

An introduction to DDCalc

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Direct detection experiments place some of the strongest constraints on the interactions of dark matter and promise unique opportunities for the discovery of a dark matter signal. The public software tool DDCalc has been developed to make this wealth of data available for reinterpretation in a wide range of dark matter models and perform calculations fast enough for the use in large-scale parameter scans. DDCalc calculates predicted event rates for a given model of dark matter and its astrophysical distribution and then determines experimental likelihoods and exclusion limits from a large database of direct detection experiments. This talk summarizes the main features of the code, some recent developments and plans for the future.

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1. Introduction

The observation of galactic rotation curves strongly suggest that spiral galaxies like the Milky Way are embedded in a dark matter (DM) halo, implying that there is a continuous flux of DM particles passing through the Earth. While most of these particles will not experience any interactions, there will be occasional scatters with ordinary nuclei. The energy transferred in this process (called the nuclear recoil energy E_R) can be observed in dedicated low-background detectors placed deep underground. These so-called direct detection experiments have achieved spectacular improvements in sensitivity over recent years and are now sensitive to a few events with recoil energy in the keV range per tonne-year exposure. As a result, they place some of the strongest bounds on the interactions of DM and tightly constrain models with electroweak-scale DM candidates such as WIMPs.

The differential event rate in a direct detection experiment with respect to nuclear recoil energy is given by

$$\frac{dR}{dE_R} = \frac{\rho}{m_N m_\chi} \int_{v_{\min}}^{\infty} v f(\mathbf{v} + \mathbf{v}_E(t)) \frac{d\sigma}{dE_R} d^3v, \quad (1)$$

where $\rho \approx 0.3 \text{ GeV cm}^{-3}$ is the local DM density, m_χ and m_N are the DM and nucleus mass and $d\sigma/dE_R$ is the scattering cross section. The DM velocity distribution $f(v)$ needs to be transformed from the Galactic rest frame into the laboratory frame by adding the (time-dependent) velocity of the Earth $\mathbf{v}_E(t)$. Finally, energy conservation implies that only DM particles with a velocity greater than $v_{\min} = \sqrt{m_N E_R / (2\mu^2)}$ can contribute to events with nuclear recoil energy E_R , where μ is the DM-nucleus reduced mass.

Calculating an exclusion limit from a given direct detection experiment then requires the following steps:

1. Obtain the scattering cross section as a function of fundamental model parameters.
2. Perform the velocity integral for a given DM velocity distribution.
3. Convert the true nuclear recoil energy E_R into the detected energy E , accounting for quenching factors and the energy resolution of the detector.
4. Multiply the resulting spectrum with the detector efficiency $\Phi(E)$ and with the exposure ϵ and integrate over energy to obtain the predicted number of events:

$$N_p = \epsilon \int_0^{\infty} \Phi(E) \frac{dR}{dE} dE. \quad (2)$$

5. Compare with the observed number of events to determine whether the parameter point is excluded or to calculate the likelihood \mathcal{L} .

The public code DDCalc, first released together with Ref. [1] and then updated together with Ref. [2], has been developed to automate all of these steps in full generality and make them sufficiently fast for the implementation of direct detection constraints in global fits. DDCalc also provides a large database of existing and planned direct detection experiments and a flexible user interfaces in several programming languages. In the following we will present the main features of the code, highlight some recent additions and discuss plans for future development.

2. General overview

2.1 Calculation of scattering cross sections

For many DM models the differential scattering cross section is independent of E_R and can be expressed in terms of spin-independent and spin-dependent interactions between DM particles and individual nucleons. The only remaining task is then to include pre-calculated nuclear form factors that account for the coherence of the scattering process.

There are however various well-motivated models for which the differential scattering cross section exhibits a more complicated dependence on E_R . This is the case for example for models with light mediators, dipole interactions or pseudoscalar exchange (see e.g. Refs. [3, 4]). All of these interactions can be described in a general non-relativistic effective theory, which can schematically be written as

$$\mathcal{L}_{\text{eff}} = \sum c_\lambda \mathcal{O}_\lambda . \quad (3)$$

DDCalc supports both the set of operators \mathcal{O}_λ introduced in Ref. [5], which includes operators relevant for spin-1 DM, and the set from Ref. [6], which includes meson pole contributions.

The remaining task is then to calculate the coefficients c_λ as a function of the fundamental interactions of DM at high energies. While this task is beyond the scope of DDCalc, we provide an interface to the public code DirectDM, which performs the required running and matching [7] (see below).

DDCalc stores all relevant properties of the DM particle under consideration (i.e. its mass, spin and couplings) in the `WIMPStruct` structure.

2.2 Calculation of velocity integrals

Constraints from direct detection experiments suffer from substantial uncertainties resulting from the unknown distribution of DM in the Milky Way, which enters both through the local DM density and the DM velocity distribution. It is therefore essential to allow the user to study these astrophysical uncertainties by varying the underlying assumptions.

By default, DDCalc employs the so-called Standard Halo Model, which assumes that the DM velocity distribution is given by a Maxwell-Boltzmann distribution with peak velocity v_0 cut off at the Galactic escape velocity v_{esc} . While numerical simulations confirm that this is a reasonable approximation, one expects deviations for example due to anisotropy or non-virialised sub-structures like streams. DDCalc therefore provides a flexible interface for alternative velocity distributions, for which the velocity integral can either be provided analytically or through tabulated values.

DDCalc stores all relevant information on the DM distribution in the `HaloStruct` structure.

2.3 Calculation of the predicted number of events

A simple cut-and-count analysis based on the total number of predicted events is typically insufficient to achieve the optimal sensitivity to a DM signal. Instead, it is essential to exploit differences in the distribution of signal and background. Ideally, this should be done using an unbinned profile likelihood ratio analysis, but the required information (in particular regarding background distributions) is typically not publicly available. DDCalc therefore calculates exclusion

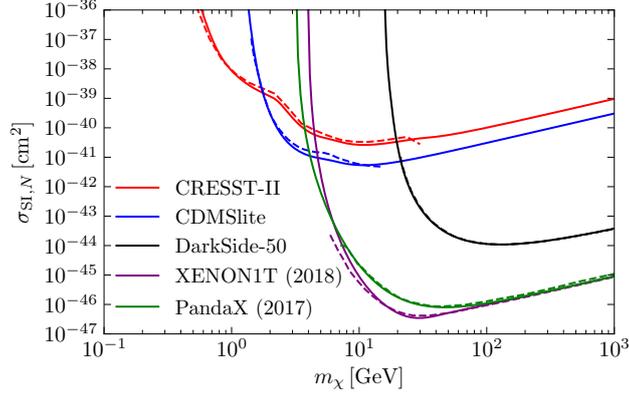


Figure 1: Comparison of bounds on the spin-independent DM-proton scattering cross section as a function of DM mass from various recent direct detection experiments as obtained from DDCalc (solid) and the corresponding official publications (dashed). Figure taken from Ref. [2].

limits based on one or more signal regions chosen in such a way that they approximately capture the results obtained from a more detailed analysis.

The details of a specific experiment are then encoded in efficiency functions $\Phi_i(E_R)$, which combine the steps 3 and 4 mentioned above and yield the probability that a nuclear recoil with energy E_R gives a signal in signal region i . This information is stored in the `DetectorStruct` structure together with details on the experimental exposure, the composition of the target, and the background expectation and observed number of events for each signal region. The actual calculation of the predicted number of signal events is then performed by calling `DDCalc_CalcRates(Detector, WIMP, Halo)`. The result of this calculation is again stored in `DetectorStruct`.

2.4 Calculation of experimental bounds

The remaining task is then to compare the predicted number of events to observations in order to calculate an exclusion limit (i.e. an upper bound on the signal strength) or a likelihood (for fixed signal strength). For the former case, DDCalc relies on either the Feldman-Cousins method (for known backgrounds) or the optimum interval method (for unknown backgrounds). For the latter case, DDCalc typically employs the Binned Poisson method, which can be extended by additional nuisance parameters for the case of unknown backgrounds (see Ref. [1] for details). In practice this is done by calling for example `DDCalc_LogLikelihood(Detector)`.

While the exclusion limit gives the most precise result for a single direct detection experiment (and is therefore used to validate DDCalc against published bounds), the likelihood has the advantage that it allows for the combination of several direct detection experiments as well as the combination with other types of constraints on DM models, using for example the GAMBIT global fitting framework [9]. Figure 1 shows a number of exclusion bounds from recent direct detection experiments as calculated by DDCalc in comparison with the literature. Figure 2 shows an example for how the combination of several direct detection experiments and complementary information can be used to strongly constrain specific DM models.

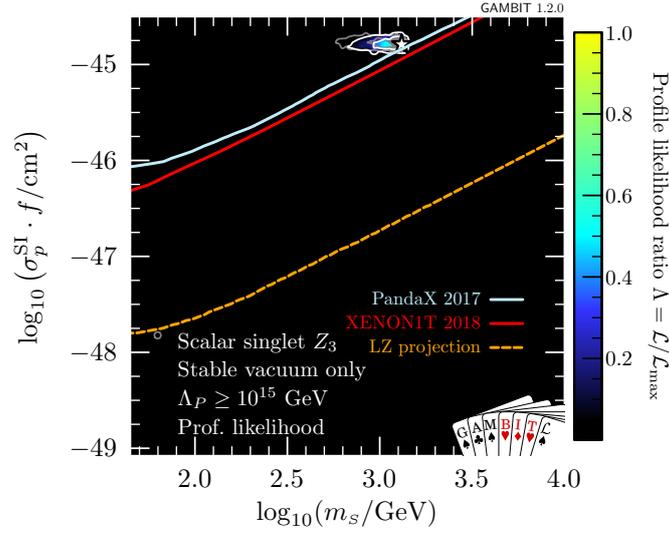


Figure 2: Viable parameter space of the scalar singlet DM model with \mathbb{Z}_3 symmetry in terms of the spin-independent DM-proton scattering cross section (rescaled by the predicted DM relic abundance) and the DM mass in comparison to bounds from direct detection experiments. Figure taken from Ref. [8].

3. Recent additions

In addition to the continuously updated database of direct detection experiments, DDCalc has recently been extended by several notable features. First of all, in addition to the existing user interfaces in Fortran and C, there is now also a user interface in Python, which provides various wrappers for the C library.

The new Python interface is particularly useful in the context of the general non-relativistic effective theory for DM (see above). While it was previously necessary to set each operator coefficient individually via the function `SetNRCoefficient`, the new function `Set_NREFT_CPT_couplings` accepts as input a Python dictionary of the form

```
cNR_coeffs = {'cNR1p' : 1e-8, 'cNR1n' : 1e-8}
```

This particular example would set the coefficient for the operator \mathcal{O}_1 to 10^{-8} GeV^{-2} for both protons and neutrons.

The Python interface has been designed in order for DDCalc to be able to interpret the output of DirectDM, which calculates the coefficients for the non-relativistic effective operators based on an effective theory at higher energies. The following example calculates the likelihood for the most recent analysis from XENON1T [10] for the case of a Dirac DM particle coupling via the magnetic dipole operator $\mathcal{O}_1^{(5)}$ with coefficient 10^{-5} GeV^{-1} :

```
import DDCalc
import DDCalc_DirectDM_Interface

halo = DDCalc.InitHalo()
detector = DDCalc.InitExperiment('Xenon1T_2018')
```

```
mDM = 50.0
DM_type = "D"
coupling_dict = {'C51' : 1e-5}

wimp = DDCalc_DirectDM_Interface.SetWIMP_Nf(coupling_dict, mDM, DM_type)

DDCalc.CalcRates(detector, wimp, halo)

print(DDCalc.LogLikelihood(detector))
```

In the example above it is implicitly assumed that the Wilson coefficients are specified in the five-flavour scheme at $\Lambda = m_Z$, i.e. after electroweak symmetry breaking. Different functions are available for the case that the Wilson coefficients are specified at different energy scales as well as before electroweak symmetry breaking.

4. Outlook

The most relevant feature currently missing in DDCalc is the support for direct detection experiments searching for annual modulations, i.e. the expected time dependence of a DM signal arising from the motion of the Earth around the Sun. Evidence for such a modulation has been claimed by the DAMA collaboration [11], in strong tension with results from other direct detection experiments. Nevertheless, the robustness of this conclusion in the light of theoretical, astrophysical and experimental uncertainties has long been a matter of debate.

With new experiments being constructed to search for annual modulations and conclusively test the DAMA signal, it becomes increasingly important to perform consistent and efficient combinations of experiments searching for annual modulations. We are therefore in the process of developing DDCalc_v3, which will extend the `HaloStruct` and `DetectorStruct` structures to allow for time-dependent DM signals.

Direct detection experiments will continue to make substantial improvements in sensitivity for the foreseeable future. Irrespective of whether or not they will make a discovery, these experiments will have profound implications for our understanding of DM. DDCalc makes this huge amount of information available to the community in a way that is easy to use and fast to run, while at the same time providing a wide range of advanced features and an up-to-date database of experimental results.

DDCalc is available via <https://ddcalc.hepforge.org/> and <https://github.com/patscott/DDCalc>.

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