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An Inhomogeneous Phase in Quark Matter without the Sign Problem

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As shown recently, the ground state of quantum chromodynamics (QCD) in sufficiently strong magnetic fields and at moderate baryon densities carries a crystalline condensate of neutral pions: the chiral soliton lattice (CSL). This phase cannot be realized from first principles using standard lattice Monte Carlo simulations due to the infamous sign problem. On the other hand, we show that CSL, or a similar inhomogeneous phase, also appears in the phase diagram of a class of vector-like gauge theories that do not suffer from the sign problem even in the presence of a baryon chemical potential and external magnetic field. Hence, we give a class of explicit counterexamples to the long-standing conjecture that positivity of the determinant of the Dirac operator (that is, absence of the sign problem) in a vector-like gauge theory precludes spontaneous breaking of translational invariance, and thus implies the absence of inhomogeneous phases in the phase diagram of the theory.

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1. Introduction: inhomogeneous phases in quark matter

The nature of QCD matter at finite temperature and baryon number remains an open questions; in particular, one of the intensly studied issues is the appearance of inhomogeneous phases in the QCD phase diagram. At high densities, cristalline color superconductivity is expected; further, it is conjectured that the chiral condensate may become spatially modulated in the part of the phase diagram where a first-order chiral phase transition occurs in the traditional picture (see [1] for a review).

The theoretical study of such inhomogeneous phases poses a challenge, since the standard lattice Monte-Carlo simulations fail at finite densities due to the infamous sign problem. While for asymptotically high densities, perturbative QCD can be used, at intermediate densities usually only model calculations are available. One of the rare exceptions is the case of the dense QCD matter in strong magnetic fields where the appearence of so-called chiral soliton lattice (CSL) phase was shown in a model independent way - using low-energy effective field theory [2]. The CSL phase is formed by the topological crystalline condensate of neutral pions which is energetically favorable due to the anomalous coupling of the neutral pion field to the magnetic field and chemical potential.

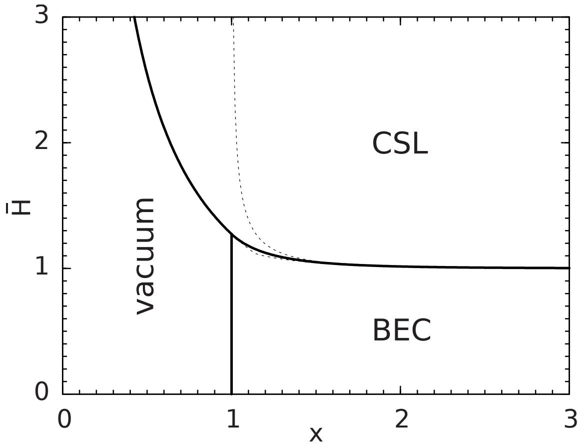
Unlike QCD itself, some of the QCD-like theories (such as two-color QCD) allow for lattice simulations even at finite density. However, it was conjectured [3] that the positivity of the Dirac determinant (i.e., the absence of the sign problem) implies absence of inhomogeneous phases in the phase diagram. In this work we give an explicit counter-example to this conjecture by showing that the CSL phase exists also for a class of QCD-like theories without the sign problem.

2. CSL phase in QCD-like theories

QCD-like theories with quarks in real or pseudoreal representations of the gauge group in strong magnetic fields were studied assuming two quark flavors (u and d) of equal masses [4, 5]. It can be shown that in this case the determinant of the Dirac operator is positive at finite baryon chemical potential provided the two quark flavors have opposite electric charges: $q_u = -q_d$.

Only electrically neutral pseudo-Goldstone modes remain light in strong magnetic fields, specifically, the neutral pion field and a diquark-antidiquark pair in case of the QCD-like theories with (pseudo-)real quarks. The low-energy effective field theory describing the interactions of these fields was used to find the ground state for different values of the baryon chemical potential μ_B and magnetic field H. The resulting phase diagram is shown in the figure below where rescaled quantities $x = \frac{\mu_B}{m_\pi}$, $\bar{H} = \frac{d(q_u - q_d)}{8\pi^2 f_\pi^2} H$ are plotted. Here m_π and f_π are pion mass and decay constant, respectively, and d is the dimension of the representation of the color gauge group that a single quark flavor transforms in.

Above $\mu_B = m_\pi$ and below certain critical magnetic field the Bose-Einstein condesation of diquarks is energetically most favorable (in this type of QCD-like theories, the diquarks are baryons obeying Bose-Einstein statistics) whereas for higher magnetic fields the inhomogeneous neutral pion condensate takes over. Again, the chiral anomaly (captured by the so-called Wess-Zumino-Witten term in the effective field theory [6]) is crucial for appearance of CSL in the phase diagram. The dashed lines are the spinodal curves of the first-order transition between the BEC and CSL phases.



3. Conclusion and outlook

We have shown that an inhomogeneous phase exists for high enough magnetic fields in certain class of QCD-like theories which do not suffer from the sign problem at finite densities, hence, can be simulated on lattice. An actual lattice simulation would be a great completion of our work.

Another issue is the stability of the inhomogeneous phases with respect to quantum fluctuations. It is known that one-dimensional modulations in pion condensates [1] are unstable at finite temperatures (although quasi-long-range order may persist). On the other hand, CSL in QCD seems to be stable under quantum fluctuations [7] and the effects of finite temperature are subject of ongoing work.

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