A gauge invariant description of phase transitions

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Phase transitions are of wide interest to be sure, whether it's in superconductors or the early universe: Beyond the Standard Model scenarios like Baryogenesis cry out for a strong first-order phase transition. So a precise description of phase transitions is vital. Phase transitions are, in field theory, studied with numerical methods (lattice) and perturbative calculations (the effective potential). Perturbative calculations are quite handy since lattice calculations are as yet rather resource expensive. But perturbative calculations face a number of obstacles, to wit the expansion breaks down at high temperatures and is gauge dependent. The former problem is often remedied by a resummation; though, this resummation isn't gauge invariant. And so we will present a gauge invariant method for describing phase transitions using the effective potential. This method also allays the high-temperature breakdown through a gauge invariant resummation

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1. The Electroweak Phase Transition

The electroweak symmetry was unbroken in the inchoate universe, not so much in our current day and age. Moments after the Big Bang the Higgs field broke the electroweak symmetry. An event known as the electroweak phase transition. But this phase transition is not merely interesting for its own sake but can also shed light on the matter-antimatter asymmetry problem. For potential solutions to the matter-antimatter asymmetry must fulfil three conditions. One of these is a loss of thermal equilibrium. Such a loss of thermal equilibrium can be initiated by a strong electroweak phase transition—occurring via bubble nucleation. The electroweak symmetry is exact outside the bubbles, while on the inside the symmetry is broken. Particles swept up by the bubbles get out of thermal equilibrium and an asymmetry can be generated.

2. Phase Transitions

Calculations of phase transitions are arduous. Especially when including higher-order corrections. Here the effective potential is the method of choice, but this quantity is notoriously slippery to work with. Common issues are infrared-divergences, gauge dependence, and a slow convergence. Many of these problems stems from not finding the different ground states consistently. In this paper we consider a strict power-counting whereupon these issues disappear.

2.1 The Effective Potential

The effective potential $V(\phi)$ is the energy density in the presence of a constant scalar background ϕ. And we consider a perturbative expansion of the form

$$V(\phi) = V_0(\phi) + \hbar V_1(\phi) + \hbar^2 V_2(\phi) + \dots$$

The leading-order contribution, $V_0(\phi)$, is here the tree-level potential. Higher-order corrections are obtained from vacuum Feynman diagrams in the ϕ background.

Yet gauge bosons require gauge-fixing; the choice of gauge fixing should not affect physical observables. Indeed, the effective potential's gauge dependence is described by the Nielsen identity [1]

$$\left(\xi \partial_\xi + C(\phi, \xi) \partial_\phi\right) V(\phi) = 0.$$

Thus extremal points of the effective potential are gauge independent order-by-order in $\hbar$. Yet there can be artificial gauge dependence if the potential is not minimized consistently. Below we describe a gauge-independent method.

Loop-Induced Phase Transitions

The schematic expansion of $V(\phi)$ in powers of $\hbar$ is not appropriate for large temperatures. For at these large temperature one-loop terms compete with tree-level ones. For high temperatures the potential is of the schematic form [2]

$$V(\phi) \sim m_{\text{eff}}^2(T)\phi^2 - g^3 T \phi^3 + \lambda \phi^4. \tag{1}$$

A first-order transition occurs when $\phi \sim \frac{e^3}{\lambda} T$ and $m_{\text{eff}}^2(T) \sim \frac{e^6}{\lambda} T^2$. There are problems if λ scales as e^2 or e^4 [2], so we choose $\lambda \sim e^3$. These assumed scalings imply that vector, and scalar,
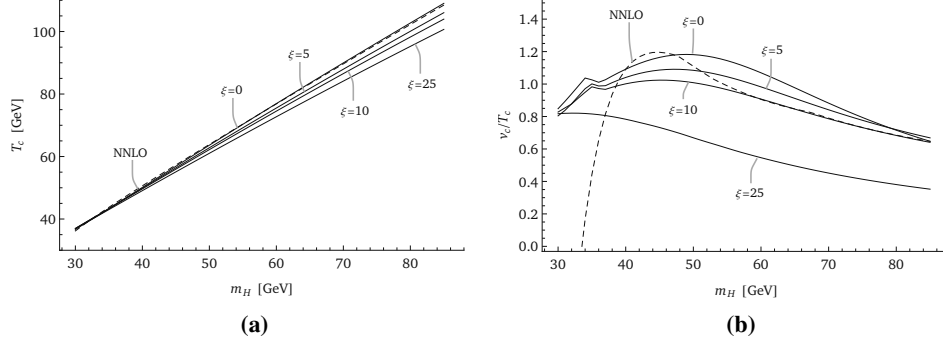
Figure 1: Comparison of the gauge-independent and gauge-dependent method in the Standard Model. The tree-level Higgs mass is set to $m_H = 45$ GeV. Panel (a) shows the critical temperature and panel (b) shows the phase transition strength.

boson self-energies are of the same order as their tree-level masses: they need to be resummed $m_X^2 \rightarrow m_X^2 + T^2 \Pi_X$. Where Π_X is the self-energy coming with T^2 for a particle X.

With this counting we write the effective potential as $V = V_{\text{LO}} + e V_{\text{NLO}} + e^2 V_{\text{NNLO}} + \ldots$, where we take e as the powercounting parameter. The gauge-invariant minimum is then [3]

$$\partial V = \partial \left(V_{\text{LO}} + e V_{\text{NLO}} + e^2 V_{\text{NNLO}} + \ldots \right) |_{\phi = \phi_m} = 0, \ \phi_m = \phi_{\text{LO}} + e \phi_{\text{NLO}} + \ldots$$

$$\partial V_{\text{LO}} |_{\phi = \phi_{\text{LO}}} = 0, \ \phi_{\text{NLO}} = -\frac{\partial V_{\text{NLO}}}{\partial^2 V_{\text{LO}}} |_{\phi = \phi_{\text{LO}}}, \ \ldots V_{\text{min}} = (V_{\text{LO}} + e V_{\text{NLO}} + \ldots) |_{\phi = \phi_{\text{LO}}}$$

This procedure is gauge-invariant and consistent if masses are appropriately resummed.

Likewise, if we denote the energy of the symmetric phase ($\phi = 0$) by V_A, and that of the broken phase by V_B, the critical temperature where these two phases overlap is

$$\Delta V \equiv V_A - V_B, \ T_c = T_{\text{LO}} + e T_{\text{NLO}} + \ldots, \ \Delta V |_{T = T_c} = 0$$

$$\Delta V_{\text{LO}} |_{T = T_{\text{LO}}} = 0, \ T_{\text{NLO}} = -\frac{\Delta V_{\text{NLO}}}{\partial_T \Delta V_{\text{LO}}}, \ \ldots$$

The critical temperature can then be used to calculate observables such as the phase-transition strength. Results of these calculations are shown for the critical temperature in figure 1. In these plots we compare the gauge-invariant method in this paper to a gauge-dependent method where the effective-potential is not minimized consistently. From this figure we see that the gauge-dependence for the critical temperature is rather small, while other observables, like the phase-transition strength or the latent heat, depend more on the gauge-fixing parameter.

References

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