89.10^{-6}$ | - |
| $\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta'}$ | $11.10^{-3}$ | - |
| $\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta}$ | $8.10^{-5}$ | - |
| $\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta'}$ | $3.10^{-5}$ | - |

**Table 2:** The partial decays of $\chi_{c0}$ into scalar glueball and mesons.

## 4. Conclusion

In the present work, we have represented a chirally invariant linear sigma model with (axial-)vector mesons in four-flavor case, $N_f = 4$, by including a dilaton field, a scalar glueball field, and describing the interaction of the pseudoscalar glueball with (pseudo-)scalar mesons. We have calculated the decay widths of the scalar charmonium ground state $\chi_{c0}$ into two- and three strange and nonstrange mesons (Table I and Table III) as well as into scalar mesons and a scalar glueball $G$ which is a mix of two resonances $f_0(1370)$ and $f_0(1500)$ and predominantly to be $f_0(1710)$ (Table II). Notice that the decays of charmonium states into the open charmed mesons are forbidden in the eLSM as the same outcomes in Ref. [33]. The parameters were determined in Refs.[18, 20, 21].

We have found that the extended linear sigma model (2.1) has not any decay channels for the (axial-)vector charmonium states where $\Gamma_{J/\psi} = 0$ and $\Gamma_{\chi_{c1}} = 0$. The hadronic decays of the ground-state pseudoscalar charmonium $\eta_c(IP)$ have been studied in Ref. [21]. The results of the decay



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| Decay Channel | theoretical result [MeV] | Experimental result [MeV] |
|---|---|---|
| $\Gamma_{\chi_{c0}\to a_0 a_0}$ | 0.004±0.0019 | - |
| $\Gamma_{\chi_{c0}\to K^* \overline{K}_0^*}$ | 0.00007±0.000049 | - |
| $\Gamma_{\chi_{c0}\to \rho\rho}$ | 0.01±0.006 | - |
| $\Gamma_{\chi_{c0}\to \eta\eta'}$ | 0.0012±0.0005 | <0.0024 |
| $\Gamma_{\chi_{c0}\to K_0^* K\eta}$ | 0.0004±0.00015 | - |
| $\Gamma_{\chi_{c0}\to K_0^* K\eta'}$ | 0.00021±0.00013 | - |

**Table 3:** The decays of $\chi_{c0}$ into light mesons.

widths of $\chi_{c0}$ and $\eta_c$ are in good agreement with the experiment [32] which highlights to notice that how much the eLSM is a successful model to study the phenomenology of hidden charmed meson and the phenomenology of open charmed mesons as seen in the Refs.[20].
The results of decay widths are reasonably compatible with the experimental data where available. On the other hand, the predictions for as yet unmeasured channels are potentially interesting for Belle II, BESIII, LHCb as well as the upcoming PANDA experiment at the FAIR facility.

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