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Green Functions of Chiral Currents within OPE

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We present new results on contributions of the QCD condensates to the two-point and the three-point Green functions of chiral currents, calculated within the means of the operator product expansion (OPE). Further, for the Green functions of the odd-intrinsic parity sector of QCD, we show up-to-date knowledge of behavior of the matching between the calculations performed in the resonance chiral theory and OPE. This matching, however, as complicated as it is, can lead to important constraints on the coupling constants of the resonance Lagrangian, relevant in the odd-sector of QCD.

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1. Introduction

Currently, Quantum chromodynamics (QCD) is believed to be the correct fundamental theory of strong interactions in terms of quarks and gluons. However, this perturbative approach fails in the low-energy region of the hadronic spectrum, i.e. for energies less than 2 GeV, where QCD becomes non-perturbative.

At low energies, a possible course of action is to use an effective field theory that would be built upon the relevant degrees of freedom, which are the mesons and the baryons. However, the situation here is not that simple, since such a theory in the full-energy region is not known from the first principles. Nevertheless, in the region of energies typically less than M_ρ, with M_ρ being the mass of the $\rho(770)$ meson, we have an effective field theory of QCD, the Chiral perturbation theory (χPT) [1–4].

Inspired by the large-N_c limit, we can construct the effective theory for an intermediate energy region that also satisfies all symmetries of the underlying theory. This effective theory, Resonance chiral theory (RχT), is relevant for energies within the bounds of $M_\rho \le E \le 2\,\mathrm{GeV}$ [5, 6], in which the dynamics is not yet affected by the higher meson states. RχT increases the number of degrees of freedom of Chiral perturbation theory by including massive U(3) multiplets of vector $V(1^{--})$, axial-vector $A(1^{++})$, scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances. Interactions within these types of channels can be studied by the means of the Green functions of the chiral currents that represent a powerful tool in order to extract physical observables of the theory.

The phenomenological Lagrangian approach based on large-N_c and the chiral symmetry was first introduced in 1989 in [5] and was further developed and enlarged both for the even-parity and the odd-parity sector, see [6] and [7] (for an extended list of references see [8]).

2. Green Functions of Chiral Currents

The Green functions are defined as the vacuum expectation values of the time ordered products of the composite local operators. The standard definition of the n-point correlator reads[1]

$$\begin{aligned}&\Pi_{O_1\ldots O_n}(p_1,\ldots,p_{n-1};p_n)\\ &\quad= \int \mathrm{d}^4x_1\ldots\mathrm{d}^4x_{n-1}\, e^{-i(p_1\cdot x_1+\ldots+p_{n-1}\cdot x_{n-1})}\left\langle 0\middle|\mathrm{T}\, O_1(x_1)\ldots O_{n-1}(x_{n-1})O_n(0)\middle|0\right\rangle, \end{aligned} \tag{1}$$

with T being the time-ordering and where we have suppressed all the indices for simplicity. Due to the translation invariance, we have set the coordinate of the n^{th} operator into the origin.

In our case, we consider the composite local operators in (1) to be represented by the octets of the chiral scalar $S^a(x) = \overline{q}(x)T^a q(x)$ and pseudoscalar $P^a(x) = i\overline{q}(x)\gamma_5 T^a q(x)$ densities, and the vector $V^a_\mu(x) = \overline{q}(x)\gamma_\mu T^a q(x)$ and axial-vector $A^a_\mu(x) = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x)$ currents, where q stands for the triplet of the lightest quarks and $T^a = \frac{1}{2}\lambda^a$ are the SU(3) generators, with λ^a being the Gell-Mann matrices ($a = 1,\ldots,8$).

Due to the Lorentz covariance and invariance of QCD with respect to parity and/or time reversal, 15 three-point Green functions of chiral currents exist in total. It is useful to introduce a

[1] In our case, the symbol $|0\rangle$ stands for the nonperturbative QCD vacuum. For clarity, however, we will omit showing the vacuum state from now on, i.e. we symbolically define $\langle 0|\bullet|0\rangle \equiv \langle\bullet\rangle$.

simple division of these correlators into two sets in such a way that the correlators of the specific set have, in the chiral limit, nonvanishing contribution from the same QCD condensates. The classification is as follows:

- Set 1: The correlators with the perturbative contribution in the chiral limit:
  - $\langle ASP\rangle$, $\langle VSS\rangle$, $\langle VPP\rangle$, $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle AAV\rangle$, $\langle VVV\rangle$.
- Set 2: The correlators that are the order parameters of the chiral symmetry breaking in the chiral limit:
  - $\langle SSS\rangle$, $\langle SPP\rangle$, $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVS\rangle$, $\langle AAS\rangle$, $\langle VAP\rangle$.

3. Operator Product Expansion and QCD Condensates

At high energies, the asymptotic freedom allows us to use the perturbative approach in terms of the strong coupling constant α_s and the asymptotics of the Green functions of chiral currents for large euclidean momenta is given by the operator product expansion (OPE) [9]. This framework allows us to rewrite the respective correlators as a sum of vacuum expectation values of gauge-invariant scalar local operators, made of quark and gluon fields, with c-number Wilson coefficients, which we call the QCD condensates. These are purely nonperturbative parameters and their numerical values can not be calculated directly from the first principles.

Considering the OPE of the Green functions in terms of the QCD condensates up to dimension 6 without derivative terms, an arbitrary three-point correlator in massless theory can be written down as follows:

$$\Pi_{O_1^a O_2^b O_3^c}(\lambda p, \lambda q; \lambda r) \xrightarrow{\lambda\to\infty} \lambda\, C^{1}_{O_1^a O_2^b O_3^c}(p,q;r) + \frac{\langle\overline{q}q\rangle}{\lambda^2} C^{\langle\overline{q}q\rangle}_{O_1^a O_2^b O_3^c}(p,q;r)$$
$$+ \frac{\langle G^2\rangle}{\lambda^3} C^{\langle G^2\rangle}_{O_1^a O_2^b O_3^c}(p,q;r) + \frac{\langle\overline{q}\sigma\cdot Gq\rangle}{\lambda^4} C^{\langle\overline{q}\sigma\cdot Gq\rangle}_{O_1^a O_2^b O_3^c}(p,q;r) + \frac{\langle\overline{q}q\rangle^2}{\lambda^5} C^{\langle\overline{q}q\rangle^2}_{O_1^a O_2^b O_3^c}(p,q;r) + \dots , \quad (2)$$

where the first term corresponds to the perturbative contribution ($D = 0$) and the subsequent ones stand for the contributions of the quark ($D = 3$), gluon ($D = 4$), quark-gluon ($D = 5$) and four-quark ($D = 6$) condensates, with the respective canonical dimension shown in the bracket.[2] So far, we have assumed the dominance of an intermediate vacuum states in the large N_c limit for the four-quark condensate and we have also neglected the contributions of both the three-gluon condensate ($D = 6$) and the higher-dimensional ($D > 6$) QCD condensates.

The Wilson coefficients, denoted generally as $C^{\mathcal{I}}_{O_1^a O_2^b O_3^c}(p,q;r)$ in (2), contain all the information about short-distance physics and are calculable in perturbative QCD by means of the technique of Feynman diagrams. They are labeled by the upper index $\mathcal{I} = 1, \langle\overline{q}q\rangle, \dots$, symbolizing which QCD condensate contribution they belong to.

[2]For clarity, we denoted $\langle G^2\rangle \equiv \langle G^a_{\mu\nu}G^{\mu\nu\,a}\rangle$ and $\langle\overline{q}\sigma\cdot Gq\rangle \equiv \langle\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle$, with $G_{\mu\nu} = G^a_{\mu\nu}T^a$ being the gluon field strength tensor.

4. Propagation of Nonlocal Condensates

The QCD condensates are vacuum expectation values of gauge-invariant scalar local operators. While calculating the contributions of the QCD condensates to the respective Green functions by the means of the corresponding Feynman diagrams, one does not obtain the local condensate in the calculations immediately since the quark or gluon fields are, generally, in different space-time points. To this end, the Fock-Schwinger gauge [10, 11] is a suitable framework the calculations can be performed in. Indeed, this choice of gauge allows us to simply expand the quark and gluon fields into the Taylor series around the origin, and then to project out the Lorentz structure and form the local condensate.

Using the Fock-Schwinger gauge, one can thus obtain expressions that convert the nonlocal condensates into local ones. In our case, we have derived the following propagation formulas for the nonlocal quark, gluon and quark-gluon condensates, that gives us contributions of the local QCD condensates [8]:[3]

$$\begin{aligned}\left\langle \overline{q}^A_{i,\alpha}(x) q^B_{k,\beta}(y)\right\rangle = &\left(\frac{\langle \overline{q}q\rangle}{2^2\cdot 3^2}\delta_{ik} - \frac{g_s\langle \overline{q}\sigma\cdot Gq\rangle}{2^5\cdot 3^2}\left[F^{\langle\overline{q}q\rangle}(x,y)\right]_{ki}\right.\\ &\left. + \frac{i\pi\alpha_s\langle\overline{q}q\rangle^2}{2^3\cdot 3^7}\left[G^{\langle\overline{q}q\rangle}(x,y)\right]_{ki} + \ldots\right)\delta_{\alpha\beta}\delta^{AB}\,,\end{aligned} \tag{3}$$

$$\alpha_s\left\langle \mathcal{A}^a_\mu(x)\mathcal{A}^b_\nu(y)\right\rangle = \frac{\alpha_s\langle G^2\rangle}{2^7\cdot 3}H^{\langle G^2\rangle}_{\mu\nu}(x,y)\delta^{ab} + \ldots\,, \tag{4}$$

$$\begin{aligned}g_s\left\langle \overline{q}^A_{i,\alpha}(x)\mathcal{A}^a_\mu(u) q^B_{k,\beta}(y)\right\rangle = &\left(\frac{g_s\langle \overline{q}\sigma\cdot Gq\rangle}{2^7\cdot 3^2}\left[F^{\langle\overline{q}\mathcal{A}q\rangle}_\mu(x,u,y)\right]_{ki}\right.\\ &\left. + \frac{\pi\alpha_s\langle\overline{q}q\rangle^2}{2^3\cdot 3^5}\left[G^{\langle\overline{q}\mathcal{A}q\rangle}_\mu(x,u,y)\right]_{ki} + \ldots\right)(T^a)_{\beta\alpha}\delta^{AB}\,,\end{aligned} \tag{5}$$

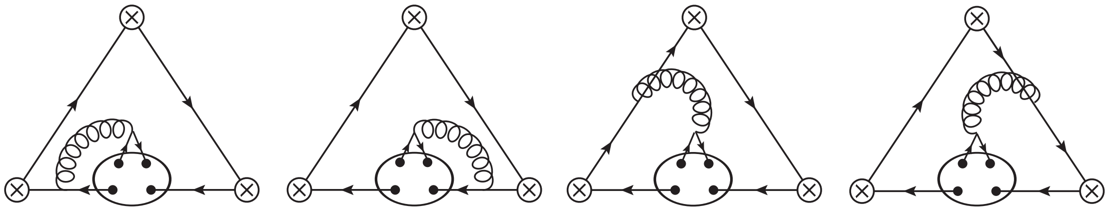
where $\mathcal{A}^a_\mu$ denotes the gluon field and the individual F and G functions being defined in Ref. [8]

We note that some parts of the propagation formulas (3)-(5) have been already presented in the literature, however, in most cases only for one of the coordinates to set into origin. See for example [12, 13]. Also, to the best of our knowledge, the OPE has been studied extensively only for some of the Green functions and mainly only with the emphasis on the quark condensate (see e.g. [14] and [15]). To this end, we have recalculated some of the known contributions independently, while the remaining contributions in this paper were calculated for the first time here.

The main result of our paper [8] rests in the fact that we have calculated not only the perturbative contribution and the contribution of the quark condensate to all the three-point Green functions, but also the contributions of the gluon, quark-gluon and four-quark condensates.

An example of Feynman diagrams giving arise to the contribution of the four-quark condensate to the three-point correlators is shown at Fig. 1. In this context, a note is in order. A calculation of these diagrams in the momentum representation is performed using the Fourier transform of the original result in the coordinate representation. Although the Fourier transform of the individual

[3]The notation regarding the indices in (3)-(5) is as follows. Spinor indices are denoted as small Latin letters (i, k). Color indices are denoted as small Greek letters (α, β). Flavor indices corresponding to the fundamental representation of the flavor group are denoted as capital Latin letters (A, B). SU(3) index of the adjoint representation is denoted as small Latin letter (a).



(a) (b) (c) (d)

Figure 1: Feynman diagrams of the contributions of the four-quark condensate to the three-point Green functions due to the effective propagation of the nonlocal quark condensate (1(a) and 1(b)) and the nonlocal quark-gluon condensate (1(c) and 1(d)). For details, see Ref. [8].

terms leads to the presence of the logarithmic terms, all logarithms cancel each other out completely after summing up all the diagrams, and we are left with a simple rational result. This can be of course expected since these contributions are given by tree-level diagrams, however, the fulfilment of such property serves as a nontrivial check of consistency of our calculations, besides the satisfaction of the respective Ward identities.

5. Matching RχT-OPE: Odd-intrinsic parity sector

Among the 15 relevant Green functions of chiral currents, there exist 5 correlators that belong to the odd-intrinsic parity sector of QCD: $\langle VVP\rangle$, $\langle VAS\rangle$, $\langle AAP\rangle$ and the anomalous pair of $\langle VVA\rangle$ and $\langle AAA\rangle$. Our present effort [16] is to try to build upon the Ref. [7], where the first three Green functions were calculated by the means of the resonance Lagrangian $\mathcal{L}_{R\chi T}^{6,\,\mathrm{odd}}$ at $O(p^6)$ and subsequently compared to the respective OPE prescription, which led to prediction of some constraints on the relevant coupling constants of the resonance Lagrangian.

In fact, such calculations by the means of $\mathcal{L}_{R\chi T}^{6,\,\mathrm{odd}}$ were performed both for $\langle VVA\rangle$ and $\langle AAA\rangle$, and have been presented by the author several times already. However, the situation for these correlators seems to be a bit more complicated, since their OPE starts with the perturbative contribution, that carries logarithmic terms as well that must be matched somehow onto the respective RχT contributions. However, there is no way to generate logarithmic terms with finite number of resonances and, therefore, the matching RχT-OPE thus must be performed in a delicate way.

As we have stated above, the topic of this matching for the $\langle VVA\rangle$ and $\langle AAA\rangle$ Green functions is an ongoing work which we will cover in detail in our forthcoming work [16].

6. Summary

In this contribution we have presented the up-to-date survey of leading order contributions of the QCD condensates up to dimension six to all three-point Green functions of the chiral currents and densities within the framework of the operator product expansion due to the effective propagation of the nonlocal quark, gluon and quark-gluon condensates. We have also discussed shortly the prospects of the matching between the OPE and contributions of the RχT at $O(p^6)$.

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