PROCEEDINGS OF SCIENCE

Magnetic monopole dominance for the Wilson loops in higher representations

Akihiro Shibata,[a,b,*] **Seikou Kato**[c] **and Kei-Ichi Kondo**[d]

[a] *Computing Research Center, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan*

[b] *Department of Accelerator Science, SOKENDAI (The Graduate Univercity for Advanced Studies), Tsukuba 305-0801, Japan*

[c] *Oyama National College of Technology, Oyama 323-0806, Japan*

[d] *Department of Physics, Graduate School of Science, Chiba University, Chiba 263-8522, Japan*

E-mail: [a,b]Akihiro.Shibata@kek.jp, [c]skato@oyama-ct.ac.jp, [d]kondok@faculty.chiba-u.jp

The dual superconductor picture is one of the most promising scenarios for quark confinement. To investigate this picture in a gauge-invariant manner, we have proposed a new formulation of Yang-Mills theory, named the decomposition method, on the lattice. The so-called restricted field obtained from the gauge-covariant decomposition plays the dominant role in quark confinement. It has been known by preceding works that the restricted-field dominance is not observed for the Wilson loop in higher representations if the restricted part of the Wilson loop is obtained by adopting the Abelian projection or the field decomposition naively in the same way as done in the fundamental representation. Recently, through the non-Abelian Stokes theorem (NAST) for the Wilson loop operator, we have proposed suitable gauge-invariant operators constructed from the restricted field to reproduce the correct behavior of the original Wilson loop averages for higher representations. We have demonstrated the numerical evidence for the restricted-field dominance in the string tension, which means that the string tension extracted from the restricted part of the Wilson loop reproduces the string tension calculated from the original Wilson loop.

In this talk, we focus on the magnetic monopole. According to this picture, magnetic monopoles causing the dual superconductivity are the dominant degrees of freedom responsible for confinement. With the help of the NAST, we define the magnetic monopole and the string tension extracted from the magnetic-monopole part of the Wilson loop in a gauge-invariant manner. We will further perform lattice simulations to measure the static potential for quarks in higher representations using the proposed operators and examine the magnetic monopole dominance in the string tension, which means that the string tension extracted from the magnetic-monopole part of the Wilson loop reproduces the proper string tension obtained from the original Wilson loop.

The 38th International Symposium on Lattice Field Theory, LATTICE2021 26th-30th July, 2021
Zoom/Gather@Massachusetts Institute of Technology

---

[*] Speaker

PoS(LATTICE2021)085

1. Introduction

The dual superconductor picture is one of the most promising scenarios for quark confinement[1]. According to this picture, magnetic monopoles causing the dual superconductivity are regarded as the dominant degrees of freedom responsible for confinement. However, it is not so easy to establish this hypothesis. Indeed, even the definition of magnetic monopoles in the pure Yang-Mills theory is not obvious. If magnetic charges are naively defined from electric ones by exchanging the role of the magnetic field and electric one according to the electric-magnetic duality, one needs to introduce singularity to obtain non-vanishing magnetic charges, as represented by the Dirac monopole. For such configuration, however, the energy becomes divergent. Avoiding this issue in defining magnetic monopoles, there are two prescriptions i.e., *the Abelian projection method* and *the gauge-covariant decomposition method*.

The Abelian projection method, which is proposed by 't Hooft [2], is the most frequently used prescription. In the Abelian projection method, the "diagonal component" of the Yang-Mills gauge field is identified with the Abelian gauge field, and a magnetic monopole is defined as the Dirac monopole. The energy density of this monopole can be finite everywhere because the contribution from the singularity of a Dirac monopole can be canceled by that of the off-diagonal components of the gauge field. For quarks in the fundamental representation, indeed, such numerical simulations were already performed within the Abelian projection using the maximal Abelian (MA) gauge in $SU(2)$ and $SU(3)$ Yang-Mills theories on the lattice[3, 4, 5, 6]. Then it was confirmed that (i) the diagonal part extracted from the original gauge field in the MA gauge reproduces the full string tension calculated from the original Wilson loop average [3, 5, 6], which is called the Abelian dominance, and that (ii) the monopole part extracted from the diagonal part of the gauge field by applying the Toussaint-DeGrand procedure [7] mostly reproduces the full string tension [4, 5, 6], which is called the monopole dominance. However, it should be noted that the MA gauge in the Abelian projection simultaneously breaks the local gauge symmetry and the global color symmetry. This defect should be eliminated to obtain the physical result by giving a procedure to guarantee the gauge invariance.

We have developed a new formulation of Yang-Mills theory, named the decomposition method, which enables us to perform the numerical simulations on the lattice in such a way that both the local gauge symmetry and the global color symmetry remain intact, in sharp contrast to the Abelian projection which breaks both symmetries (See [8] for review). The gauge-covariant decomposition was first proposed for $SU(2)$ by Cho [9] and Duan and Ge [10] independently, and later readdressed by Faddeev and Niemi [11], and developed by Shabanov [12] and the Chiba University group [13, 14, 15]. In this method, the gauge field is decomposed into two parts; A part called restricted field transforms under the gauge transformation just like the original gauge field, while the other part called the remaining field transforms like an adjoint matter. The key ingredient in this decomposition is the Lie-algebra valued field with a unit length that we call the color field. The decomposition is constructed in such a way that the field strength of the restricted field is "parallel"to the color field. Then monopoles can be defined by using the gauge-invariant part proportional to the color field in the field strength just as the Abelian field strength in the Abelian projection. While the main advantage of the field decomposition is its gauge covariance, another advantage is that, through a version of the non-Abelian Stokes theorem (NAST) invented originally by Diakonov and Petrov[23] and extended in a unified way in [18, 19, 20, 21, 22], the restricted field naturally appears in the surface-integral representation of the Wilson loop. By virtue of this method, we understand how monopoles contribute to the Wilson loop at least classically. The field decomposition was extended to $SU(N)$ ($N \geq 3$) gauge field in [13, 14, 16, 16, 17]. By way of the non-Abelian Stokes theorem for the Wilson loop operator [21, 22], indeed, it was found that the different type of decomposition called the minimal option is available for $SU(3)$ and $SU(N)$ for $N \geq 4$ [14, 17].

For quarks in the fundamental representation, we have presented the lattice formulation and demonstrated some numerical evidences for the dual superconductivity by using the gauge-covariant decomposition method [24, 25, 26, 27, 28, 29, 30]. This framework improves the Abelian projection in the gauge-independent manner. Moreover, the MA gauge in the Abelian projection is not the only way to recover the string tension in the fundamental representation. Even for the minimal option, we have demonstrated the restricted field dominance and monopole dominance in the string tension for quarks in the fundamental representation [27, 28, 29]. Thus, our method enables one to extract various degrees of freedom to be responsible for quark confinement by combining the option of gauge-covariant field decomposition and the choice of the reduction condition, which is not restricted to the Abelian projection and the MA gauge, respectively.

As for quarks in higher representations, it was known by preceding works that the restricted field dominance is not observed for the Wilson loop in higher representations if the restricted part of the Wilson loop is extracted by adopting the Abelian projection or the field decomposition naively in the same way as in the fundamental representation [34]. This is because, in higher representations, the diagonal part of the Wilson loop does not behave in the same way as the original Wilson loop. Poulis heuristically found the correct way to extend the Abelian projection approach for the adjoint representation in the $SU(2)$ Yang-Mills theory [35]. Recently, the NAST for the Wilson loop operator are extended to any representations [33]. By virtue the NAST, we have proposed suitable gauge-invariant operators constructed from the restricted field to reproduce the correct behavior of the original Wilson loop averages for higher representations. We have further demonstrated the numerical evidence that the proposed operators well reproduce the behavior of the original Wilson loop average, namely, the linear part of the static potential with the correct value of the string tension [32].

In this talk, we focus on the magnetic monopole. With the help of the NAST, we can define the magnetic monopole and the string tension extracted from the magnetic-monopole part of the Wilson loop in a gauge-invariant manner. We will further perform lattice simulations to measure the static potential for quarks in higher representations using the proposed operators and examine the magnetic monopole dominance in the string tension.

2. The gauge-covariant field decomposition

We decompose the gauge link variable $U_{x,\mu}$ into the product of the two variables $V_{x,\mu}$ and $X_{x,\mu}$ in such a way that the new variable $V_{x,\mu}$ is transformed by the full $SU(N)$ gauge transformation Ω_x as the gauge link variable $U_{x,\mu}$, while $X_{x,\mu}$ transforms as the site variable:

$$U_{x,\mu} = X_{x,\mu}V_{x,\mu} \in G = SU(N), \qquad U_{x,\mu} \longrightarrow U'_{x,\nu} = \Omega_x U_{x,\mu}\Omega^{\dagger}_{x+\mu}, \tag{1a}$$

$$V_{x,\mu} \longrightarrow V'_{x,\nu} = \Omega_x V_{x,\mu}\Omega^{\dagger}_{x+\mu}, \qquad X_{x,\mu} \longrightarrow X'_{x,\nu} = \Omega_x X_{x,\mu}\Omega^{\dagger}_{x}. \tag{1b}$$

From the physical point of view, $V_{x,\mu}$ which we call the restricted field, could be the dominant mode for quark confinement, while $X_{x,\mu}$ is the remainder part. The possible options of the decomposition are discriminated by the stability subgroup of the gauge group. Here, we only consider the maximal option.

The maximal option is obtained for the stability subgroup of the maximal torus subgroup of G: $\tilde{H} = U(1)^{N-1} \subset SU(N)$. The resulting decomposition is the gauge-invariant extension of the Abelian projection in the MA gauge. We introduce color fields as,

$$\mathbf{n}^{(k)}(x) = \Theta(x) H_k \Theta^\dagger(x) \in Lie[G/\tilde{H}] \qquad (k = 1, \ldots, N-1), \tag{2}$$

which are expressed using a common $SU(N)$-valued field $\Theta(x)$ with the Cartan generators H_k. The decomposition is obtained by solving the defining equations:

$$D^\epsilon_\mu[V]\mathbf{n}^{(k)}_x := \frac{1}{\epsilon}\left[V_{x,\mu}\mathbf{n}^{(k)}_{x+\mu} - \mathbf{n}^{(k)}_x V_{x,\mu}\right] = 0\,, \tag{3a}$$

$$g_x := e^{i2\pi q/N} \exp\left(-i\sum_{j=1}^{N-1} a^{(j)}_x \mathbf{n}^{(j)}_x\right), \tag{3b}$$

where, the variable g_x is the $U(1)^{N-1}$ part which is undetermined from Eq.(3a) alone, $a^{(j)}_x$ are coefficients, and q is an integer. Note that the above defining equations correspond to the continuum version ($g_x = \mathbf{1}$): $D_\mu[\mathscr{V}]\mathbf{n}^{(k)}(x) = 0$ and $\mathrm{tr}(\mathscr{X}_\mu(x)\mathbf{n}^{(k)}(x)) = 0$, respectively. These defining equations can be solved exactly [26], and the solution is given by

$$\begin{aligned}
X_{x,\mu} &= \widehat{K}^\dagger_{x,\mu} \det(\widehat{K}_{x,\mu})^{1/N} g_x^{-1}, & V_{x,\mu} &= X^\dagger_{x,\mu} U_{x,\mu}, \\
\widehat{K}_{x,\mu} &:= \left(K_{x,\mu}K^\dagger_{x,\mu}\right)^{-1/2} K_{x,\mu}, & K_{x,\mu} &:= 1 + 2N \sum_{k=1}^{N-1} \mathbf{n}^{(k)}_x U_{x,\mu} \mathbf{n}^{(k)}_{x+\mu} U^\dagger_{x,\mu}.
\end{aligned} \tag{4}$$

In the naive continuum limit, we can reproduce the decomposition in the continuum theory.

3. Non-Abelian Stokes theorem and magnetic monopole

It is known that if we adopt the Abelian projection naively to higher representations, the monopole contributions do not reproduce the correct behavior [34]. Thus, we have to find a more appropriate way to extract the monopole contributions. We can relate the decomposed field variables to a Wilson loop operator through a version of the NAST which was proposed by Diakonov and Petrov [23]. Recently, the NAST is extended to the Wilson loop operator in any representations for $SU(N)$ Yang-Mills theory [33]. Through the NAST, therefore, we have proposed suitable gauge-invariant operators constructed from the restricted field to reproduce the correct behavior of the original Wilson loop averages for higher representations, and demonstrated the restricted field dominance in the string tension [31, 32]. Here, we focus on the magnetic monopole contribution. We quickly summarize the result of the Ref. [33],

The Wilson loop operator in a representation R of $SU(N)$ Yang-Mills theory is rewritten into the surface integral form by introducing a functional integral on the surface Σ surrounded by the loop C as

$$\begin{aligned}
W_C[\mathscr{A}] &= \int [d\mu(g)]_\Sigma \exp\left[-ig_{\mathrm{YM}}\sqrt{\frac{N-1}{2N}}\int_{\Sigma:\partial\Sigma=C} f^g\right], \quad f^g := \frac{1}{2} f^g_{\mu\nu}(x) dx^\mu \wedge dx^\nu, \\
f^g_{\mu\nu}(x) =& \kappa(\partial_\mu \mathrm{tr}\{n(x)\mathscr{A}_\nu(x)\} - \partial_\nu \mathrm{tr}\{n(x)\mathscr{A}_\mu(x)\} + ig^{-1}_{\mathrm{YM}}\mathrm{tr}\{n(x)[\partial_\mu n_k(x), \partial_\nu n_k(x)]\}), \\
=& \partial_\mu\{n^A(x)\mathscr{A}^A_\nu(x)\} - \partial_\nu\{n^A(x)\mathscr{A}^A_\mu(x)\} - g^{-1}_{\mathrm{YM}} f^{ABC} n^A(x)\partial_\mu n^B_k(x)\partial_\nu n^C_k(x), \\
n(x) =& \sqrt{\frac{2N}{N-1}}\Lambda_j n_j(x), \quad n_j(x) = g(x) H_j g^\dagger(x) \quad (j = 1, \ldots, r).
\end{aligned} \tag{5}$$

where $[d\mu(g)]_\Sigma$ is the product of the Haar measure over the surface Σ, Λ_k is the k-th component of the highest-weight of the representation R. It should be noticed that the Wilson loop operator is represented by using the restricted field $\mathscr{V}$. We choose the highest-weight state as the reference state. For $G = SU(2)$ the highest-weight vector of the representation with the spin J is given by $\vec{\Lambda}_{SU(2)} = (\Lambda_3) = (J) = (\frac{m}{2})$, while for $G = SU(3)$ the highest-weight vector of the representation with the Dynkin indices $[m, n]$ is given by $\vec{\Lambda} = (\Lambda_3, \Lambda_8) = \left(\frac{m}{2}, \frac{m+2n}{2\sqrt{3}}\right)$.

Through the NAST we can define the gauge-invariant magnetic-monopole from the field strength in the same manner as the Dirac monopoles for the Abelian-like gauge-invariant field. By using the Hodge decomposition for the field strength, the Wilson loop operator in arbitrary representation is written in terms of the electric current j and the magnetic current k:

$$W_C[\mathscr{A}] = \int [d\mu(g)] \exp\left\{-ig_{\text{YM}}\sqrt{\frac{N-1}{2N}}[(\omega_{\Sigma_C}, k) + (N_{\Sigma_C}, j)]\right\}, \tag{6}$$

where we have defined the $(D-3)$-form k and the one-form j in D space-time dimensions:

$$k := \delta^* f^g, \quad j := \delta f^g, \tag{7}$$

we have introduced an antisymmetric tensor Θ_{Σ_C} of rank two which has the support only on the surface Σ_C spanned by the loop C:

$$\Theta^{\mu\nu}_{\Sigma_C}(x) := \int_{\Sigma_C:\partial\Sigma_C=C} d^2S^{\mu\nu}(x(\sigma))\delta^D(x - x(\sigma)),$$

and we have defined the $(D-3)$-form ω_{Σ_C} and one-form N_{Σ_C} using the Laplacian Δ by

$$\omega_{\Sigma_C} := {}^*d\Delta^{-1}\Theta_{\Sigma_C} = \delta\Delta^{-1*}\Theta_{\Sigma_C}, \quad N_{\Sigma_C} := \delta\Delta^{-1}\Theta_{\Sigma_C}, \tag{8}$$

with the inner product for two forms being defined by

$$\begin{aligned}(\omega_{\Sigma_C}, k) &= \frac{1}{(D-3)!}\int d^D x k^{\mu_1\cdots\mu_{D-3}}(x)\omega_{\Sigma_C}^{\mu_1\cdots\mu_{D-3}}(x),\\ (N_{\Sigma_C}, j) &= \int d^D x j^\mu(x) N^\mu_{\Sigma_C}(x).\end{aligned} \tag{9}$$

Therefore we can construct the proper Wilson-loop operator by using the decomposed variable V in Eq(4) on the lattice. Incidentally, we can show [8] that the gauge-invariant field strength $F^g_{\mu\nu}$ is equal to the component of the non-Abelian field strength $\mathscr{F}[\mathscr{V}]$ of the restricted field $\mathscr{V}$ (in the decomposition $\mathscr{A} = \mathscr{V} + \mathscr{X}$) projected to the color field n:

$$\begin{aligned}W_R[\mathscr{A}; C] &:= \int [d\mu(g)]_\Sigma \exp\left(ig\int_{\Sigma:\partial\Sigma=C} dS_{\mu\nu}\sum_{k=1}^{N-1}\Lambda_k F^{(k)}_{\mu\nu}\right),\\ F^g_{\mu\nu} &= \text{tr}\{m\mathscr{F}_{\mu\nu}[\mathscr{V}]\} = \Lambda_j f^{(j)}_{\mu\nu}, \quad f^{(j)}_{\mu\nu} = \text{tr}\{n_j\mathscr{F}[\mathscr{V}]\}.\end{aligned} \tag{10}$$

This relation is useful in calculating the contribution from magnetic monopoles to the Wilson loop average from the viewpoint of the dual superconductor picture for quark confinement. We might identify the restricted field $\mathscr{V}$ and the color field n_j in Eq(10) with the restricted field and the color field in Eq(4), respectively. However, since the NAST Eq(10) includes the integration over the measure $[d\mu(g)]_\Sigma$, which indicates the integral over the whole directions of color fields, the field correspondences are not simple. In the gauge-covariant decomposition method, it should be remined that the color fields are introduced as auxiliary fields and the decomposition is defined in the space with extended symmetry $G \times G/\tilde{H}$. In order to make the theory written by the decomposed fields equivalent to the original Yang-Mills theory with the same gauge symmetry, it is necessary to impose a reduction condition between the color field and the Yang-Mills field. Thus, this requirement can be formulated as a path integral with the reduction condition imposed as a constraint. While in the lattice measurements this means that the operator is evaluated by using the color field which is obtained as the solution of the reduction condition in place of performing an integral over the color field.

Thus we define the magnetic monopole by using the restricted field:

$$
\begin{aligned}
&k_\mu = \sum_{j=1}^{r} \Lambda_j k_\mu^{(j)} , \qquad k_\mu^{(j)} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu \mathscr{F}_{\alpha\beta}^{(j)}[V] , \\
&V_{x,\mu}V_{x+\hat{\mu},\nu}V^\dagger_{x,+\hat{\nu},\mu}V^\dagger_{x,\nu} = \exp\left(-i\mathscr{F}_{\alpha\beta}[V]\right) = \exp\left(-i\sum_{k=1}^{r}\mathscr{F}_{\alpha\beta}^{(k)}[V]\mathbf{n}_x^{(k)}\right) .
\end{aligned} \tag{11}
$$

4. Lattice data

It should be examined on the lattice whether or not these monopoles can reproduce the expected infrared behavior of the original Wilson loop average in the higher representation. As the first step, we examine the case of the $SU(2)$ Yang-Mills theory. We set up the gauge configuration for the standard Wilson action at $\beta = 2.5$ on the 24^4 lattice for $SU(2)$ case. We prepare 500 configurations every 100 sweeps after 3000 thermalization by using the pseudo heat-bath method. The restricted field is obtained by using the decomposition formula Eq(4) for a given set of configurations of the Yang-Mills field $\{U_{x,\mu}\}$ and the color field $\{\mathbf{n}_x ; \mathbf{n}_x := \Theta_x T^3 \Theta_x^\dagger \}$, where the color field is determined by minimizing the reduction condition functional:

$$
R[U,\{n\}] = \sum_{x,\mu} \mathrm{tr}[(D_\mu[U]\mathbf{n}_x)^\dagger (D_\mu[U]\mathbf{n}_x)] \quad . \tag{12}
$$

We calculate the Wilson loop average $W(R,T)$ for the rectangular loop with length T and width R to derive the potential $V(R,T)$ through the formula $V(R,T) := -\log(W(R,T+1)/W(R,T)$. In the measurement of the Wilson loop average we apply the APE smearing to reduce noises and eliminate exciting modes. The Wilson-loop operator for the spin-J representation is given by

$$
W_{[J]}^{SU(2)}[U;C] = \sum_{k=0}^{\lfloor J \rfloor} \mathrm{tr}(U_C^{2(j-k)}) \qquad \text{with } U_C := \prod_{\langle x,\mu\rangle \in C} U_{x,\mu} , \tag{13a}
$$

$$
W_{[J]}^{SU(2)}[V;C] = \frac{1}{2}\mathrm{tr}(V_C^{2J}) \qquad \text{with } V_C := \prod_{\langle x,\mu\rangle \in C} V_{x,\mu} . \tag{13b}
$$

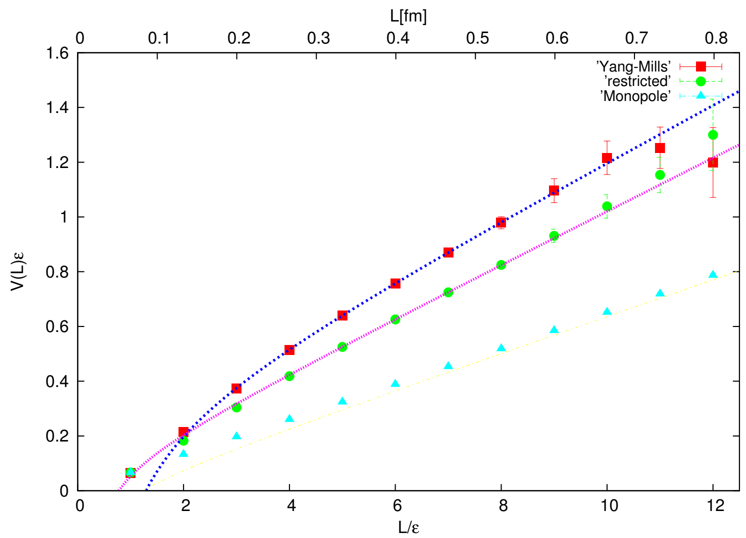

Figure 1: The static potential for quarks in the adjoint representation.

For $SU(2)$ case the magnetic monopole part of Eq(6) can be calculated as

$$
\begin{aligned}
W^{SU(2)}_{[J]}[k;C] &= \exp\left\{2\pi i J \sum_{x,\mu} k_\mu(x) N_\mu(x)\right\}, \\
k_\mu(x) &= \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu \Theta_{\alpha\beta}(x), \qquad \Theta_{\alpha\beta} = \arg\operatorname{tr}\left((\mathbf{1}+2\mathbf{n}_x)V_{x,\mu}V_{x+\hat{\mu},\nu}V^\dagger_{x,+\hat{\nu},\mu}V^\dagger_{x,\nu}\right), \\
N_\mu(x) &= \sum_y \Delta^{-1}(x-y)\frac{1}{2}\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha S^C_{\beta\gamma}(y),
\end{aligned} \tag{14}
$$

where $\Delta^{-1}(x-y)$ is the inverse of the Laplacian, and $S^C_{\beta\gamma}(y)$ is the surface element with the Wilson loop (C) as boundary which satisfies the equation $\partial_\beta S^C_{\beta\gamma}(y) = C_\gamma(y)$ with $C_\gamma(y)$ being the tangent vector of the path C.

Figure 1 shows the preliminary result of the statical potential for a pair of quark and antiquark in the adjoint representation ($J = 1$), where we simultaneously plot the the statical potential calculated from the magnetic monopole Eq(14), the original Yang-Mills field Eq(13a), and the restricted field Eq(13b). We find the magnetic monopole play a dominant role in the string tension ($\sigma_{mono}/\sigma_{full} \simeq 68\%$) for the quarks in the adjoint representation as well as the restricted field dominance in the string tension which we have already shown in Ref.[31, 32].

5. Summary and Discussion

We have investigated the magnetic monopole contribution for quark confinement in the higher representation. Through the non-Abelin Stokes theorem for the Wilson loop in the higher representation, we have defined the magnetic monopole and the string tension extracted from the magnetic-monopole part of the Wilson loop in the higher representation in a gauge-invariant manner. We have performed lattice simulations for the $SU(2)$ Yang-Mills theory and measured the Wilson loop average in the adjoint representation calculated from the magnetic monopole to examine the magnetic-monopole dominance in the string tension. We have found that the magnetic monopole plays a dominant role in the string tension for the quarks in the adjoint representation.

To examine the restricted-field dominance and the magnetic-monopole dominance in the string tension for the quarks in the higher representations, we need to investigate the Wilson loops in various gauge groups and various representations. We will further investigate large size Wilson loops to examine whether or not the string breaking for Wilson loops in the higher representation calculated from the original Yang-Mills field can be reproduced for the Wilson loops calculated from the restricted field as well from the magnetic monopole.

.

Acknowledgement

This work was supported by Grant-in-Aid for Scientific Research, JSPS KAKENHI Grant Number (C) No.19K03840. The numerical calculations were performed in part using Cygnus and Oakforest-PACS at the CCS, University of Tsukuba.

References

[1] Y. Nambu, Phys. Rev. D10 4262 (1974) ; G. 't Hooft, in High Energy Physics, Editorice Compositori, Bologna, 1975; S. Mandelstam, Phys. Rept. 23 245 (1976) .

[2] G. 't Hooft, Nucl. Phys. B 190 455 (1981) .

[3] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D 42, 4257 (1990).

[4] J. D. Stack, S. D. Neiman and R. J. Wensley, Phys. Rev. D 50, 3399 (1994); H. Shiba and T. Suzuki, Phys. Lett. B 333, 461 (1994).

[5] J. D. Stack, W. W. Tucker and R. J. Wensley, Nucl. Phys. B 639, 203 (2002).

[6] N. Sakumichi and H. Suganuma, Phys. Rev. D 90, 111501 (2014).

[7] T. DeGrand and D. Toussaint, Phys. Rev. D 22, 2478 (1980).

[8] K.-I. Kondo, S. Kato, A. Shibata and T. Shinohara, Phys. Rept. 579 (2015) 1 [arXiv:1409.1599]

[9] Y.M. Cho, Unpublished preprint, MPI-PAE/PTh 14/80 (1980); Y.M. Cho, Phys. Rev. Lett. 44, 1115 (1980).

[10] Y.S. Duan and M.L. Ge, Sinica Sci., 11, 1072 (1979).

[11] L. Faddeev and A.J. Niemi, Phys. Lett. B449, 214 (1999). [hep-th/9812090]; L. Faddeev and A.J. Niemi, Phys. Lett. B464, 90 (1999). [hep-th/9907180]; T.A. Bolokhov and L.D. Faddeev, Theoretical and Mathematical Physics, 139, 679 (2004).

[12] S.V. Shabanov, Phys. Lett. B458, 322 (1999). [hep-th/0608111] ; S.V. Shabanov,Phys. Lett. B463, 263 (1999). [hep-th/9907182]

[13] K.-I. Kondo, T. Murakami and T. Shinohara, Eur. Phys. J. C42, 475 (2005). [hep-th/0504198]

[14] K.-I. Kondo, T. Murakami and T. Shinohara, Prog. Theor. Phys. 115, 201 (2006). [hep-th/0504107]

[15] K.-I. Kondo, Phys. Rev. D74, 125003 (2006). [hep-th/0609166

[16] W.S. Bae, Y.M. Cho and S.W. Kimm, Phys. Rev. D65, 025005 (2002). [hep-th/0105163] ;Y.M. Cho, [hep-th/0301013]

[17] K.-I. Kondo, T. Shinohara and T. Murakami, Prog. Theor. Phys. 120, 1 (2008). arXiv:0803.0176 [hep-th]

[18] K.-I. Kondo, Phys. Rev. D 58, 105016 (1998). [hep-th/9805153]

[19] K.-I. Kondo and Y. Taira, Mod. Phys. Lett. A 15, 367 (2000); [hep-th/9906129]

[20] K.-I. Kondo and Y. Taira, Prog. Theor. Phys. 104, 1189 (2000). [hep-th/9911242]

[21] K.-I. Kondo, Phys. Rev. D 77, 085029 (2008). arXiv:0801.1274 [hep-th]

[22] K.-I. Kondo, J. Phys. G: Nucl. Part. Phys. 35, 085001 (2008). arXiv:0802.3829 [hep-th]

[23] D. Diakonov and V. Petrov, Phys. Lett. B 224 131 (1989).

[24] S. Ito, S. Kato, K.-I. Kondo, T. Murakami, A. Shibata and T. Shinohara, Phys. Lett. B 645, 67 (2007). [hep-lat/0604016]

[25] K.-I. Kondo, A. Shibata, T. Shinohara, T. Murakami, S. Kato, S. Ito, Phys. Lett. B 669, 107 (2008). arXiv:0803.2451 [hep-lat]

[26] A. Shibata, K.-I. Kondo and T. Shinohara, Phys. Lett. B 691, 91 (2010). arXiv:0911.5294 [hep-lat]

[27] K.-I. Kondo, A. Shibata, T. Shinohara and S. Kato, Phys. Rev. D 83, 114016 (2011).

[28] A. Shibata, K.-I. Kondo, S. Kato and T. Shinohara, PoS LATTICE 2015, 320 (2016).

[29] S. Kato, K.-I. Kondo and A. Shibata, Phys. Rev. D 91, 034506 (2015).

[30] N. Cundy, Y. M. Cho, W. Lee and J. Leem, Nucl. Phys. B 895, 64 (2015).

[31] A. Shibata, R. Matsudo, S. Kato, and K.-I. Kondo, PoS LATTICE2018 254 (2018)

[32] R. Matsudo, A. Shibata, K.-I. Kondo, and S. Kato, Phys. Rev. D 100 1,014505(2019)

[33] R. Matsudo and K.-I. Kondo, Phys. Rev. D 92, 125038 (2015).

[34] L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, Phys. Rev. D 53, 5891 (1996).

[35] G. Poulis, Phys. Rev. D 54, 6974 (1996).

[36] M. N. Chernodub, K. Hashimoto and T. Suzuki, Phys. Rev. D 70, 014506 (2004).