Transverse momentum dependence of fragmentation functions from Belle data

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In this paper, we present new sets of transverse momentum dependence of fragmentation functions (TMD FFs) along with their uncertainties. We include the most recent TMD production cross sections of $\pi^\pm$, $K^\pm$ and $p/\bar{p}$ measured in single inclusive $e^+e^-$ annihilation by Belle Collaboration. In this analysis, the common Gaussian distribution is applied for transverse momentum dependence ($P_{HT}$) of the cross section. Our theory calculations over a wide range of transverse momentum $P_{HT}$ are in good agreement with Belle experimental data analyzed in this study.
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1. Introduction

In Quantum ChromoDynamic (QCD), the Fragmentation Functions (FFs) is one of the essential ingredients for the theory predictions in Semi Inclusive Deep Inelastic scattering (SIDIS), Single Inclusive electron-positron annihilation (SIA) and hadron-hadron collisions [1]. The Transverse momentum dependent (TMD) FFs provides deeper insight on the hadronization processes, where hadron \( H \) carries the \( z \) fraction of fragmenting parton’s momentum and also depends on the transverse momentum of hadron \( P_{HT} \). One of the hard processes which is commonly used to determine the TMD FFs is single or double inclusive \( e^+ e^- \) annihilation [2–5]. Most recently, the Belle collaboration has provided the first measurements of cross sections for pions, kaons and protons productions in SIA process at 10.58 GeV [6]. In this work, we present the first unpolarized TMD FFs for \( \pi^\pm, K^\pm \) as well as \( p/\bar{p} \) and their uncertainties based on the recent Belle experimental data.

2. QCD framework and TMD FFs

According to the TMD factorization theorem, the differential cross section which depends on the fractional energy variable \( z \) and transverse momentum \( P_{HT} \), can be written as follows [7, 8],

\[
\frac{d\sigma^H}{dz dP_{HT}} = L_{\mu\nu}(W_{\text{TMD}}^{\mu\nu} + W_{\text{coll}}^{\mu\nu}).
\]

There is one observed hadron \( H \) at the final state and \( W_{\text{TMD}}^{\mu\nu} \) and \( W_{\text{coll}}^{\mu\nu} \) are hadronic tensors and \( L_{\mu\nu} \) is leptonic tensor. The differential cross section can be written as TMD FFs as the following equation [9]

\[
\frac{d\sigma^H}{dz dP_{HT}} = 2\pi P_{HT} \frac{4\alpha^2}{3s} \sum_q e_q^2 ND_{H/f}(z, P_{HT}, Q^2) h^H(P_{HT})
\]

This equation is expressed at the leading order (LO), and hence, one can see just the contribution of TMD FFs \( D_{H/f}(z, P_{HT}, Q^2) \). The hard scale of the process is \( Q \) and \( N \) is an appropriate normalization parameter. The TMD FFs \( D_{H/f}(z, P_{HT}, Q^2) \) can be expressed based on two terms. The first term \( D_{H/f}(z, Q^2) \) is related to the unpolarized collinear FF and the second term \( h^H(P_{HT}) \) is the hadron transverse momentum which produced in the final state. Then the TMD FFs can be written as \( D_{H/f}(z, P_{HT}, Q^2) = D_{H/f}(z, Q^2) h^H(P_{HT}) \). The most recent calculations are performed for the unpolarized FFs by using the experimental data from SIA, SIDIS and hadron-hadron collisions. [10–13]. For the unpolarized collinear term of FFs, we use of NNFF1.0 FFs sets for pion, kaon and proton [12]. Since our theory calculations are performed at LO perturbative QCD, we also use the LO FFs sets from NNFF1.0 Collaboration. For determination of \( h^H(P_{HT}) \) for charged pion, kaon and proton/antiproton, we parameterize TMD FFs for the phenomenological study of TMD distributions.

We apply the Gaussian form which is common parametrization for the transverse momentum dependent distributions such as parton distribution functions (TMD PDFs) and TMD FFs in...
Table 1: The best-fit parameters for TMD FFs into $\pi^\pm$, $k^\pm$ and $p/\bar{p}$. The values labeled by (*) have been fixed.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\pi^\pm$</th>
<th>$k^\pm$</th>
<th>$p/\bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.105</td>
<td>0.002</td>
<td>0.240</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.413</td>
<td>1.077</td>
<td>4.648</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.854</td>
<td>0.739</td>
<td>1.153</td>
</tr>
<tr>
<td>$N$</td>
<td>0.290</td>
<td>0.166</td>
<td>0.335</td>
</tr>
</tbody>
</table>

Refs. [9, 14] and it has been used in the QCD analysis of SIDIS, Drell-Yan and SIA processes,

$$D_{H/f}(z, P_{HT}, Q^2) = D_{H/f}(z, Q^2) e^{-P_{HT}^2/(\langle P_{HT}^2 \rangle)}.$$  \hspace{1cm} (3)

Also, the Belle datasets depend on the $z$ parameter and transverse momentum of detected hadron in final state. Hence we consider the following functional form for $\langle P_{HT}^2 \rangle$

$$\langle P_{HT}^2 \rangle = \alpha + z^\beta (1 - z)^\gamma,$$  \hspace{1cm} (4)

while $\alpha$, $\beta$ and $\gamma$ are free parameters and they will be calculated from QCD fit to the Belle experimental data. The parameters are calculated from the QCD fit for charged pion, kaon and proton separately in which have been presented in Table. 1. The details of the determination of best fit values are described in our paper [15]. The uncertainty of FFs are obtained using the standard Hessian method [16, 17].

3. Results and discussions

We apply $z$ and $P_{HT}$ kinematical cuts for various hadrons in our QCD analyses. In one hand, the experimental data with $P_{HT} > 1$ GeV are excluded for different identified light charged hadrons because the uncertainties of observables increase for the high $P_{HT}$ value. Then we investigate the sensitivity of $\chi^2$ of analyses to the variations of $P_{HT}$ and in Fig. 1 the $\chi^2$ scans for our TMD FFs are presented to find the $P_{HT}^{max}$. On the other hand, the Belle experimental data also depend on the $z$ parameter and we study the $\chi^2$ scanning to find the value of $z_{max}$ in Fig. 1. Our final data selection considering the $P_{HT}$ and $z$ kinematical cuts applied for pion, kaon and proton are presented in Table. 2. In order to compare the theory predictions with Belle experimental data, we use our results of unpolarized TMD FFs at LO for pion, kaon as well as proton. In Fig. 2, the differential cross section of Belle datasets as a function of $P_{HT}$ for all three light charged hadrons are shown. The cross sections presented in this figure depend on the $z$ parameter which are shown in various bins. The scales of energy for Belle experiment are fixed for all kinds of hadrons which are equal to 10.58 GeV. As we mentioned in Table. 2, our theory predictions are restricted to the range of $0.275 \leq z \leq 0.675$ and $P_{HT} < 0.9$ GeV for pion, $0.275 \leq z \leq 0.625$ and $P_{HT} < 0.8$ GeV for kaon and $0.275 \leq z \leq 0.775$ and $P_{HT} < 1$ GeV for proton.
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<table>
<thead>
<tr>
<th>Hadron</th>
<th>$z$ cut</th>
<th>$P_{HT}$ cut</th>
<th>data points</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm$</td>
<td>[0.275 - 0.675]</td>
<td>[0 - 0.9]</td>
<td>63</td>
<td>1.053</td>
</tr>
<tr>
<td>$K^\pm$</td>
<td>[0.275 - 0.625]</td>
<td>[0 - 0.8]</td>
<td>48</td>
<td>1.154</td>
</tr>
<tr>
<td>$p/\bar{p}$</td>
<td>[0.275 - 0.775]</td>
<td>[0 - 1]</td>
<td>88</td>
<td>0.755</td>
</tr>
</tbody>
</table>

Table 2: The input datasets used in the three individual analyses for $\pi^\pm$, $K^\pm$ and $p/\bar{p}$. For each hadron, we indicate the kinematical cuts of $z$ and $P_{HT}$, number of data points in the fits and the $\chi^2$/dof values for each dataset.

Figure 1: Dependence of $\chi^2$/dof on the maximum cut values of $P_{HT}$ and $z$ for $0.3 < P_{HT} < 1.1$ GeV and $0.5 < z_{max} < 0.85$ datasets of pion, kaon and proton used in the analyses.

For the pion analysis, the observable measurements agree well with the theory predictions for the different bins of $P_{HT}$ and the $z$ values. However, a very small shift for our predictions at the highest value of $z$ can be seen. For the case of kaon, as one expects an excellent fit from $\chi^2$/dof value in Table 2, the agreements between data and theory are excellent for all range of $P_{HT}$ and $z$. Finally, our theory prediction for the proton has the lack of agreement with the data only to the highest $z$ bins $0.75 < z < 0.80$ and the smallest $z$ bins $0.25 < z < 0.30$, both for the $P_{HT} < 0.30$ GeV regions.

4. Summary

In this article, we have presented the first determination of unpolarized TMD FFs from a QCD fit of recent Belle Collaboration measurements at KEK from $e^+e^- \rightarrow HX$ for differential cross sections for charged pion, kaon and proton/antiproton. We use these data as only available observables, to determine the unpolarized TMD distributions of parton to identified light charged hadrons for the first time. In the current study, according to the cross section factorization, we assume two terms for TMD FFs. We consider the unpolarized collinear FF $D_{H,f}(z, Q^2)$ and a TMD term $h^H(P_{HT})$. We have assumed a Gaussian form for TMD term and we have used NNFF1.0 FFs sets for pion, kaon and proton as the collinear FFs. In terms of directions for the future project, we would be performed the effect of the higher order correction. In addition, we would be to add...
Figure 2: Comparisons between the differential cross sections measurements from Belle Collaboration [6] for pion, kaon and proton and our theory predictions as a function of $P_{HT}$ for different $z$ bins.

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References


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