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Finite temperature phase transition for three flavor QCD with Möbius-domain wall fermions

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The nature of the finite temperature phase transition in (2+1)-flavor QCD depends on the quark mass, and the order and universal class of the phase transition are shown in a diagram called the Columbia plot. The region of light quark masses in this diagram is not yet fully understood. We present preliminary results of a three-flavor QCD study using Möbius-domain wall fermions to search for the critical endpoint in the light quark mass region on the diagonal line of the Columbia plot.

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1. Introduction

Quantum Chromodynamics (QCD) is one of the parts of the Standard Model of elementary particles, whose dynamics produce 99% of the mass of visible matter in the universe. The quarks that make up the protons and neutrons responsible for most of that mass are not found alone due to a special property of QCD called confinement. This property, however, is expected to be lost at sufficiently high temperatures. In fact, heavy-ion collisions at RHIC in the United States and the LHC in Europe have observed a gas state of free quarks and gluons at high temperatures, a phenomenon that seems to be characteristic of quark-gluon plasmas. In the process of decreasing the temperature of the universe through the expansion of the universe, a major change in the thermodynamics of the universe occurs when confinement is triggered to create protons and neutrons from the quark-gluon plasma state. Whether this change is a true phase transition or a divergence-free crossover will have a major impact on solving the mystery of matter formation. Also, if a phase transition actually occurred, detection of its remnants by gravitational waves is expected.

For quantitative analysis of QCD behavior at finite temperatures, numerical methods based on lattice gauge theory are powerful. The results of previous calculations suggest that there is no phase transition at the transition temperature between the plasma and hadronic phases, but rather a crossover transition [1], but their theoretical background is not satisfactory because the symmetry of the system is recovered only in the continuum limit. No sign of a phase transition has been found experimentally, but it would be difficult to detect a phase transition when it is weak.

QCD is a theory that is completely defined by determining the quark mass, which is a parameter of the Standard Model, and there is only one unique set of parameters that describe the real world, which is called the "physical point". When we change the mass parameters from the physical point for theoretical interest, which is impossible in experiments, we would find a phase transition somewhere. This is because the low-temperature phase, in which the chiral symmetry is spontaneously broken, and the high-temperature phase, in which it is recovered, cannot be analytically connected in the limit of zero quark mass. It is expected that a phase transition is necessary at least at the zero quark mass point, and that a phase transition can occur even around that point [2]. If there is no phase transition in the real world (physical point), it means that the quark mass of the physical point happens to be in such a region. Naturally, understanding the physics around the physical point will lead to a deeper understanding of the real world.

To summarize the finite temperature phase transition of QCD with various quark masses the "Columbia plot" is often used (Fig. 1). Since the three heaviest of the six quarks have only a small perturbative effect on the phase transition, it is sufficient to treat only the three lightest (u, d, s quarks) in the simulations. Furthermore, since the isospin symmetry (the exchange of u and d) is very good, we denote the mass $m_u = m_d = m_{ud}$ and consider it as one parameter. The other parameter is the mass of the s quark, m_s. The Columbia University collaboration [3] first illustrated the phase boundary between the phase transition and crossover with lattice QCD calculations. Since their study more precise calculations have been made, and a generally appreciated Columbia plot now is Fig. 1. Only the upper right corner ($m_{ud} = m_s = \infty$) and the physical point(★) are studied to be in the continuum limit. All other information contains systematic errors, and in fact, the position and shape of the phase boundary is undetermined, or there is no consensus between different approaches. Another unsatisfactory point is that the position of the physical point is calculated by staggered fermions, which may be questionable, although the continuum limit is taken. As a recent progress, the possibility of a significant reduction of the first order transition region or even the disappearance of the region in the continuum limit has been discussed with Wilson or staggered actions for three flavor ($N_f = 3$) case [4, 5].

In this context, we employ chiral fermions as a lattice QCD action to treat the spontaneous breaking and recovery of chiral symmetry more precisely in numerical simulations.

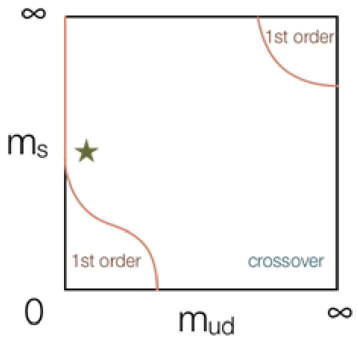

Figure 1: Columbia plot. ★ denotes the physical point.

2. Numerical setup

We generate gauge ensembles with three flavor Möbius domain-wall fermions, and the tree-level Symanzik improved gauge action at $\beta = 4.0$ by using Grid [6]. We use a version of Grid optimized for A64FX which is the CPU of Fugaku [7]. The gauge links for Möbius domain-wall fermion are smeared by three steps of stout smearing with a coefficient $\alpha = 0.1$. The fifth dimension for domain-wall fermion is $L_s = 16$. This L_s is slightly larger than the value used by JLQCD collaboration in zero temperature simulations with larger β. This is because, in our finite temperature simulations, we want to keep the temperature low with a small temporal lattice size, N_T, and keep the residual mass, m_{res}, low at the same time. The lattice spacing is computed by Wilson flow method [8] with BQCD [9]. The inverse of lattice spacing a is $1/a = 1.57(16)$ GeV estimated through $N_f = 2 + 1$ theory [10], corresponding $a \sim 0.127$ fm and m_{res} in the lattice unit is $m_{res} \sim 0.0062$ through mid-point correlation function measured using Hadrons [11].

We take two values of N_T in our simulations. One is temporal lattice size $N_T = 6$ and the spatial lattice sizes $N_L = 12, 16$. The temperature T is $T = 262(27)$ MeV at $N_T = 6$. The other is $N_T = 8$, $N_L = 16$, and $T = 196(20)$ MeV. We have only performed simulations with one N_L for $N_T = 8$ so far. When we find a sign of a phase transition, we plan to perform simulations at a larger N_L around that point. Statistics is 100,000 trajectories for each parameter. We measure observables every 10 trajectories. Our main simulations have been done on the supercomputer “Fugaku” at RIKEN, and the supercomputer system “ITO” at Kyushu University was also used in the preparatory research, as selected projects in regular calls for application proposal High Performance Computing Infrastructure (HPCI) resources in Japan and HOKUSAI at RIKEN.

3. Numerical results

We compute plaquette, plaquette susceptibility, and topological charge susceptibility in the range of quark mass in the lattice unit $0 \le m \le 0.4$ both at $N_T = 6, 8$. The topological charge susceptibility is measured by using Wilson flow method at flow time $\tau = 5$. Figs. 2 and 3 show plaquette, plaquette susceptibility, and topological charge susceptibility as a function of m. We see a peak of plaquette susceptibility around $m \sim 0.15$ at $N_T = 6$, and a rapid change for plaquette and topological charge susceptibility. Since there is no significant difference in these values when the spatial volume is changed, we consider this to be a crossover. We can see similar behavior in range $0.05 \le m \le 0.1$ at $N_T = 8$. There are no other spatial volume simulations at $N_T = 8$, so it is too early to discuss whether it is a phase transition or crossover.

4. Summary

We are studying the finite temperature phase transition for three-flavor QCD with Möbius-domain wall fermions at $N_T = 6$ and 8, and measured plaquette and topological charge so far. We found a peak in the plaquette susceptibility at $N_T = 6$, $\beta = 4.0$, $m \sim 0.15$, which is around 240 MeV. The transition seems to be a crossover. We also found a peak at $N_T = 8$, $\beta = 4.0$ around $0.05 < m < 0.1$, but we have analyzed it only in one spatial volume, so we can not tell the order of the phase transition or whether it is a crossover or not. The quark masses that show signs of these phase transitions are as heavy as or heavier than the strange quark mass. We are measuring fermionic observables and also performing $N_T = 12$ simulations corresponding to $T = 131$ MeV.

This work used computational resources of the supercomputers Fugaku provided by RIKEN (Project ID : hp210032) and Ito at Kyushu University (Project IDs : hp200050 and hp190124) through the HPCI System Research Project and HOKUSAI (Project ID : G20018) at RIKEN. This work is supported in part of the JSPS KAKENHI grant No.20H01907.

References

[1] Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz and K.K. Szabo, *The Order of the quantum chromodynamics transition predicted by the standard model of particle physics*, *Nature* **443** (2006) 675 [`hep-lat/0611014`].

[2] R.D. Pisarski and F. Wilczek, *Remarks on the chiral phase transition in chromodynamics*, *Phys. Rev. D* **29** (1984) 338.

[3] F.R. Brown, F.P. Butler, H. Chen, N.H. Christ, Z.-h. Dong, W. Schaffer et al., *On the existence of a phase transition for QCD with three light quarks*, *Phys. Rev. Lett.* **65** (1990) 2491.

[4] Y. Kuramashi, Y. Nakamura, H. Ohno and S. Takeda, *Nature of the phase transition for finite temperature* $N_{\mathrm{f}} = 3$ *QCD with nonperturbatively O(a) improved Wilson fermions at* $N_{\mathrm{t}} = 12$, *Phys. Rev. D* **101** (2020) 054509 [`2001.04398`].

[5] F. Cuteri, O. Philipsen and A. Sciarra, *On the order of the QCD chiral phase transition for different numbers of quark flavours*, *JHEP* **11** (2021) 141 [`2107.12739`].

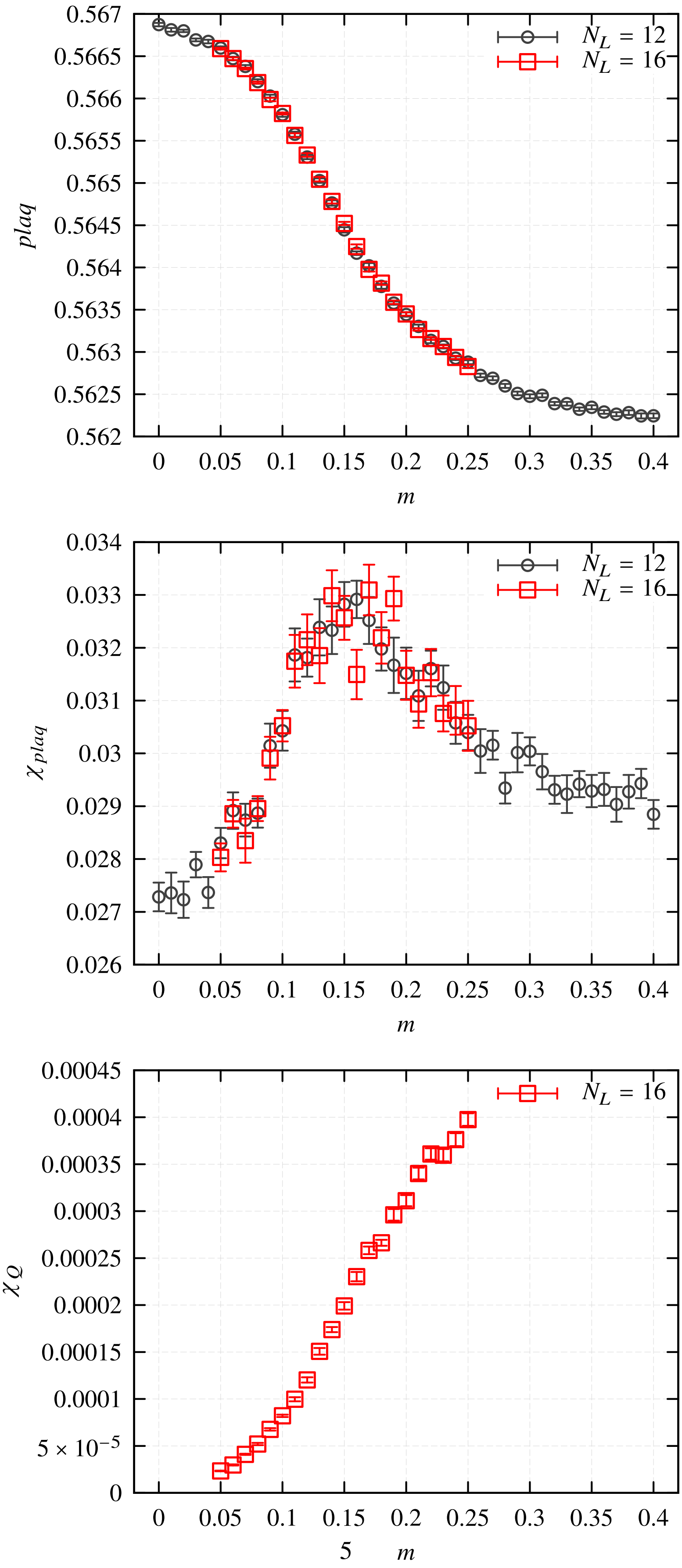

Figure 2: Plaquette (top), plaquette susceptibility (middle), and topological charge susceptibility (bottom) as a function of m at $N_T = 6$, $\beta = 4.0$ ($T = 262(27)$ MeV).

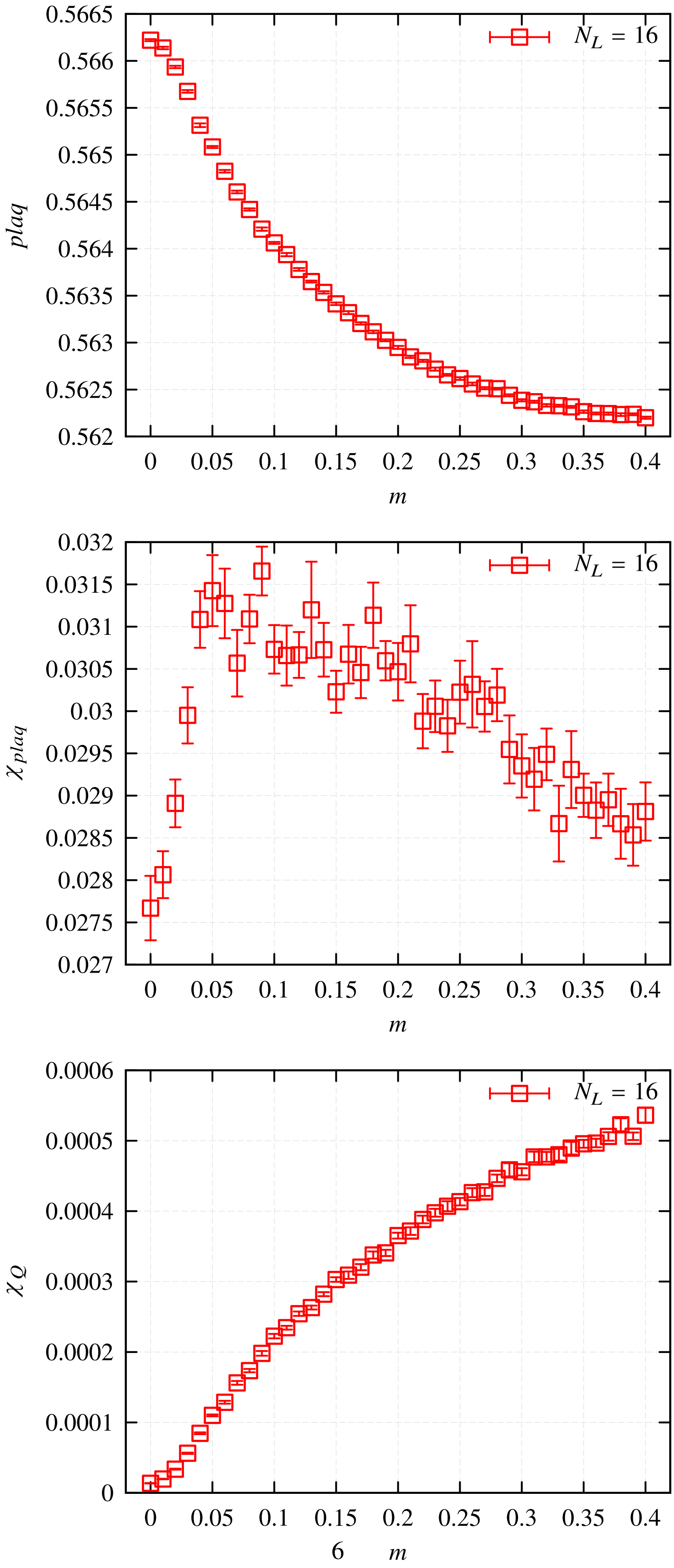

Figure 3: Plaquette (top), plaquette susceptibility (middle), and topological charge susceptibility (bottom) as a function of m at $N_T = 8$, $\beta = 4.0$ ($T = 196(20)$ MeV).

[6] P.A. Boyle, G. Cossu, A. Yamaguchi and A. Portelli, *Grid: A next generation data parallel C++ QCD library*, *PoS* **LATTICE2015** (2016) 023.

[7] N. Meyer, D. Pleiter, S. Solbrig and T. Wettig, *Lattice QCD on upcoming Arm architectures*, *PoS* **LATTICE2018** (2019) 316 [`1904.03927`].

[8] M. Luscher, *Properties and uses of the Wilson flow in lattice QCD*, *JHEP* **08** (2010) 071 [`1006.4518`].

[9] Y. Nakamura and H. Stuben, *BQCD - Berlin quantum chromodynamics program*, *PoS* **LATTICE2010** (2010) 040 [`1011.0199`].

[10] S. Aoki, Y. Aoki, H. Fukaya, S. Hashimoto, I. Kanamori, T. Kaneko et al., *2+1 flavor fine lattice simulation at finite temperature with domain wall fermions*, *PoS* **LATTICE2021** 609.

[11] A. Portelli, N. Asmussen, P. Boyle, F. Erben, V. Gülpers, R. Hodgson et al., *aportelli/hadrons: Hadrons v1.2*, Nov., 2020. 10.5281/zenodo.4293902.