PROCEEDINGS OF SCIENCE

The Evolution of Lattice Field Theory: a Statistical Study

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Researchers working in lattice field theory constitute an established community since the early 1990s, and around the same time the online open-access e-print repository `arXiv` was created. The fact that this field has a specific arXiv section, `hep-lat`, provides a unique opportunity for a statistical study of its evolution over the last three decades. We present data for the number of entries, E, published papers, P, and citations, C, in total and separated by nations. We compare them to 6 other `arXiv` sections, and to socio-economic indices of the nations involved, namely the Gross Domestic Product (GDP) and the Education Index (EI). We present rankings, which are based either on the Hirsch Index H, or on the linear combination $\Sigma = E + P + 0.05C$. We consider both extensive and intensive national statistics, *i.e.* absolute and relative to the population or to the GDP.

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1. Outline

The conceptual basis of lattice field theory was elaborated in the 1970s and 1980s, and since the early 1990s physicists working in this field are a well-established, intercontinental community. The latter is related to the fact that around the same time computational resources became accessible more easily — computing was "democratized" — and they attained a level which allows for precise non-perturbative studies of some quantum field theoretic models, by means of Monte Carlo simulations in the lattice regularization.

It happened around the same time, more precisely in 1991/2, that the online e-print repository `arXiv` [1] became operational. Ever since it has contributed very significantly to the scientific communication. In particular, since 1992 its `hep-lat` section captures practically the entire activity of the lattice community.[1] Lattice researchers use it comprehensively, which is not the case to the same extent in other branches of physics, like condensed matter and optics.

The `hep-lat` section provides a perfect opportunity to statistically monitor the evolution of lattice field theory over the last three decades. In contrast, a similar study in other specific lines of physical research would require a tedious and less reliable search for keywords in titles and abstracts.

Taking advantage of this opportunity, we study the number of articles which were submitted to the `arXiv` with `hep-lat` as the "primary archive", `primarch` (we do not consider articles which are cross-listed to `hep-lat`; usually they only have an indirect link to lattice field theory). We used the open access digital library `INSPIRE` [2] to count the following quantities:

E	Entries, all article submitted to the `arXiv` with primary archive `hep-lat`
P	The subset of E, which was later published as regular papers (this excludes proceeding contributions and unpublished preprints)
C	Citations to all articles in E, which were registered by `INSPIRE` until summer 2020
H	Hirsch Index [3] of a set of articles which appear in E, considering all the citations to them included in C.

In order to compare lattice field theory to other fields of physical research, we considered the same quantities in 6 other arXiv sections, which are thematically somewhat related, namely `hep-ph` (high-energy physics, phenomenology), `hep-th` (high-energy physics, theory), `gr-qc` (general relativity and quantum cosmology), `nucl-th` (nuclear theory), `quant-ph` (quantum physics), and `cond-mat`$\star$ (condensed matter; the symbol $\star$ indicated the sum over all subsections).

In the national statistics, an article counts for a nation if at least one author has a working address there (we are not concerned with the authors' nationalities). Hence it can count for several countries. The data were taken in July 2020 from the "old" INSPIRE version, `https://old.inspirehep.net`, which is deactivated now, unfortunately.[2]

We are going to present data for *global* and for *national* statistics, first from an *extensive* and then from an *intensive* perspective, *i.e.* absolute and relative to the population, respectively. (We do

[1] Also since 1992, the annual Lattice Conference (or Symposium) attained an extent of $\gtrsim 200$ contributions.

[2] The new `INSPIRE` version [2] is not useful for such a study: if we fix *e.g.* "date", much larger numbers appear than in the old version, due to multiple counting. The command "de" (data earliest) is supposed to overcome this effect, but it returns weird results, which are much too low (*e.g.* just one `hep-lat` entry in 2019). Moreover, the new version does not have the option "country code", hence it does not provide data for national statistics.

not count contributions by specific authors or collaborations.) In the special case of Switzerland, we exclude CERN as an affiliation. As a single parameter, the Hirsch Index H [3] can be applied in extensive rankings, but not in intensive ones. As another single parameter for the scientific activity and achievement, we define

$$\Sigma := E + P + 0.05C\,, \tag{1}$$

where the coefficients ("weights") are motivated by the statistical trends to be presented below, see in particular Table 1. The index Σ does have an intensive counterpart, see Section 3.

We compare these data with two socio-economic parameters: as an economic index, we consider the Gross Domestic Product (GPD), given in 10^9 US dollars (more precisely: the value of its purchasing parity in 2011). Its intensive version is the GPDpp (per capita). As another intensive quantity, which seems likely to be related to the scientific performance, we consider the Education Index (EI),[3] which is defined as

$$\mathrm{EI} := \frac{1}{2}\Big(\frac{\mathrm{EYS}}{18} + \frac{\mathrm{MYS}}{15}\Big)\,. \tag{2}$$

EYS means "expected years of schooling" for children (normalized by the duration for a Master's degree), while MYS are the "mean years of schooling" of adults (normalized to the projected maximum in 2025). For each nation and year one obtains $\mathrm{EI} \in [0,1]$. Our source for the annual GDP, GPDpp and EI is the United Nations Development Programme [5]; we average over the years from 1992 to 2019. The population is averaged over the same period; it is expressed in millions of inhabitants (we do not include tiny countries with less than 10^5 inhabitants). We also intended to include the percentage of "skilled labor force", but the data for different countries do not seem to be based on consistent criteria.[4]

2. Extensive Statistics

Figure 1 shows global data for the annual evolution from 1992 to 2019 in the `hep-lat`, and in all the 7 `arXiv` sections under consideration (in 1991 there were only a few sporadic entries and 2020 we could only capture the first half). The total statistics, summed up from 1991 to July 2020, for each of the 7 arXiv sections is displayed in Table 1.

Figure 2 shows the individual time evolutions of `hep-lat`-entries E for the 12 leading nations in this respect, with $E > 400$. Table 2 contains the national sums of E, P and C from 1991 to July 2020, along with the population in millions and the GDP (both averaged from 1992 to 2019). For comparison, we add the European Union (with 28 nations, still including the UK) and CERN, but the ranking only refers to nations. It is based on the Hirsch Index, H-rank, but we also show the Σ-rank, which is very similar. This confirms that the definition of Σ of eq. (1) is sensible.

3. Intensive Statistics

Figure 3 shows scatter plots for $\sigma := \Sigma/\mathrm{pop}$ (pop: population in millions of inhabitants) vs. GPDpp and vs. EI. We include the 101 nations with $H > 5$. The symbols for the 45 dominant

[3] In another statistical study, which deals with High Energy Physics in Latin America [4], we considered separately the EI and the Human Development Index (HDI). The HDI is the geometric mean of indices of income (in purchasing parity), health and the EI. However, the EI and HDI tend to be similar in most countries, so here we refrain from a separate consideration.

[4] For instance, Japan reports 99.9% "skilled labor force", but Italy and Portugal only 69.6% and 54.1%, respectively.

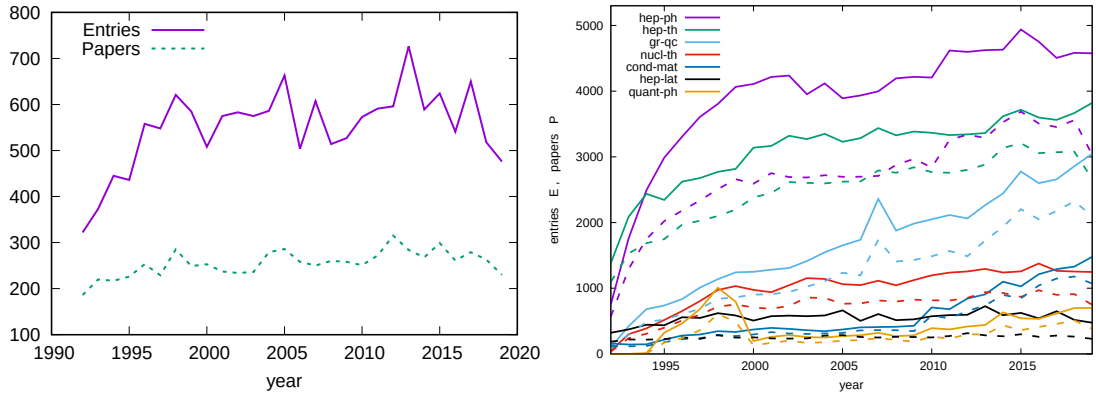

Figure 1: Research activity from 1992 to 2019. Left: `hep-lat` entries (E) and papers (P). Right: E [solid] and P [dashed] in the `arXiv` sections `hep-ph` (purple), `hep-th` (green), `gr-qc` (cyan), `nucl-th` (red), `cond-mat` (blue), `hep-lat` (black) and `quant-ph` (orange). They are hierarchically ordered, cf. Table 1. We see some peaks of E and P, in particular for `quant-ph` in the period from 1995 to 2000. Condensed matter physics has a high productivity, but only part of its articles are submitted to the `arXiv`; we see, however, that this is becoming more frequent since 2008.

	hep-ph	hep-th	gr-qc	nucl-th	cond-mat	hep-lat	quant-ph
E	111515	89279	48927	28522	16969	15610	11602
P	76520	70561	35703	20215	13677	7165	7484
C	3960720	2857462	1043823	682874	247734	402121	106901
P/E	0.69	0.79	0.71	0.71	0.81	0.46	0.65
C/E	35.5	32.0	21.3	23.9	14.6	25.8	9.21

Table 1: Total parameters E, P and C, for 7 arXiv sections, summed up from 1991 to July 2020, along with the publication fraction P/E and the citation rate C/E. Note the peculiarity $P < E/2$ for `hep-lat`, which might be related to the particular importance of the proceedings of the annual Lattice Conference; for a number of results, the authors are satisfied if the appear in these "lattice proceedings". Also the citation rate varies strongly between the different sections.

nations in these plots can be identified from Tables 4 and 5 for the intensive `hep-lat` statistics and summed over the 7 `arXiv` sections, respectively. These tables for intensive quantities further include the GDPpp, the EI and $(e, p, c) := (E, P, C)/\text{pop}$. Here the ranking is based on the parameter $\sigma = \Sigma/\text{pop}$, but we also display the economic rank (e-rank) based on $\Sigma/\text{GPD} \propto \sigma/\text{GPDpp}$.

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References

[1] https://arxiv.org/

[2] https://inspirehep.net/

[3] J. E. Hirsch, PNAS 102 (2005) 16569-72.

[4] G. Urrutia Sánchez, L. Prado and W. Bietenholz, Scientometrics 116 (2018) 125-146.

[5] http://hdr.undp.org/

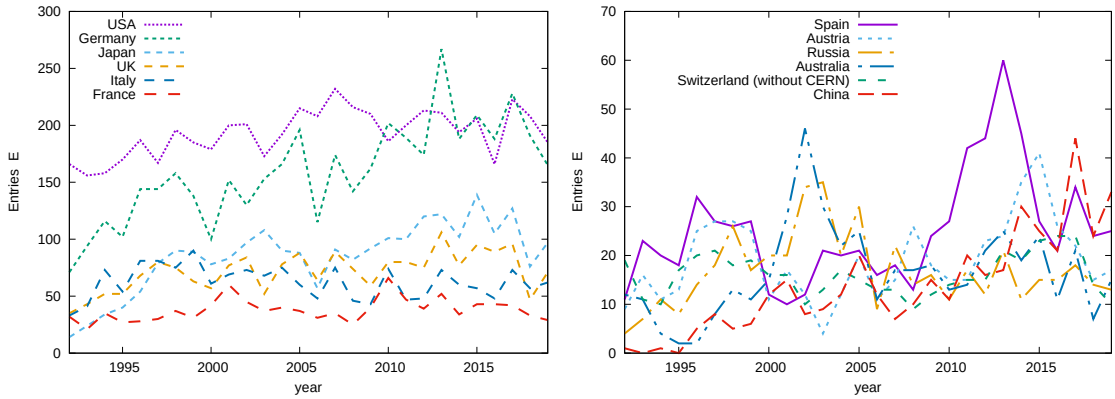

Figure 2: Evolution of the number of `hep-lat` entries (E) of the 6 leading nations (left), and of the nations ranking 7 to 12 (right), cf. Table 2. We see a slight general trend up. During the last decade, Germany has caught up with the USA as the most productive countries, followed by Japan, UK, Italy, France, etc. There are remarkable peaks for Germany, Spain, Austria and Australia. Lately China is moving up.

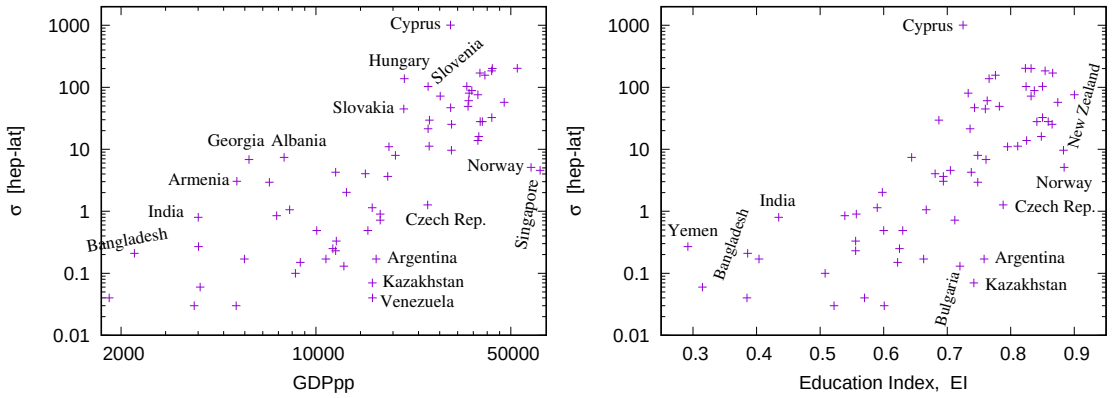

Figure 3: Scatter plots for the 66 nations which contributed any `hep-lat` entries. We show $\sigma := \Sigma/\text{pop}$ (pop = population in millions) vs. GDPpp (left), and vs. EI (right). Monotonic trends are visible, but not as clearly as one might expect. The top 45 nations can be identified from Table 4. We indicate some nations which are clearly off the dominant trend (above or below).

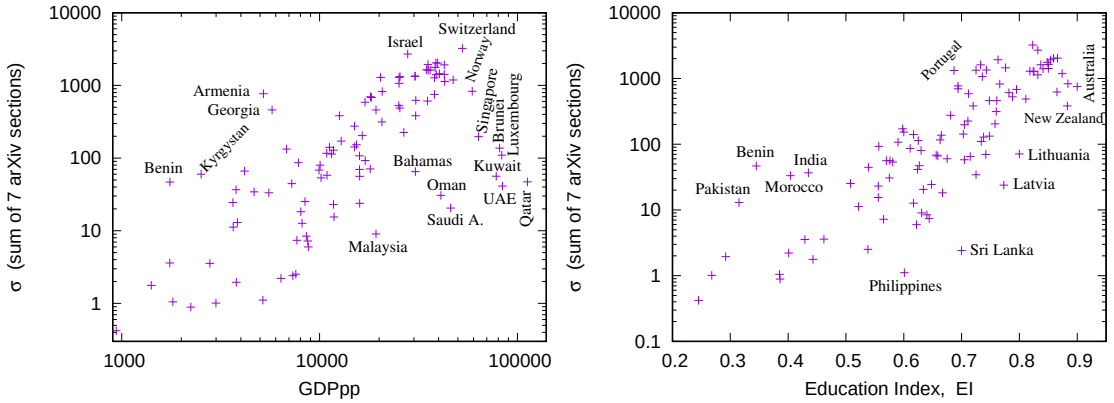

Figure 4: Like Figure 3, but for the sum over 7 `arXiv` sections. The plots capture the 101 nations with $H > 5$, the leading 45 can be identified from Table 5. The monotonic trend is somewhat clearer than in Figure 3, which is lattice-specific. Again we indicate some nations significantly off the dominant trend.

	Σ-rank	population	GDP	E	P	C	Σ	H
European Union		495.0	15900.4	9020	4265	249236	**25746.8**	**182**
1. USA	1	294.3	14050.1	5472	2565	175639	**16818.9**	**167**
2. Germany	2	81.3	3167.4	4523	2107	143667	**13813.3**	**155**
3. UK	3	61.4	2136.2	2026	950	67386	**6345.3**	**121**
4. Japan	4	127.6	4458.1	2419	1078	55729	**6283.5**	**103**
5. France	6	61.1	2232.9	1070	540	42060	**3713.0**	**101**
6. Italy	5	58.5	2066.6	1765	877	41252	**4704.6**	**89**
CERN				615	306	27553	**2298.7**	**79**
7. Switzerland	8	7.5	400.7	451	228	16744	**1516.2**	**75**
8. Spain	7	43.6	1326.8	708	370	19310	**2043.5**	**71**
9. Australia	9	18.9	801.9	471	250	14208	**1431.4**	**64**
10. Hungary	10	10.1	208.5	292	151	18824	**1384.2**	**61**
11. Cyprus	14	1.0	23.1	389	193	8794	**1021.7**	**54**
12. Austria	11	8.3	333.8	534	254	10300	**1303.0**	**51**
13. China	13	1326.5	9638.8	415	223	9708	**1123.4**	**50**
14. Denmark	15	5.4	233.1	322	174	10188	**1005.4**	**50**
15. Russia	12	145.5	2822.8	475	252	8608	**1157.4**	**48**
16. Canada	17	32.5	1269.4	325	164	8387	**908.3**	**46**
17. India	16	1142.0	4509.3	316	183	8198	**908.9**	**45**
18. Ireland	18	4.2	183.7	292	117	8565	**837.3**	**45**
19. Taiwan	20	22.5	702.2	196	100	5491	**570.6**	**40**
20. Finland	23	5.3	191.6	165	81	4460	**469.0**	**36**
21. Netherlands	19	16.3	696.5	203	98	4544	**528.2**	**35**
22. Israel	22	6.6	197.8	149	96	4622	**476.1**	**35**
23. South Korea	21	48.3	1242.6	238	82	4448	**542.4**	**34**
24. Poland	24	38.3	696.8	181	82	3192	**422.6**	**30**
25. Brazil	25	184.4	2397.1	117	58	3939	**372.0**	**29**
26. Sweden	27	9.2	365.9	99	50	2162	**257.1**	**27**
27. Slovenia	30	2.0	51.1	67	33	2174	**208.7**	**26**
28. Portugal	26	10.4	264.0	138	55	2252	**305.6**	**25**
29. Slovakia	28	5.4	111.5	82	42	2354	**241.7**	**25**
30. Greece	29	10.9	273.9	96	49	1756	**232.8**	**22**
31. Belgium	31	10.7	409.5	54	28	1792	**171.6**	**20**
32. Mexico	33	106.6	1710.8	58	26	760	**122.0**	**15**
33. Ukraine	32	47.5	319.9	85	34	417	**139.9**	**11**
34. Turkey	34	68.2	1192.0	25	19	347	**61.3**	**10**
35. New Zealand	36	4.1	127.7	11	8	420	**40.0**	**8**
36. Bangladesh	39	137.0	317.0	10	6	267	**29.4**	**7**
37. Belarus	35	9.7	113.6	9	6	527	**41.3**	**6**
38. Iran	37	69.9	1088.1	17	11	126	**34.3**	**6**
39. Georgia	38	4.4	22.6	11	7	236	**29.8**	**5**
40. Norway	40	4.7	280.4	14	6	83	**24.2**	**5**
41. Albania	41	3.0	22.9	15	2	107	**22.4**	**5**
42. Singapore	43	4.5	297.6	10	5	113	**20.7**	**4**

Table 2: Extensive `hep-lat` statistics: ranking according to the Hirsch Index H, which is very similar to the Σ-rank. Both ranks refer to nations only; in case of identical H-indices, Σ decides; further nations with $H = 4$ are Croatia, Uruguay and the Czech Republic. According to the Σ-rank, Thailand is at position 42. Population in millions of inhabitants, Gross Domestic Product (GDP) in 10^9 US $ (purchasing parity in 2011), both averaged from 1992 to 2019. We add the European Union — with 28 nations, still including the UK — and CERN. Here and throughout, the data for Switzerland exclude CERN.

	Σ-rank	E	P	C	Σ	H
1. USA	1	85601	63422	4006414	**349343.7**	**583**
European Union		147464	106804	4961069	**502321.4**	**548**
2. France	3	28598	21090	1368170	**118096.5**	**394**
3. Germany	2	45242	35533	1713406	**166445.3**	**390**
4. UK	4	26174	19927	1089933	**100597.7**	**339**
CERN		9836	7253	682730	**51225.5**	**333**
5. Italy	5	27034	19875	949801	**94399.1**	**309**
6. Spain	9	16466	12223	601111	**58744.6**	**267**
7. Russia	7	22429	15260	586236	**67000.8**	**257**
8. Canada	10	12364	9696	469849	**45552.5**	**250**
9. Japan	6	24611	18620	690235	**77742.8**	**249**
10. Switzerland	14	5734	4142	290056	**24378.8**	**227**
11. Netherlands	16	5634	4307	262786	**23080.3**	**207**
12. Poland	13	8205	5660	245108	**26120.4**	**191**
13. China	8	20475	16372	444642	**59079.1**	**186**
14. India	11	14374	10925	332859	**41942.0**	**186**
15. Sweden	18	4973	3753	198321	**18642.1**	**178**
16. Belgium	17	4955	3798	201122	**18809.1**	**171**
17. Israel	19	5085	3943	176081	**17832.1**	**166**
18. Brazil	12	11460	8961	222670	**31554.5**	**146**
19. South Korea	15	7627	6096	196954	**23570.7**	**142**
20. Portugal	22	4166	3087	128890	**13697.5**	**139**
21. Austria	23	3885	2563	112541	**12075.1**	**137**
22. Greece	24	3505	2748	107266	**11616.3**	**134**
23. Australia	20	4185	3219	136125	**14210.3**	**133**
24. Denmark	27	3006	2248	103328	**10420.4**	**133**
25. Taiwan	21	4110	3186	134243	**14008.2**	**129**
26. Finland	29	2460	1818	86101	**8583.1**	**128**
27. Hungary	30	2274	1586	88347	**8277.4**	**121**
28. Chile	28	3089	2567	77183	**9515.2**	**108**
29. Argentina	31	2576	2179	63899	**7950.0**	**104**
30. Mexico	25	4450	3232	75687	**11466.4**	**101**
31. Ireland	36	1476	1078	43298	**4718.9**	**97**
32. Iran	26	4054	3398	64526	**10678.3**	**89**
33. South Africa	34	1903	1483	44556	**5613.8**	**85**
34. Slovenia	41	718	484	28219	**2613.0**	**85**
35. Norway	37	1242	923	35198	**3924.9**	**84**
36. Ukraine	33	2508	1674	42411	**6302.6**	**82**
37. Czech Rep.	35	2106	1501	37754	**5494.7**	**78**
38. Bulgaria	40	1040	684	24849	**2966.5**	**73**
39. Croatia	38	1035	778	25216	**3073.8**	**71**
40. Estonia	47	425	336	20056	**1763.8**	**71**
41. Turkey	32	2571	2112	32563	**6311.2**	**62**
42. Georgia	45	693	512	16033	**2006.7**	**62**
43. Armenia	42	835	635	17331	**2336.6**	**61**
44. Slovakia	48	642	394	13173	**1694.7**	**61**
45. New Zealand	49	495	362	14576	**1585.8**	**60**

Table 3: Like Table 2, but with extensive, joint statistics for the 7 arXiv sections under consideration, for all nations with $H \geq 60$. Based on the Σ-rank, Romania (39), Colombia (43) and Pakistan (44) enter the top 45.

	e-rank	GDPpp	EI	e	p	c	Σ/GDP	σ
1. Cyprus	1	30386	0.725	383.32	190.18	8665.6	**44.17**	**1006.78**
2. Switzerland	8	52698	0.823	60.13	30.40	2232.5	**3.78**	**202.16**
3. Ireland	3	42903	0.832	70.27	28.16	2061.1	**4.56**	**201.48**
4. Denmark	5	42671	0.854	59.22	32.00	1873.8	**4.31**	**184.02**
5. Germany	4	38719	0.866	55.60	25.90	1766.1	**4.36**	**169.81**
6. Austria	7	40348	0.776	64.31	30.59	1240.4	**3.90**	**156.92**
7. Hungary	2	20738	0.766	29.04	15.02	1872.0	**6.64**	**137.66**
8. UK	9	34770	0.850	32.99	15.47	1097.4	**2.97**	**103.33**
9. Slovenia	6	25235	0.824	33.01	16.26	1071.1	**4.08**	**102.83**
10. Finland	10	36205	0.837	31.18	15.30	842.7	**2.45**	**88.61**
11. Italy	12	35411	0.733	30.19	15.00	705.6	**2.28**	**80.47**
12. Australia	14	38067	0.900	24.95	13.25	752.8	**1.79**	**75.84**
13. Israel	11	27906	0.832	22.58	14.55	700.3	**2.41**	**72.14**
14. France	15	35303	0.763	17.52	8.42	688.8	**1.66**	**60.80**
15. USA	19	47324	0.874	18.59	8.72	596.9	**1.20**	**57.15**
European Union		31966	0.792	18.22	8.62	503.5	**1.62**	**52.01**
16. Japan	17	35125	0.782	18.96	8.45	436.8	**1.41**	**49.25**
17. Spain	16	30406	0.743	16.24	8.48	442.8	**1.54**	**46.86**
18. Slovakia	13	20657	0.760	15.19	7.78	435.9	**2.17**	**44.76**
19. Netherlands	24	42689	0.850	12.48	6.02	279.3	**0.76**	**32.46**
20. Portugal	20	25515	0.687	13.33	5.31	217.5	**1.16**	**29.52**
21. Canada	25	38828	0.841	10.01	5.05	258.4	**0.72**	**27.98**
22. Sweden	26	39526	0.859	10.77	5.44	235.3	**0.70**	**27.98**
23. Taiwan	23	30654	0.865	8.70	4.44	243.7	**0.81**	**25.33**
24. Greece	22	25240	0.736	8.84	4.51	161.7	**0.85**	**21.43**
25. Belgium	31	38398	0.848	5.07	2.63	168.1	**0.42**	**16.10**
26. Iceland	34	38087	0.825	3.33	3.33	143.3	**0.36**	**13.83**
27. South Korea	30	25473	0.811	4.93	1.70	92.1	**0.44**	**11.24**
28. Poland	27	18267	0.795	4.72	2.14	83.3	**0.61**	**11.02**
29. New Zealand	35	30601	0.883	2.66	1.93	101.4	**0.31**	**9.66**
30. Russia	32	19274	0.748	3.27	1.73	59.2	**0.41**	**7.96**
31. Albania	21	7683	0.644	4.96	0.66	35.4	**0.98**	**7.39**
32. Georgia	18	5752	0.761	2.52	1.60	54.0	**1.32**	**6.82**
33. Norway	43	59066	0.884	2.96	1.27	17.6	**0.09**	**5.11**
34. Singapore	46	63641	0.705	2.22	1.11	25.0	**0.07**	**4.58**
35. Belarus	33	11763	0.738	0.93	0.62	54.4	**0.36**	**4.27**
36. Uruguay	36	15032	0.681	2.11	1.51	8.1	**0.27**	**4.03**
37. Croatia	37	18091	0.694	2.05	0.91	13.9	**0.20**	**3.65**
38. Armenia	28	5207	0.694	2.30	0.66	1.6	**0.60**	**3.05**
39. Ukraine	29	6806	0.748	1.79	0.72	8.8	**0.44**	**2.95**
40. Brazil	39	12856	0.598	0.63	0.31	21.4	**0.16**	**2.02**
41. Czech Rep.	48	25149	0.788	0.48	0.48	6.3	**0.05**	**1.27**
42. Mexico	45	15910	0.590	0.54	0.24	7.1	**0.07**	**1.14**
43. Jordan	40	8043	0.667	0.46	0.46	2.6	**0.13**	**1.06**
44. Turkey	47	16994	0.557	0.37	0.28	5.1	**0.05**	**0.90**
45. China	41	7231	0.539	0.31	0.17	7.3	**0.12**	**0.85**

Table 4: Intensive `hep-lat` statistics, with a ranking according to $\sigma = \Sigma/\text{pop}$, for all nations with $\sigma > 0.8$. We add further intensive quantities: GDPpp, EI, $(e, p, c) := (E, P, C)/\text{pop}$. We also display the economic rank, e-rank, based on $\Sigma/\text{GDP} \propto \sigma/\text{GPDpp}$. In that regard, India (38), Bangladesh (42) and Yemen (44) enter the top 45.

	e-rank	e	p	c	Σ/GDP	σ
1. Switzerland	5	760.78	549.55	38484.1	**60.84**	**3234.53**
2. Israel	2	770.45	597.42	26678.9	**90.16**	**2701.83**
3. Germany	8	556.15	436.80	21062.6	**52.55**	**2046.09**
4. Sweden	11	541.20	408.43	21582.7	**50.95**	**2028.76**
5. France	7	468.31	345.36	22404.5	**52.89**	**1933.89**
6. Denmark	16	552.87	413.46	19004.5	**44.70**	**1916.56**
7. Belgium	13	464.85	356.31	18868.3	**45.93**	**1764.58**
8. United Kingdom	12	426.24	324.50	17749.2	**47.09**	**1638.20**
9. Finland	15	464.80	343.50	16268.2	**44.79**	**1621.71**
10. Italy	14	462.38	399.94	16245.2	**45.68**	**1614.58**
11. Austria	27	467.86	308.66	13553.1	**30.09**	**1454.18**
12. Netherlands	24	346.27	264.71	16151.2	**33.14**	**1418.55**
13. Canada	22	380.91	298.71	14475.0	**35.89**	**1403.37**
14. Spain	17	377.60	280.30	13784.6	**44.28**	**1347.12**
15. Cyprus	6	489.74	267.04	11504.6	**58.43**	**1332.02**
16. Portugal	9	402.44	298.21	12450.9	**51.88**	**1323.19**
17. Estonia	4	310.98	245.85	14675.1	**64.03**	**1290.59**
18. Slovenia	10	353.76	238.47	13903.5	**51.14**	**1287.40**
19. Iceland	25	476.67	380.00	8336.7	**33.02**	**1273.50**
20. United States	30	290.89	215.52	13614.6	**24.86**	**1187.14**
21. Ireland	29	355.19	259.41	10419.3	**25.69**	**1135.56**
22. Greece	18	322.66	252.97	9874.5	**42.41**	**1069.35**
European Union		297.91	215.77	10022.5	**31.59**	**1014.81**
23. Norway	42	262.81	195.31	7447.9	**14.00**	**830.50**
24. Hungary	19	226.14	157.72	8785.9	**39.70**	**823.16**
25. Armenia	1	274.94	209.09	5706.5	**150.46**	**769.35**
26. Australia	39	221.73	170.55	7212.3	**17.72**	**752.90**
27. Croatia	20	235.43	176.97	5735.7	**39.12**	**699.18**
28. Poland	21	214.02	147.64	6393.5	**37.49**	**681.34**
29. Taiwan	34	182.44	141.43	5959.0	**19.95**	**621.82**
30. Japan	40	192.89	145.94	5409.8	**17.44**	**609.32**
31. Chile	23	190.64	158.42	4763.3	**33.86**	**587.22**
32. Czech Rep.	33	201.85	143.87	3618.6	**21.02**	**526.65**
33. South Korea	36	158.01	126.29	4080.2	**18.97**	**488.31**
34. Russia	31	154.18	104.90	4029.9	**23.74**	**460.58**
35. Georgia	3	158.70	117.25	3671.7	**88.78**	**459.54**
36. New Zealand	46	119.54	87.42	3520.1	**12.42**	**382.98**
37. Bulgaria	26	134.16	88.24	3205.6	**30.90**	**382.68**
38. Slovakia	41	118.89	72.96	2439.4	**15.20**	**313.82**
39. Uruguay	38	92.61	76.32	2119.0	**18.14**	**274.89**
40. Malta	54	82.50	72.50	1405.0	**8.18**	**225.25**
41. Argentina	44	66.17	55.97	1641.4	**12.29**	**204.21**
42. Singapore	69	82.24	62.29	1055.8	**2.99**	**197.32**
43. Brazil	43	62.14	48.59	1207.4	**13.16**	**171.11**
44. Iran	50	58.02	48.63	923.5	**9.81**	**152.83**
45. Romania	51	58.77	42.72	823.5	**9.69**	**142.67**

Table 5: Like Table 4, but joint statistics for the 7 `arXiv` sections. The e-rank strongly deviates from the σ-rank for Georgia, Armenia, Estonia, Bulgaria (up), and for Singapore, Norway, Austria, Malta, Australia, Netherlands (down). According to the e-rank, Benin (28), Kyrgyzstan (32), Ukraine (35), Moldova (37) and Lebanon (45) are among the top 45.