

Nonlinear particle reacceleration by successive shocks

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Multiple shock acceleration is thought to take place in various environments, including chaotic winds, accretion disks and superbubbles. In the latter, multiple supernovae may explode within small timescales in compact regions and successively reaccelerate the distribution of cosmic rays which do not have time to escape far away from the star cluster. Reacceleration by successive shock waves is therefore expected to take place. We develop a model of nonlinear shock reacceleration and show that the downstream distribution of particles rapidly reach an asymptotic concave solution, such that the cosmic ray pressure saturates at the level of a few percent of the hydrodynamic pressure.

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1. Introduction

The problem of particle reacceleration by successive shocks has been early tackled in the test-particle regime [1–4]. It was shown that the spectrum hardens until it reaches an asymptotic p^{-3} power-law. However, the pressure of the reaccelerated particles is expected to quickly rise above the ram pressure of the shocks, which implies that the backreaction of the particles onto the shocks must be taken into account. While many models have been devoted to the modelling of cosmic ray acceleration by nonlinear shocks (e.g. [5–9]), there have been very few attempts to account for the presence of preexisting “seed” particles [10–12]. We therefore seek to develop an up-to-date semi-analytical modelling in order to compute the reacceleration of particles by many successive shocks.

2. Nonlinear model of shock reacceleration

2.1 Kinetic equation

We consider a plane infinite shock propagating along the x axis with a flow profile $u(x)$. The distribution of seed particles upstream of the shock is denoted f_∞ while the total distribution of particles, including the reaccelerated seeds as well as the particles accelerated from the thermal bath, is denoted f . The transport equation integrated around the shock reads [7]:

$$\frac{p}{3} (u_2 - u_p) \frac{df_1}{dp} = \left(u_p + \frac{p}{3} \frac{du_p}{dp} \right) f_1 - u_0 f_\infty - Q_1 \delta(x) \delta(p - p_0), \quad (1)$$

with u_p defined as:

$$u_p(p) \equiv u_1 - v_{A,1} - \frac{1}{f_1(p)} \int_{-\infty}^0 dx \, d_x (u - v_A) f(x, p). \quad (2)$$

The indices “0”, “1” and “2” respectively denote quantities evaluated at upstream infinity, immediately upstream of the shock and immediately downstream of the shock. v_A is the Alfvén velocity which is introduced in order to account for the Alfvénic drift [13]. Q_1 is the injection rate of particles from the thermal bath, determined such that the non-thermal distribution f matches the thermal (Maxwell Boltzmann) distribution at the injection momentum, which eventually provides the following boundary condition [14]:

$$f_1(p_0) = \frac{n_0 R_{tot}}{\pi^{3/2} p_0^3} \xi^3 e^{-\xi^2}, \quad (3)$$

where n_0 is the density of the plasma at upstream infinity, $R_{tot} = u_0/u_2$, p_0 is the injection momentum and $\xi \approx 2.5 - 4$ the injection parameter.

2.2 Fluid equation

The hydrodynamic profile of the shock is driven by the conservation of mass and momentum. The latter reads:

$$1 + P_{g,0} + P_{c,0} = U(x) + P_g(x) + P_c(x) + P_B(x), \quad (4)$$

where the pressures have been normalised with respect to the ram pressure $\rho_0 u_0^2$ and $U = u/u_0$. $P_c(x) = 4\pi/(3\rho_0 u_0^2) \int_{p_0}^{\infty} dp p^3 v(p) f(x, p)$ is the pressure of the particles. The gas pressure P_g is obtained assuming an adiabatic equation of state of index $\gamma (= 5/3)$: $P_g(x) = U^{-\gamma}(x)/(\gamma M_0^2)$. The magnetic pressure p_B is obtained by solving the transport equation of the magnetic field amplified by resonant streaming instability, which provides $P_B(x) = (2/25) (1 - U(x)^{5/4})^2 U(x)^{-3/2}$ [13].

2.3 Method of resolution

An approximate solution of the problem is obtained by introducing the typical distance $x_p(p)$ probed by particles of momentum p ahead of the shock [10, 15]. The spatial dependency of the distribution function can be approximately explicated as:

$$f(x, p) = [f_1(p) - f_{\infty}(p)] \vartheta[x - x_p(p)] + f_{\infty}(p), \quad (5)$$

such that the expression of the cosmic ray pressure simplifies to:

$$P_c(x_p) \approx \frac{4\pi}{3\rho_0 u_0^2} \left\{ \int_{p_0}^p dp' p'^3 v(p') f_{\infty}(p') + \int_p^{\infty} dp' p'^3 v(p') f_1(p') \right\}. \quad (6)$$

Defining $\zeta(p) \equiv U(x_p(p))$, the problem reduces to a system of coupled ordinary differential equations:

$$\frac{p}{3} \frac{df_1}{dp} \left(\frac{1}{R_{tot}} - \frac{7}{5}\zeta + \frac{2}{5}\zeta^{-1/4} \right) = \frac{f_1 - f_{\infty}}{5} \left(7\zeta - 2\zeta^{-1/4} + \frac{p}{6} \left(14 + \zeta^{-5/4} \right) \zeta'(p) \right), \quad (7)$$

$$\zeta'(p) \left[\frac{27}{25} - \frac{\zeta^{-\gamma-1}}{M_0^2} + \frac{\zeta^{-5/4}}{25} - \frac{3\zeta^{-5/2}}{25} \right] = \frac{4\pi}{3\rho_0 u_0^2} p^3 v(p) [f_1(p) - f_{\infty}(p)], \quad (8)$$

which can be solved iteratively with suitable boundary conditions determined by the Rankine-Hugoniot jump conditions at the subshock [8]. The initial value of the distribution function f is given by Equation 3 while the hydrodynamic equation 8 is supplemented by the boundary condition $\zeta(p_{max}) = u_0$, for the flow should not be modified far upstream. Because nonlinear shocks generally produce hard spectra in the high energy bands, it is mandatory to introduce a maximum momentum p_{max} which is equivalent to assume a free escape boundary in the upstream region. The solution is eventually found following the procedure detailed in [10].

2.4 Adiabatic decompression and escape flux

The medium is compressed by a factor u_0/u_2 during the passage of a shock. In between two shocks, the decompression must be taken into account. This is properly computed according to Liouville's theorem as discussed in [4].

Because we introduce a maximum momentum p_{max} , a non-negligible fraction of particles may escape at upstream infinity. The escape flux is computed as in [16], including the additional pressure of the seeds at upstream infinity.

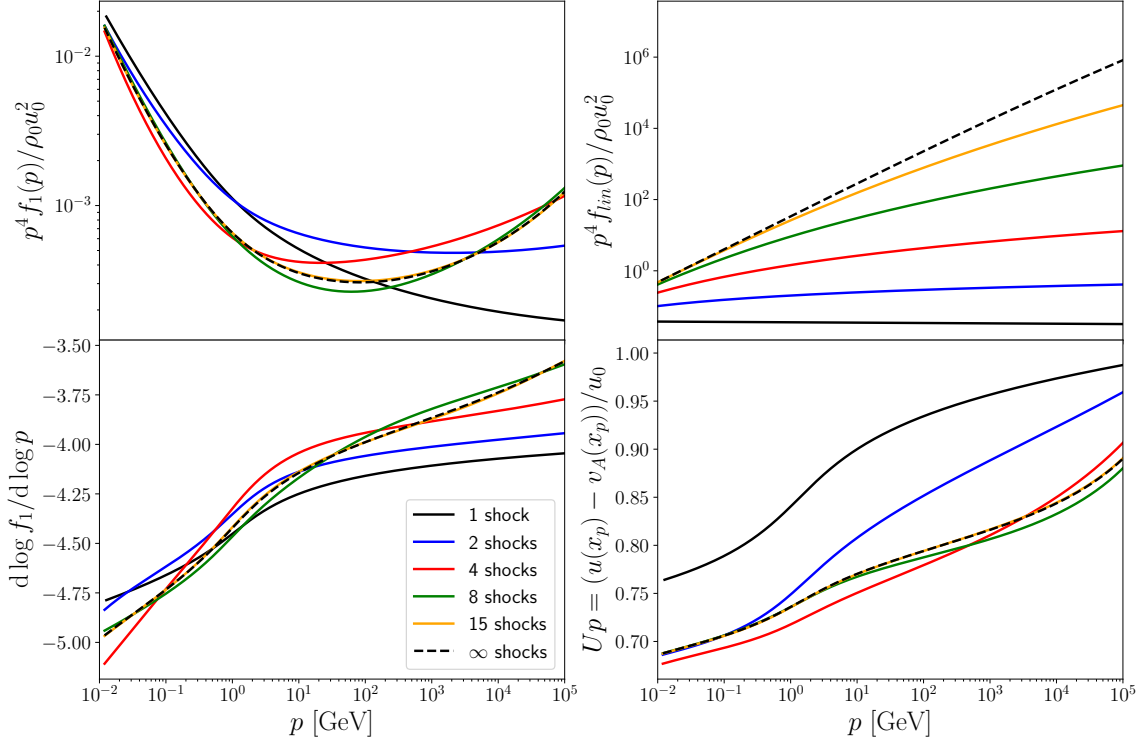


Figure 1: Evolution of the downstream distribution of particles (left panels) as the shocks succeed each other, compared to the test-particle solution (top right panel). The bottom right panel displays the profile of the flow effectively experienced by particles of momentum p . The Mach number is set to 20, the upstream temperature to 10^6 K, the injection parameter $\xi = 3$ and the maximum momentum $p_{max} = 1$ PeV.

3. Results

3.1 Asymptotic solution

Figure 1 shows the evolution of the cosmic ray spectrum and flow profile as the shocks successively sweep-up the medium. It is well-known that the pressure of cosmic rays slows down the flow ahead of the shock, as shown in the bottom right panel. As the shocks succeed each other, the pressure of the particles increases and the flow is more and more modified. This implies that low energy particles feel a reduced compression ratio which makes their acceleration inefficient, while high energy particles feel an enhanced compression ratio and are very efficiently (re)accelerated. This results in a more and more concave spectrum, in contrast to the test-particle case (top right panel of Figure 1). The asymptotic solution has a spectral index of about 5 at the injection momentum and 3.5 at the maximum momentum.

Remarkably, the asymptotic solution is nearly universal. As shown in Figure 2, it does not depend on the Mach number, even for weak shocks. Neither does it depend on the maximum momentum, providing the spectrum is rescaled accordingly. Only low energy bands do depend on the injection efficiency, which is expected since the fresh injection of particles is introduced as a boundary condition.

Figure 3 shows how the pressure of the downstream distribution of cosmic rays evolves as the

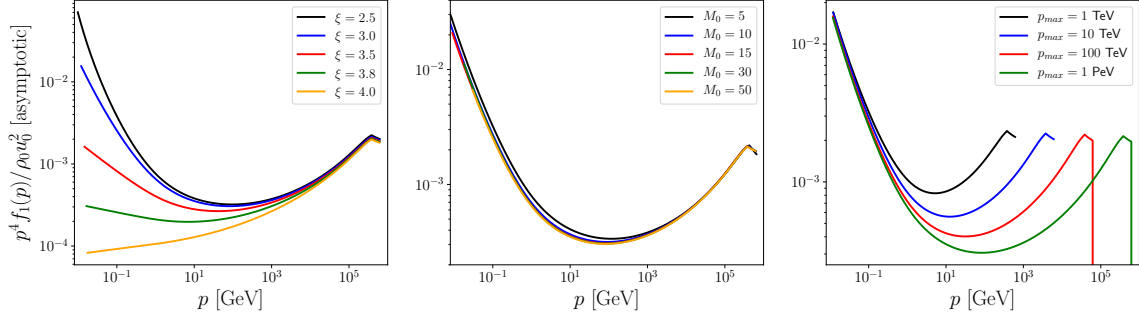


Figure 2: Asymptotic solutions for various sets of parameters. Left: the Mach number is set to 20, the maximum momentum to 1 PeV. Middle: $\xi = 3$, $p_{max} = 1$ PeV. Right: the Mach number is set to 20, $\xi = 3$.

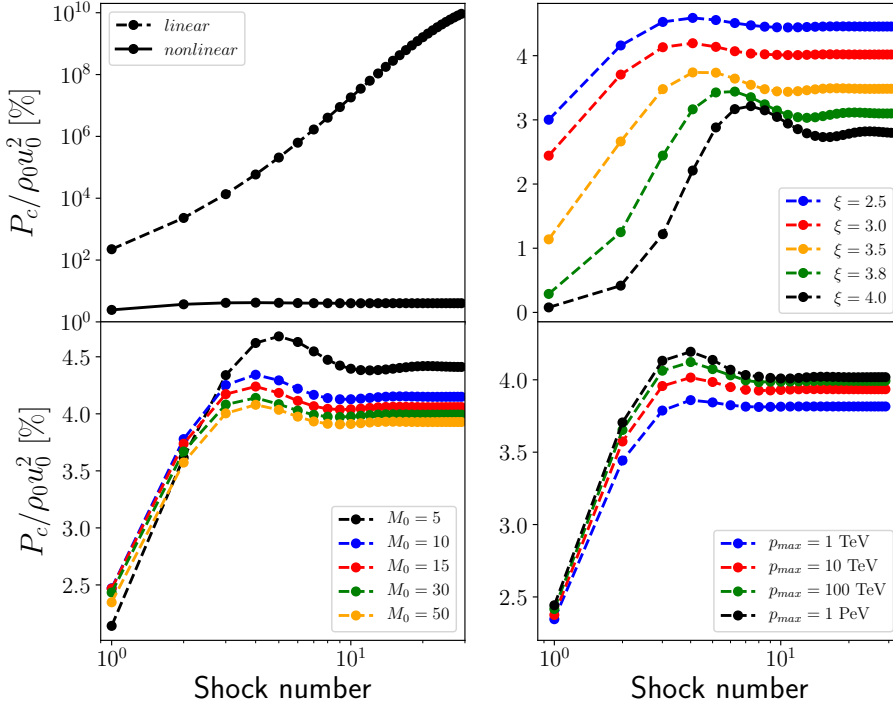


Figure 3: Evolution of the cosmic ray pressure in the downstream region. The parameters are similar to that set in Figure 2.

shocks succeed each other. The top left panel demonstrates that nonlinearities must be taken into account even when considering a small number of shocks. The pressure asymptotically converges to about a few percent of the shock ram pressure, for any set of parameters. In particular, even assuming a very small acceleration efficiency (e.g. 10^{-10} percent of thermal particles injected in the mechanism), the pressure saturates after about 10 reaccelerations at the level of a few percent of the hydrodynamic pressure. In the saturated regime, only a fraction of the seeds are reaccelerated and most of the energy is carried by the escape flux at the maximum momentum.

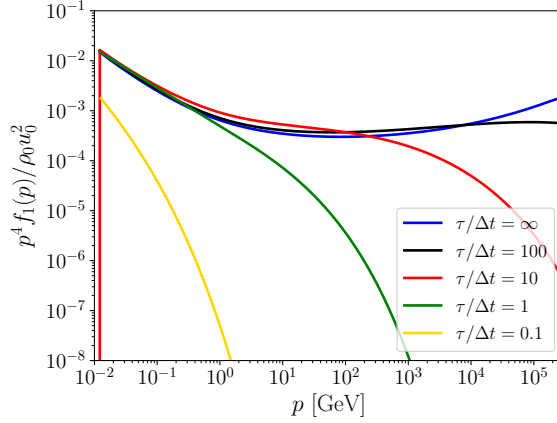


Figure 4: Asymptotic spectrum accounting for the expansion of the medium and particle escape. We assume a Mach number of 20 and $\xi = 3$.

3.2 Particle escape

The previous computation was performed in an idealistic scenario where the properties of the swept-up medium are constant in time and the particles do not escape the region of acceleration. In reality, the medium must be heated by the successive shocks, unless it expands. A more realistic model can be implemented by accounting for an adiabatic expansion of the region, which is typically the case in superbubble environments. We assume that the density decreases in between the passage of each shock according to Weaver’s scaling [17] and we allow for the escape of particles in a timescale $\tau(p) \propto p^{-1/3}$ as expected in the Kolmogorov regime of turbulence [18]. Figure 4 shows how the asymptotic solution is modified. Although the rarefaction of the medium does not have a major impact on the spectrum of particles, the escape of cosmic rays steepens the high energy bands. If the escape time of low energy particles is smaller than the typical time interval between supernovae Δt , the spectrum is very steep. The asymptotic solution is in fact only approximately retrieved up to the momentum p_* such that $\tau(p_*) = \Delta t$. The concave spectrum from the injection momentum to 1 PeV would only be realised in very massive compact clusters containing more than a thousand massive stars, such as the ones observed in the galactic centre.

4. Conclusion

Successive shocks efficiently accelerate cosmic rays such that the pressure of the particles quickly raises at the level of the ram pressure of the shocks. This implies that the test-particle approximation breaks down. The cosmic rays react onto the shock and in particular slow down the upstream flow. We have computed the reacceleration of particles in such setup using a semi-analytical model including the resonant excitation of hydromagnetic waves and self-consistently accounting for the injection of particles. The asymptotic spectrum obtained after a few reaccelerations is very concave. In the nonlinear regime the cosmic ray saturates at the level of a few percent of the shock ram pressure and most of the energy is carried by the escaping flux. Remarkably, the asymptotic solution is nearly universal. In particular, the asymptotic pressure weakly depends on the injection efficiency. The model is eventually applied to a more realistic case considering

the expansion of the medium as well as the energy-dependent escape of the particles, which is a step towards a nonlinear model of acceleration in superbubbles. The asymptotic spectrum is only retrieved if the shocks succeed each other very quickly. Eventually, the framework developed in this analysis can be used to compute the reacceleration of seed particles in various environments, not limited to superbubbles.

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