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Neutrino Oscillation in Dense Matter

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As the increasing of neutrino energy or matter density, the neutrino oscillation in matter may undergo "vacuum-dominated", "resonance" and "matter-dominated" three different stages successively. Neutrinos endure very different matter effects, and therefore present very different oscillation behaviors in these three different cases. In this talk, we focus on the less discussed matter-dominated case (i.e., $|A_{\rm CC}| \gg |\Delta m^2_{31}|$), show that as the matter parameter $|A_{\rm CC}|$ growing larger, the effective mixing matrix in matter $\tilde{V}$ evolves approaching a fixed 3×3 constant real matrix which is free of CP violation and can be described using only one simple mixing angle $\tilde{\theta}$ which is independent of $A_{\rm CC}$. As for the neutrino oscillation behavior, ν_e decoupled in the matter-dominated case due to its intense charged-current interaction with electrons while a two-flavor oscillation are still presented between ν_μ and ν_τ. At the end of this talk, we discussed the oscillation probabilities when neutrinos/anti-neutrinos passing through a typical white dwarf to give some embryo thoughts on under what circumstances these studies could be applied and put forward the interesting idea of possible "neutrino lensing" effect.

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1. Introduction

When neutrinos pass through a medium, the interactions with the particles in the background give rise to modifications of the properties of neutrinos as well as the oscillation behaviors. This is well known as the matter effect which have been playing important roles in understanding various neutrino oscillation data. In the standard three neutrinos framework, the effective Hamiltonian $\tilde{\mathcal{H}}$ in the flavor basis responsible for the propagation of neutrinos in matter, differs from the Hamiltonian in vacuum $\mathcal{H}$,

$$\tilde{\mathcal{H}} \;=\; \mathcal{H} + \mathcal{H}' \;=\; \frac{1}{2E}\left[\left(m_1^2 + A_{\rm NC}\right)\cdot \mathbb{1} + V\begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31}\end{pmatrix}V^\dagger + \begin{pmatrix} A_{\rm CC} & & \\ & 0 & \\ & & 0\end{pmatrix}\right] , \quad (1)$$

where $\mathcal{H}'$ describes the forward coherent scattering of neutrinos with the constituents of the medium (i.e., electrons, protons and neutrons) via the tree level weak charged-current (CC) and neutral-current (NC) interactions [1–4]. Here $A_{\rm CC} = 2EV_{\rm CC}$, $A_{\rm NC} = 2EV_{\rm NC}$ are parameters of the same unit as the mass-squared difference Δm^2_{ji} that measure the strength of the matter effect. For anti-neutrino oscillation in matter, one may simply replace V by V^* and $A_{\rm CC}$ by $-A_{\rm CC}$ (i.e., $A_{\rm CC}$ is negative in the case of anti-neutrino oscillation). The intriguing matter effect is a result of the interplay between the vacuum Hamiltonian $\mathcal{H}$ and the matter term $\mathcal{H}'$.

According to the relative magnitude of Δm^2_{21}, $|\Delta m^2_{31}|$ and $|A_{\rm CC}|$, the various possible values of $A_{\rm CC}$ can be laid in three main different regions: **the vacuum-dominated region** ($|A_{\rm CC}| \ll \Delta m^2_{21},\ |\Delta m^2_{31}|$), **the resonance region** ($A_{\rm CC} \sim \Delta m^2_{21},\ \Delta m^2_{31}$)and **the matter-dominated region** ($|A_{\rm CC}| \gg \Delta m^2_{21},\ |\Delta m^2_{31}|$). Recently interests have been shown in exploring the less discussed matter-dominated case [5–9]. Such studies are applicable in the case of neutrinos having extremely high energy or going through extremely dense object. Further to these works, we explore in this paper the effective neutrino mass and mixing parameters as well as the neutrino oscillation probabilities in dense matter using the perturbation theory.

2. Neutrino oscillation in the matter-dominated case

The effective neutrino masses $\tilde{m}_i$ and flavor mixing matrix $\tilde{V}$ in mater can be defined by diagonalizing the effective Hamiltonian $\tilde{\mathcal{H}} \equiv \frac{1}{2E}\tilde{V}\ \mathrm{Diag}\left\{\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2\right\}\tilde{V}^\dagger$. The exact analytical relations between $\{\tilde{V},\ \tilde{m}_i\}$ and $\{V,\ m_i\}$ have been established in many works using different approaches [10–17]. For any realistic profile of the matter density, it is also possible to numerically calculate the neutrino oscillation probabilities. However, in the matter-dominated region we are concerning, some useful and more transparent analytical approximations could be obtained by regarding both $\Delta m^2_{21}/|A_{\rm CC}|$ and $|\Delta m^2_{31}/A_{\rm CC}|$ as small parameters and performing the diagonalization of $\tilde{\mathcal{H}}$ using the perturbation theory. For the detailed results of the diagonalization, one may refer to reference [18].

As the increase of $|A_{\rm CC}|$, terms proportional to $1/A_{\rm CC}$ are all approaching zero, three eigenvalues of $\tilde{\mathcal{H}}$ are approaching a set of fixed values, which is straightforwardly given in the zeroth order expansion,

$$\begin{aligned}
\tilde{\lambda}_1^{fixed} &= \frac{1}{2E}\left(m_1^2 + A_{\rm NC} + A_{\rm CC} + \Omega_{11}\right) , \\
\tilde{\lambda}_2^{fixed} &= \frac{1}{2E}\left(m_1^2 + A_{\rm NC} + \Omega_{22}\cos^2\tilde{\theta} + \Omega_{33}\sin^2\tilde{\theta} - |\Omega_{23}|\sin 2\tilde{\theta}\right) , \\
\tilde{\lambda}_3^{fixed} &= \frac{1}{2E}\left(m_1^2 + A_{\rm NC} + \Omega_{33}\cos^2\tilde{\theta} + \Omega_{22}\sin^2\tilde{\theta} + |\Omega_{23}|\sin 2\tilde{\theta}\right) . \qquad (2)
\end{aligned}$$

Here the Hermitian matrix Ω is defined as $\Omega \equiv V\ {\rm Diag}\left\{0, \Delta m_{21}^2, \Delta m_{31}^2\right\} V^\dagger$. Apparently, in this matter-dominated case, $\tilde{\lambda}_2^{fixed}$ and $\tilde{\lambda}_3^{fixed}$ are nearly degenerate and both of them have strong hierarchies with $\tilde{\lambda}_1^{fixed}$. In the same time the effective mixing matrix in matter $\tilde{V}$ evolves towards a fixed 3×3 real matrix

$$\tilde{V}^{fixed} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\tilde{\theta} & \sin\tilde{\theta} \\ 0 & -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} , \qquad (3)$$

which has the two-flavor-mixing structure and therefore can be parametrized using just one mixing angle $\tilde{\theta}$ defined by

$$\tan 2\tilde{\theta} = \frac{2\left|\Delta m_{21}^2 V_{\mu 2} V_{\tau 2}^* + \Delta m_{31}^2 V_{\mu 3} V_{\tau 3}^*\right|}{\Delta m_{21}^2\left(|V_{\tau 2}|^2 - |V_{\mu 2}|^2\right) + \Delta m_{31}^2\left(|V_{\tau 3}|^2 - |V_{\mu 3}|^2\right)} . \qquad (4)$$

It means $\tilde{V}$ asymptotically conserves intrinsic CP and the effective Jarlskog $\tilde{\mathcal{J}}$ in matter is vanishing in the matter-dominated case.

In the case the matter density can be regarded as a constant along the path neutrinos propagate, the neutrino/anti-neutrino oscillation probabilities in the matter-dominated region can be concisely expressed as

$$\begin{aligned}
\tilde{P}(\nu_e \to \nu_e) &\approx \tilde{P}(\bar{\nu}_e \to \bar{\nu}_e) \approx 1 , \\
\tilde{P}(\nu_e \to \nu_\mu) &\approx \tilde{P}(\bar{\nu}_e \to \bar{\nu}_\mu) \approx 0 , \\
\tilde{P}(\nu_e \to \nu_\tau) &\approx \tilde{P}(\bar{\nu}_e \to \bar{\nu}_\tau) \approx 0 , \\
\tilde{P}(\nu_\mu \to \nu_e) &\approx \tilde{P}(\bar{\nu}_\mu \to \bar{\nu}_e) \approx 0 , \\
\tilde{P}(\nu_\mu \to \nu_\mu) &\approx \tilde{P}(\bar{\nu}_\mu \to \bar{\nu}_\mu) \approx 1 - \sin^2 2\tilde{\theta}\sin^2\frac{\Delta\tilde{m}_{32}^2 L}{4E} , \\
\tilde{P}(\nu_\mu \to \nu_\tau) &\approx \tilde{P}(\bar{\nu}_\mu \to \bar{\nu}_\tau) \approx \sin^2 2\tilde{\theta}\sin^2\frac{\Delta\tilde{m}_{32}^2 L}{4E} , \\
\tilde{P}(\nu_\tau \to \nu_e) &\approx \tilde{P}(\bar{\nu}_\tau \to \bar{\nu}_e) \approx 0 , \\
\tilde{P}(\nu_\tau \to \nu_\mu) &\approx \tilde{P}(\bar{\nu}_\tau \to \bar{\nu}_\mu) \approx \sin^2 2\tilde{\theta}\sin^2\frac{\Delta\tilde{m}_{32}^2 L}{4E} , \\
\tilde{P}(\nu_\tau \to \nu_\tau) &\approx \tilde{P}(\bar{\nu}_\tau \to \bar{\nu}_\tau) \approx 1 - \sin^2 2\tilde{\theta}\sin^2\frac{\Delta\tilde{m}_{32}^2 L}{4E} , \qquad (5)
\end{aligned}$$

where

$$\Delta\tilde{m}_{32}^2 \approx \left[\Delta m_{21}^2\left(|V_{\tau 2}|^2 - |V_{\mu 2}|^2\right) + \Delta m_{31}^2\left(|V_{\tau 3}|^2 - |V_{\mu 3}|^2\right)\right]\cos 2\tilde{\theta} . \qquad (6)$$

Here $\Delta\tilde{m}^2_{32}$ has the same sign as Δm^2_{31}.

One may immediately find that $\tilde{P}(\nu_\alpha \to \nu_\beta)$ and $\tilde{P}(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ approach to a same set of fixed values in the limit $|A_{\rm CC}| \to \infty$ [1]. These analytical approximations give us a clear picture of neutrino oscillation in the matter-dominated region: ν_e ($\bar{\nu}_e$) are decoupled (due to its intense charged-current interaction with electrons in the medium), while oscillation can still happened between ν_μ ($\bar{\nu}_\mu$) and ν_τ ($\bar{\nu}_\tau$) [2]. This two-flavor oscillation can be described by one effective mixing angle $\tilde{\theta}$ and one effective mass-squared difference $\Delta\tilde{m}^2_{32}$ whose expressions are given in Eqs. (4) and (6) respectively. Taking into account the strong hierarchy of $\Delta m^2_{21} \ll |\Delta m^2_{31}|$ and the smallness of s_{13}, we can then obtain $\Delta\tilde{m}^2_{32} \approx \Delta m^2_{32}$ (or Δm^2_{31}) together with $\tilde{\theta} \approx \theta_{23}$. Note that, both the oscillation parameters are independent of $A_{\rm CC}$ and can be easily calculated once the neutrino oscillation parameters in vacuum are well determined. It means as long as the "matter-dominated" condition is satisfied, above simple formulas are applicable no matter how the matter density varies along the path, and the resulting conversion probability between ν_μ and ν_τ is just a simple function of L/E.

3. An application and outlook

It is interesting to ask under what circumstances these studies of neutrino oscillation in dense matter could be applied. Here we bring our embryo thoughts by considering the oscillation probabilities of neutrinos passing through a typical white dwarf (with $M \sim 0.7 M_\odot$, $R \sim 10^4$ km and $\rho \sim 2 \times 10^6$ g/cm^3). The white dwarf is very dense therefore can give rise to significant matter effect, but at the same time the material in it no longer undergoes fusion reactions which means it does not radiate large amount of neutrinos on its own. For neutrinos with energy $E \lesssim 10$ MeV, the white dwarf can be approximately regarded as transparent. While if neutrinos with energy higher than 10 MeV are taken into account, the attenuation need to be carefully studied.

Due to the extremely high density of the white dwarf, neutrino oscillation experiences the resonances and then enter the matter-dominated region at very low energies (below MeV). One can clearly see from Fig. 1 that at around $E \sim 0.4$ keV (solar resonance) the oscillation probabilities start to markedly differ from the vacuum oscillation probabilities and change towards their fixed points. For neutrino oscillation in the normal mass ordering case or anti-neutrino oscillation in the inverted mass ordering case, there is a significant resonance hump at around $E \sim 20$ keV (atmospheric resonance). In the energy range shown in these two figures, L/E is extremely large, the oscillatory frequencies are all extremely high, therefore only the average oscillatory magnitude can be observed, which is a constant and is markedly different from the vacuum probabilities in the matter-dominated region. Noting that all these interesting features of the probabilities we discussed above will finally be embodied in the neutrino/anti-neutrino spectrum we observed. The signal of a change of the slope and a subsequent hump could be very helpful to uncover the matter effect of these compact objects.

We are now observing neutrinos with a broad range of energies from distant objects using varieties of neutrino detectors. If there happen to be a compact object sitting in between the source and the observer, this compact object can not only bend the light and produce the gravitational lensing effect, but also "lens" the neutrinos from the source by distorting its spectrum. But

[1] This is in agreement with the vanishing of CP-violation in the limit $|A_{\rm CC}| \to \infty$.

[2] This is in agreement with the near degeneracy of $\tilde{\lambda}_2$ and $\tilde{\lambda}_3$ in the limit $|A_{\rm CC}| \to \infty$.

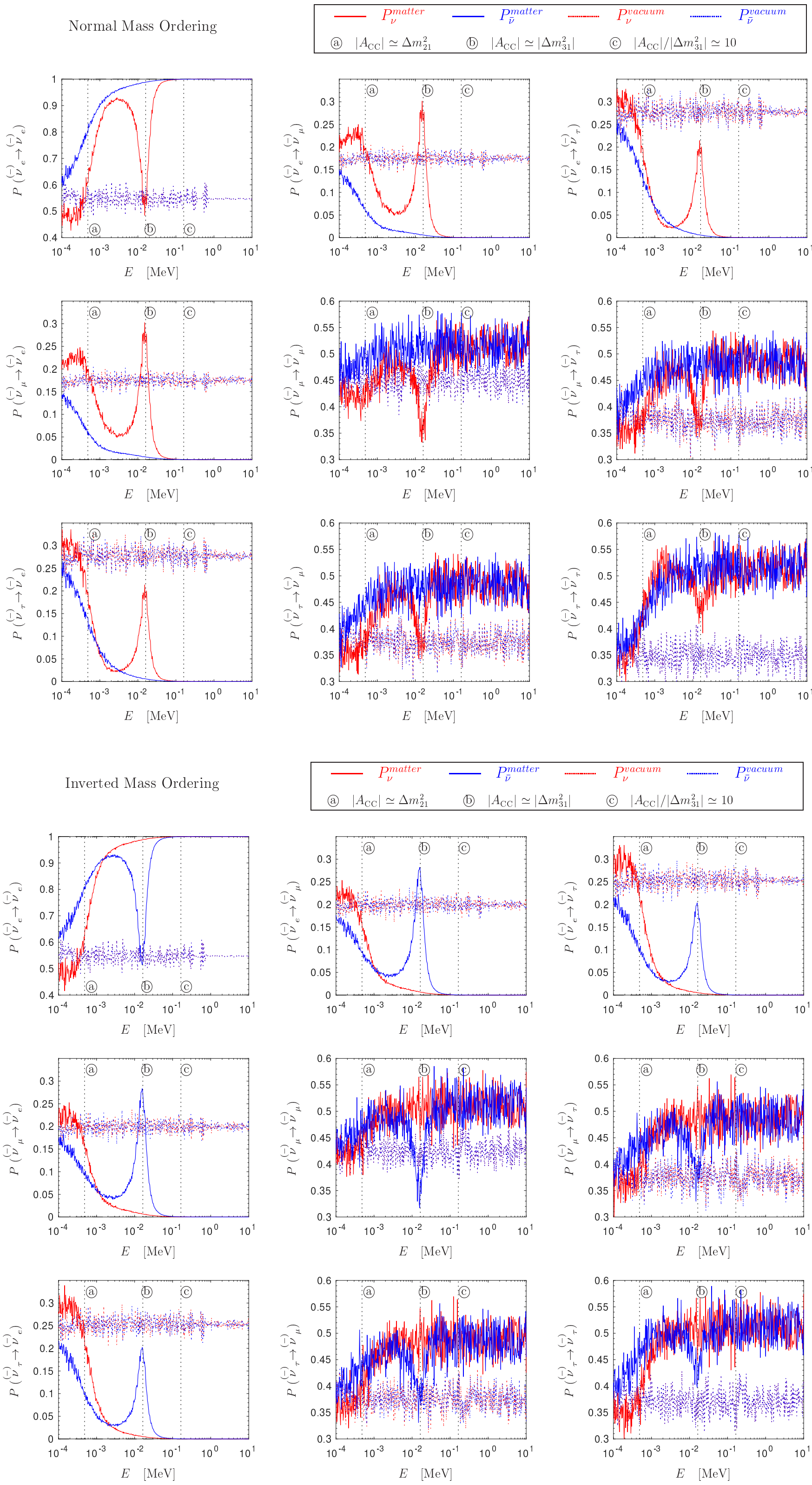

Figure 1: The comparison of the oscillation probabilities with or without the matter effect as a neutrino (antineutrino) beam of energy E go through a typical white dwarf along its diameter, where the normal neutrino mass ordering and the inverted neutrino mass ordering are assumed respectively. All the probabilities are averaged over a Gaussian energy resolution of 5%.

different from the gravitational lensing effect which is capable of uncovering the mass distribution in our universe, this “neutrino lensing” effect could be sensitive to the distribution of electrons (or positrons) in the space.

Of course, the discussion so far is just an immature and inaccurate thought. Lots of details such as the spectrum and the flavor composition of the neutrino source, the properties of the compact objects and their distribution in the space, the capability of the detector have to be carefully studied before we can finally draw the conclusion if this kind of “neutrino lensing” effect can be actually observed. However, it is worthwhile to concentrate more efforts on this topic, for it may open a new window to the universe via the weak interaction of neutrinos. With the improving of the detector capabilities and the data analysis techniques, it is possible to site experiments some day to located the hidden compact objects in the space via this “neutrino lensing” effect.

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