Overview of the Path to 0.01% Theoretical Luminosity Precision for the FCCee and Its Possible Synergistic Effects for Other FCC Precision Theory Requirements

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To exploit properly the precision physics program at the FCC-ee, the theoretical precision tag on the respective luminosity will need to be improved from the 0.054% (0.061%) results at LEP to 0.01%, where the former (latter) LEP result has (does not have) the pairs correction. We present an overview of the roads one may take to reach the required 0.01% precision tag at the FCC-ee and we discuss possible synergistic effects of the walk along these roads for other FCC precision theory requirements.

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1. Introduction

It is well documented [1–3] by now the new era of precision for Higgs and EW observables which will obtain at the proposed precision physics factories such as the FCC-ee, CLIC, ILC and CEPC. These scenarios all require that the theoretical luminosity precision be improved from its LEP era value to the regime of 0.01%, or better if possible[1]. Accordingly, the general context of our discussion can be set by recalling the situation that existed at the end of LEP. At that time, as we show here in Table 1, the error budget for the BHLUMI4.04 MC used by all LEP collaborations to simulate the luminosity process was calculated in Ref. [6]. In this table, we cite the published

	LEP1		LEP2	
Type of correction/error	1996	1999	1996	1999
(a) Missing photonic $O(\alpha^2)$ [7, 8]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $O(\alpha^3 L_e^3)$ [9]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [10, 11]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [12, 13]	0.03%	0.03%	0.05%	0.05%
(e) Z and s-channel γ [14, 15]	0.015%	0.015%	0.0%	0.0%
Total	0.11% [15]	0.061% [6]	0.25% [15]	0.12% [6]

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within $1°–3°$ (18–52 mrads), and for LEP2 energies up to 176 GeV and an angular range within $3°–6°$. Total uncertainty is taken in quadrature. Technical precision included in (a).

works upon which the various error estimates are based as they are discussed in Ref. [6]. From what we show in this table, we can chart a path to 0.01% luminosity theory error. In discussing this path, we will use the FCC-ee as our prototype but it will be clear that the arguments apply as well to the other similar precision EW factories.

As an example, consider the role of the Bhabha luminosity($\mathcal{L}$) measurement in neutrino counting where we use N_ν to denote the number of light neutrinos. From the LEP Z-peak measurements [16], we have

$$N_\nu = 2.9840 \pm 0.0082$$

$$\delta N_\nu = 10.5\frac{\delta n_{had}}{n_{had}} \oplus 3.0\frac{\delta n_{lept}}{n_{lept}} \oplus 7.5\frac{\delta\mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = 0.61\% \Rightarrow \delta N_\nu = 0.0046.$$

Recently, Janot and Jadach [17], building on the results in Ref. [18], use the current status of the LEP luminosity theory error together with improved measurement error analysis correcting for so-far ignored systematic beam-beam effects to show that $N_\nu = 2.9963 \pm 0.0074$ with $\delta N_\nu = 0.0028$ from $\frac{\delta\mathcal{L}}{\mathcal{L}}$. This resolves what had been a lingering "2σ" deviation from the predictions of the Standard Theory [19].

2. Path to 0.01% Luminosity Theory Uncertainty

The 0.01% precision tag needed for the luminosity theory error for the FCC-ee can be realized by developing the corresponding improved version of BHLUMI. This problem is discussed in some

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[1]In addition to this improvement, these new precision scenarios may require a completely new methodology of the QED "deconvolution" and its related new definition of the EW pseudo-observables (EWPO's) [4, 5]

detail in Ref. [20], wherein the path to 0.01% theory precision for the FCC-ee luminosity is presented. In Table 2, the results of this latter reference are shown, wherein we also present the current state of the art for completeness, as it is discussed in more detail in Refs. [20]. In arriving at

Type of correction / Error	Update 2018	FCC-ee forecast
(a) Photonic $[O\left(L_e\alpha^2\right)]$ $O\left(L_e^2\alpha^3\right)$	0.027%	0.1×10^{-4}
(b) Photonic $[O\left(L_e^3\alpha^3\right)]$ $O\left(L_e^4\alpha^4\right)$	0.015%	0.6×10^{-5}
(c) Vacuum polariz.	0.014% [21]	0.6×10^{-4}
(d) Light pairs	0.010% [22, 23]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [14]	0.1×10^{-4}
(f) Up-down interference	0.009% [24]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

Table 2: Anticipated total (physical+technical) theoretical uncertainty for a FCC-ee luminosity calorimetric detector with the angular range being 64–86 mrad (narrow), near the Z peak. Description of photonic corrections in square brackets is related to the 2nd column. The total error is summed in quadrature.

Table 2 the main points are as follows. The errors of the photonic corrections in lines (a) and (b) in the LEP results in Table 1 are due to known effects from Refs. [7–9] that were not implemented into BHLUMI. We show in Table 2 what these errors will become after these known results are included in BHLUMI as discussed in Ref. [20]. Similarly, in line (c) of Table 1 the error is due to the LEP era uncertainty on the hadronic contribution to the vacuum polarization for the photon at the respective momentum transfers for the luminosity process; in Table 2 we show the expected improvement of this error for the FCC-ee as discussed in Refs. [21, 25, 26].

In line (d) in Table 2, proceeding in a similar way, we show the expected [20] improvement, relative to the LEP time for Table 1, in the light pairs error for the FCC-ee . As we explain in Ref. [20], because non-photonic graphs can contribute as much as 0.01% for the cut-off $z_{\text{cut}} \sim 0.7$, the complete matrix element for the additional real e^+e^- pair radiation should be used. This can be done with the MC generators developed for the $e^+e^- \rightarrow 4f$ processes for LEP2 physics - see Ref. [20] for further discussion. The contributions of light quark pairs, muon pairs and non-leading, non-soft additional $e^+e^- + n\gamma$ corrections can be controlled, with known methods [20], such that the error on the pairs contribution is as given in line (d) for the FCC-ee.

We turn next to line (e) in Table 2, wherein we show the improvement of the error on the Z and s-channel γ exchange for the FCC-ee as well as its current state of the art. In Ref. [20], a detailed discussion is presented of all of the six interference and three additional squared modulus terms that result from the s-channel γ, s-channel Z, and t-channel Z exchange contributions to the amplitude for the luminosity process. It is shown that, if the predictions of BHLUMI for the luminosity measurement at FCC-ee are combined with the ones from BHWIDE [27] for this Z and s-channel γ exchange contribution, then the error in the second column of line (e) of Table 2 could be reduced to 0.01%. In order to reduce the uncertainty of this contribution practically to zero we would include these Z and γ_s exchanges within the CEEX [28] type matrix element at $O(\alpha^1)$ in BHLUMI. Here, CEEX stands for coherent exclusive exponentiation which acts at the level of the amplitudes as compared the original Yennie–Frautschi–Suura [29](YFS) exclusive exponentiation (EEX) that is used in BHLUMI4.04 and that acts at the level of the squared amplitudes. It is expected to be enough to add the EW corrections to the LABH process in the form of effective couplings in the Born amplitudes. This leads to the error estimate shown in Table 2 in line(e) for the FCC-ee.

For completeness, we note that for our discussion of the Z and s-channel γ exchanges we made in Ref. [20] a numerical study using BHWIDE for the the calorimetric LCAL-type detector, as described in ref. [30], for the symmetric angular range 64–86 mrad without any cut on acoplanarity. The pure weak corrections were calculated with the ALIBABA EW library [31, 32]. The results, shown in Table 3, were obtained for three values of the centre-of-mass (CM) energy: $E_{\text{CM}} = M_Z$, $M_Z \pm 1$ GeV, where the latter two values have Z contributions that are close to maximal in size. The results in the second column for the total size of the Z and γ_s exchanges are consistent with our expectations as

E_{CM} [GeV]	Δ_{tot} [%]	$\delta_{O(\alpha)}^{QED}$ [%]	$\delta_{h.o.}^{QED}$ [%]	δ_{tot}^{weak} [%]
90.1876	+0.642 (12)	−0.152 (59)	+0.034 (38)	−0.005 (12)
91.1876	+0.041 (11)	+0.148 (59)	−0.035 (38)	+0.009 (12)
92.1876	−0.719 (13)	+0.348 (59)	−0.081 (38)	+0.039 (13)

Table 3: Results from `BHWIDE` for the Z and γ_s exchanges contribution to the FCC-ee luminosity with respect to the $\gamma_t \otimes \gamma_t$ process for the calorimetric LCAL-type detector [30] with the symmetric angular range 64–86 mrad; no acoplanarity cut was applied. MC errors are marked in brackets.

explained in ref. [20]: the contribution is positive below the Z peak where it reaches a size $\sim 0.64\%$, is close to zero near the peak, and changes sign above the peak where it reaches a size $\sim -0.72\%$. The third column, which features the fixed-order (non-exponentiated) $O(\alpha)$ QED correction, shows that it is sizable and up to a half of the size of the Born level effect, but with the opposite sign. The fourth column shows the size of the higher-order QED effects from YFS exponentiation, which also change their sign near the Z-peak, oppositely to the corresponding change of the $O(\alpha)$ corrections. The size of the former effects is about a quarter of that of the latter. From the effects in the fourth column we make a conservative estimate of the size of the missing higher-order QED effects in `BHWIDE` using the big log factor $\gamma = \frac{\alpha}{\pi} \ln \frac{|\bar{t}|}{m_e^2} = 0.042$ of Section 4 of Ref. [20] and a safety factor of 2 of Ref. [14] together with the largest higher-order effect in Table 3, 0.081%, as $0.081\% \times \gamma \times 2 \simeq 0.007\%$. The last column shows that the size of the pure weak corrections is at the level of 0.01% below and at M_Z and increases up to $\sim 0.04\%$ above M_Z. We use the same factor as we did for the higher order corrections to estimate the size of the missing higher order pure weak corrections in `BHWIDE` as $\sim 0.003\%$. Altogether, by adding the two estimates of its massing effects, we obtain a conservative estimate of 0.01% for the physical precision of `BHWIDE` to justify our earlier remarks concerning the error in line (e) of Table 2 that would result from the combination of the prediction of `BHLUMI` and that of `BHWIDE` for this contribution.

In line(f) in Table 2 we show the estimate of the error on the up-down interference between radiation from the e^- and e^+ lines. While it was negligible in LEP1, for the FCC-ee this effect, calculated in Ref. [24] at $O(\alpha^1)$, is 10 times larger and has to be included in the upgraded `BHLUMI`. Once this is done, the error estimate shown in line(f) for the FCC-ee obtains [20].

This brings us to the issue of the technical precision. In order to get the upgraded `BHLUMI` technical precision at the level 10^{-5} for the total cross section and 10^{-4} for single differential distributions, in an ideal situation, one would need to compare it with another MC program developed independently, which properly implements the soft-photon resummation, LO corrections up to $O(\alpha^3 L_e^3)$, and the second-order corrections with the complete $O(\alpha^2 L_e)$. An extension of a program like `BabaYaga` [33–35], which is currently exact at NLO with a matched QED shower, to the level of NNLO for the hard process, while keeping the correct soft-photon resummation, would in principle provide the best comparison to the upgraded `BHLUMI` to establish the technical precision of both programs at the 10^{-5} precision level[2]. However, a very good test of the technical precision of the upgraded `BHLUMI` would follow from the comparison between its results with EEX and CEEX matrix elements; for, the basic multi-photon phase space integration module of `BHLUMI` was already well tested in Ref. [37] and such a test can be repeated at an even higher-precision level. An alternative soluton, perhaps using the results in Refs. [38, 39], should also be pursued.

3. Synergies

Historically, our exact $O(\alpha^2 L)$ corrections were done for BHLUMI4 precision. They were then combined via crossing with CEEX amplitude-based resummation in the MC KKMC for state-of-the-art 2f production in e^+e^- annihilation. This realization of KKMC was then extended to KKMC-hh

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[2]The upgrade of the `BHLUMI` distributions will be relatively straightforward because its multi-photon phase space is exact [36] for any number of photons.

for Z production in pp and $\bar{p}p$ collisions. This is now being extended to MG5_aMC@NLO/KKMC-hh [40] which would realize exact QCD NLO $\otimes$ exact $O(\alpha^2 L)$ EW with matrix element matched parton showers. This chain of development illustrates the synergistic nature of progress in precision quantum field theory. Indeed, when we add to BHLUMI the QED matrix element corrections of $O(\alpha^2 L)$ and $O(\alpha^3 L^3)$ we already reduce δN_ν from $\delta \mathcal{L}/\mathcal{L}$ to 0.0015.

As a technical precision solution, we need to take CEEX to BHLUMI. Synergistically, for FCC-ee, this suggests taking CEEX to all the EEX YFS realizations for LEP: for YFSWW3 & KORALW [41], this is discussed in Refs. [42, 43]; for YFSZZ [44] and BHWIDE, the work is in progress – we do need sufficient theory resources.

The pay-off is a good one. For example, for A_{FB} Ref. [45] use CEEX via KKMC state-of-the-art 2f production to achieve already the precision tag $\Delta [A_{FB}]_{IFI} \sim 10^{-4}$ on the IFI contribution to A_{FB} which shows that the FCC-ee requirement is accessible. Similarly, as discussed in Ref. [42], the FCC-ee requirement of a precision tag $\Delta M_W \sim 0.3 MeV$ in threshold and reconstruction impllies the CEEX extension of YFSWW & KORALW is need in both cases. We note that, in Ref. [43] this latter extension is in progress: the CEEX formalism is applied to to $e^+e^- \to WW + n\gamma \to 4f + n'\gamma$. We note that contact between the results in Ref. [43] and the usual Kleiss-Stirling spinor product based photon helicity infrared factors in CEEX proceeds via

$$e j_X^\mu (k_i) = e Q_X \theta_X \frac{2p_X^\mu}{2p_X k_i} \to \mathfrak{s}_{\sigma_i}(k_i) = e Q_X \theta_X \frac{b_{\sigma_i}(k_i, p_X)}{2p_X k_i}, \tag{1}$$

with

$$b_\sigma (k, p) = \sqrt{2} \frac{\bar{u}_\sigma (k) \not{p} u_\sigma (\zeta)}{\bar{u}_{-\sigma}(k) u_\sigma (\zeta)}, \tag{2}$$

where we follow the definitions in Refs. [28, 43]. The way forward is entirely open.

4. Summary

In summary, we conclude that, with the appropriate resources, the path to 0.01% precision for the FCC-ee luminosity (and the CLIC, ILC and CEPC luminosities) at the Z peak is open via an upgraded version of BHLUMI. It remains a matter for the appropriation of the required resources to support the attendant theoretical research – a matter beyond our control.

Acknowledgments

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