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Semiclassical ensembles of instanton-dyons describe the deconfinement and chiral phase transitions, in the usual and deformed QCD

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Instanton-dyons are topological solutions of YM equations at finite temperatures. Their semiclassical ensembles were studied by a number of methods, including direct Monte-Carlo simulation, for SU(2) and SU(3) theories, with and without fermions. We present these results and compare them with those from lattice studies. We also consider two types of QCD deformations. One is by adding operators with powers of the Polyakov line, affecting deconfinement. Another is changing quark periodicity condition, affecting the chiral transition. Another paper is using inverse direction, from lattice configurations (with realistic quark masses) looking at zero and near-zero Dirac modes. It turned out that those revealing the shape of the modes, In excellent agreement with analytic instanton-dyon theory. Summarizing both we conclude that QCD phase transitions are well described in terms of such semiclassical objects.

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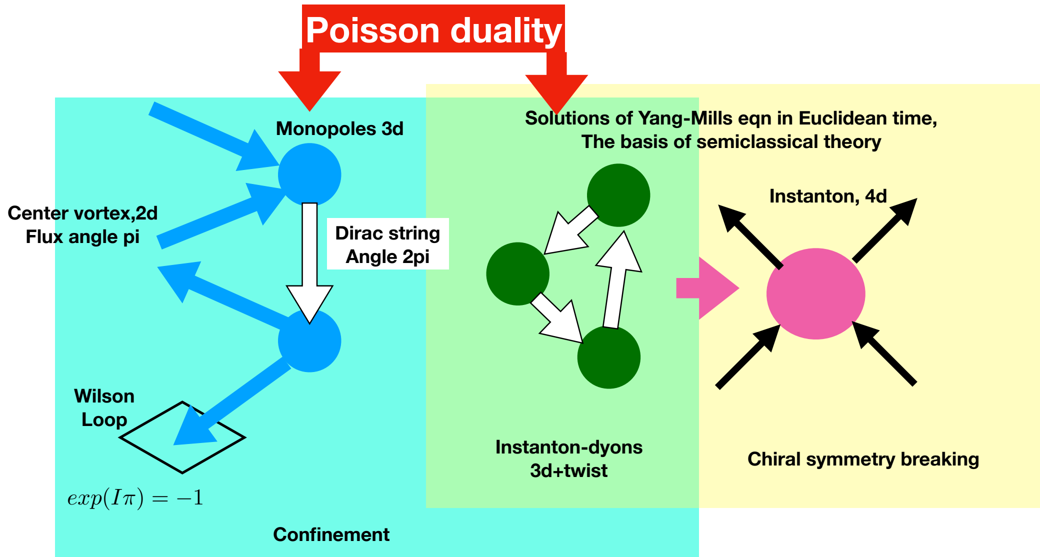

Figure 1: Map of gauge topology, explanation in the text.

1. Brief overview of gauge topology

Confinement phenomenon (left blue region in Fig.1) was first related with *center vortices*, associated with phase π for a quark going around it and thus changing the sign of the Wilson line it may pierce. Two such vortices combined together lead to a singularity with phase 2π, shown by white arrow, known as the *Dirac string*. Their ends are identified as *magnetic monopoles* (blue disks). Confinement is their *Bose-Einstein condensation*, perhaps the most physical signature of this phenomenon.

The right yellow region of Fig.1 is associated with another major nonperturbative phenomenon, *chiral symmetry breaking*. Pink disk indicate *instantons*, 4D solitons in Euclidean space-time. Fermions in its field have *zero modes*, elevating each instanton into t' Hooft multi-fermion operator. Four black arrows correspond to the case of two quark flavors, u, d. The resulting 4-fermion vertex is similar to Nambu-Jona-Lasinio hypothetical interaction, and also breaks spontanously $SU(N_f)_a$ chiral symmetry provided the instanton density is sufucuently large.

At finite temperature the *Polyakov line* has certain nonzero expectation values, or *holonomies* $\mu_i(T)$ (see below). Instanton solution amended by such asymptotics of A_0 fields splits into instanton constituents, known as *instanton-dyons* or *instanton-monopoles*[1] . Like instantons, they have topological charges. Unlike instantons, those are *not quantized to integer* Q, which is possible because they are connected by Dirac strings due to their magnetic charges. Ensemble of those will be the main focus of this talk.

[1]Both names were criticized: they are neither the dyons of Schwinger nor monopoles of Dirac, but Euclidean self-dual objects, with equal E and B up to a sign. Perhaps a new name, emphasizing their Euclidean nature, is needed: can it e.g. be *instantino*?

Last but not least is phenomenon of *Poisson duality*, a technical observation relating partition functions in terms of monopoles and in terms of instanton-dyons. Their equality rather bring to mind two other great names, as two partition functions correspond to dynamical descriptions *a la* Hamilton and Lagrange, respectively.

The detailed discussion of all of that, plus discussion of QCD flux tubes and holographic QCD etc, one can find in my recent book [1].

2. Instanton-dyons on the lattice

Here there is no place to introduce Kraan-van Baal solution for an instanton, containing N_c instanton-dyons, or formalizm leading to zero modes. Let me just remind that eigenvalues of Polyakov line, designated as $\mu_i(T), i = 1..N_c$ can be pictured as locations on an unit circle. Their differences $\nu_i = \mu_{I+1} - \mu_i$ are lengths of corresponding fractions of the circle, also giving the fractions the dyon actions make of the instanton action $S_i = \nu_i S$ where

$$S = 8\pi^2/g^2 = (\frac{11}{3}N_c - \frac{2}{3}N_f)log(\frac{T}{\Lambda_{QCD}})$$

Another fact is that if quarks are given different periodicity phases $z_f, f = 1..N_f$ over the Matsubara circle, the normalizable physical zero mode belongs to the dyon in which sector z_f belongs, namely $z_f \in [\mu_i, \mu_{i+1}]$. So, using different z_f one can see all dyon types.

Since this text is written for proceedings of a lattice conference, let me start with efforts to identify these objects on the lattice. I am sure everyone is aware of "cooling" techniques of vacuum configurations, identifying esemble of instantons in 1990's. Perhaps first lattice observation of instanton-dyons was via "constrained minimization" by Langfeld and Ilgenfritz [2], conserving $\langle P \rangle$, in which selfdual clusters with non-integer topological charge were seen. Gattringer et al and Ilgenfritz et al have introduced and refined the "fermionic filter" allowing to identify them via distinct zero modes and variable periodicity phases.

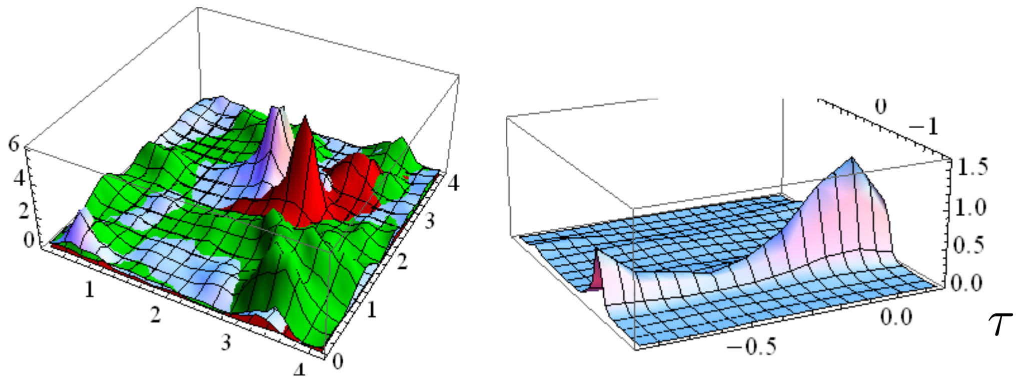

Figure 2: (Left) Space slice of density of an exact zero mode from QCD simulaiton at $T = T_c$, three colors show dyons of three different types. (Right) Tau dependence of a dyon, perturbed by an interference with other dyons.

Let me just comment on recent progress in this direction, in which I was involved [3, 4]. QCD simulation with realsitic masses were performed at and near T_c using domain wall fermions with qood chiral symmetry. Using overlap fermions, in which chiral symmetry is exact, we focused on exact zero modes (and near-zero ones). The left Fig.2 shows a typical landscape of the zero mode densities. There are three different dyon types as $N_c = 3$. One result, which we were able to make stronger, is that the shape of isolated peaks are well described by analytic formulae from van Baal and collaborators derived for a single dyon. Apparently, millions of gluons in the ensemble do not perturbed it.

Previous works however have not analyzed the "topological clusters", the situations in which two or three dyons overlap strongly. We did so, as Kraan-van Baal solution allows to consider such cases, and found that agreement is very good in such cases as well. The right figure is an example of (Euclidean time) τ-dependence of the density. An isolated dyon should show no such dependence at all, and what is seen is a result of an intereference with overlapping dyons. Locating those and using analytic expressions for zero mode density, we found agreement in those cases as well. We thus conclude that *semiclassical description of zero and near-zero Dirac modes on the lattice is quite accurate, at least in terms of the zero mode shapes.*

3. Numerical simulation of instanton-dyon ensembles

The simplest limiting case is weak coupling, $T \to \infty$, in which the dyon density is exponentially small, with their interactions and back reaction being negligible. In QCD with N_f fermionic quarks all $z_f = \pi$, and L dyon becomes t'Hooft vertex with $2N_f$ legs. Either $N_f = 1$, there is only $U(1)_a$ chiral symmetry explicitly broken at any T, with exponentially small $\langle \bar{q}q \rangle$, or $N_f > 1$ and $SU(N_f)_a$ chiral symmetry is unbroken. So there should be "molecules" $\bar{L}L$, similar to $\bar{I}I$ molecules originally discussed in [5–7], also called "bions" by Unsal and collaborators.

The opposite case is dense ensemble at $T \sim T_c$, discussed by mean field approximation in a number of settings [8–10]. Here we will only discuss numerical simulations for the $SU(2)$ [11, 12] and $SU(3)$ [13, 14] color groups, without ($N_f = 0$) and with two quark flavors ($N_f = 2$) .

The first physics issue is $deconfinement$ phase transition. Recall that Gross-Pisarski-Yaffe (GPY) perturbative potential for Polyakov line favors trivial $\langle P \rangle = 1$ case and disfavors confinement $1\, \langle P \rangle = 0$. Therefore, in order to have it the nonperturbative effects – sufficient density of the dyons in our simulations – should *overcome* the GPY. In Fig.3 one can see that it happens differently for SU(2) and SU(3) pure gauge theories. In the former the minimum gradually shifts to the confining value $\nu = 1/2$, and stay there at high densities. In the latter there is a jump, indicating first order transition. There is no place here for comparison with lattice data, let me just not that in SU(3) case $\langle P \rangle$ jumps to a value 0.4, same as on the lattice. We also discussed $deformation$ of SU(3) gauge theory by an operator $\sim P^2$, and, in agreement with lattice, observe that deconfinement temperature can be moved higher, with a jump visibly decreasing.

The second nonperturbative issue is chiral symmetry breaking at low T. Again, it requires sufficient density of the dyons, so that their zero modes can get $collectivized$[2]

[2]Note that already in 1961 NJL model has shown that one needs large enough 4-fermion coupling to break chiral symmetry.

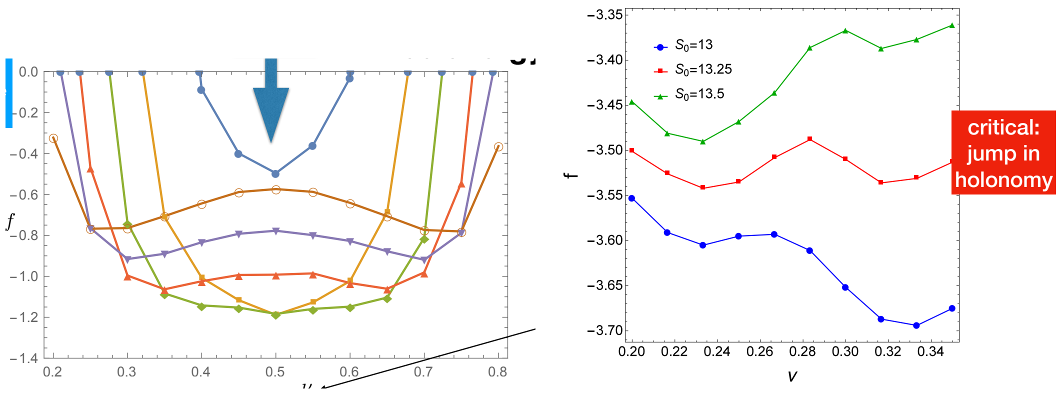

Figure 3: Free energy f versus the holonomy parameter ν, for SU(2) (left) and SU(3) (right) pure gauge theories. Different curves are for different instanton densities (or temperatures).

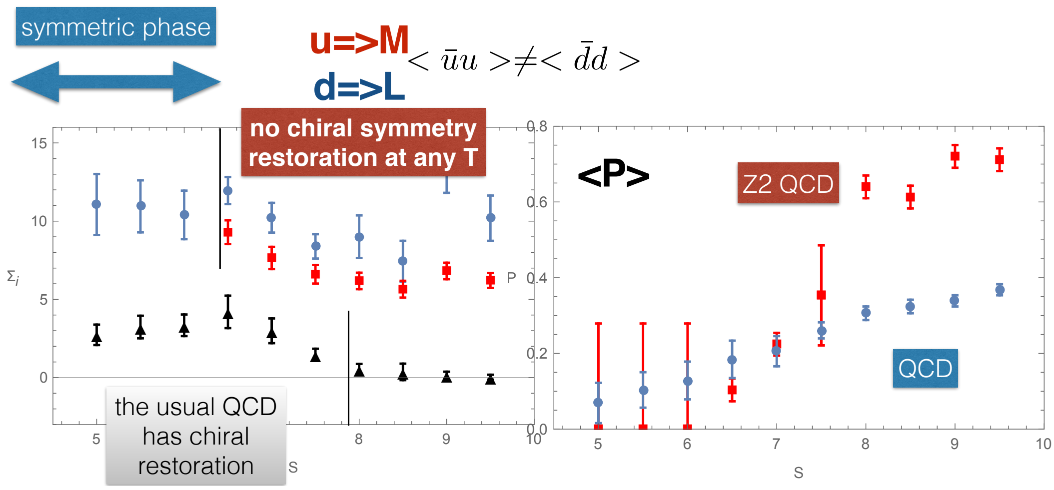

Figure 4: Quark condensates (left) and Polyakov line (right) versus the action parameter $S(T)$, higher temperatures are at the right side. Blue circles in the right plot are for QCD, $N_c = N_f = 2$, and red squares are for Z_2QC_2D: one finds *crossover* in the former changes to first order transition in the latter theory. Black triangles on the left are for QCD $N_c = N_f = 2$, possessing restoration of chiral $SU(2)_a$ symmetry above $S > 8$. The blue discs and red squares show two chiral condensates for Z_2QC_2D: one can see a transition from symmetric to asymmetric phase, but no chiral symmetry restoration.

4. QCD deformation by modified quark periodicity phases

This deformation came first under the name of "imaginary chemical potentials". in [15]. Its usage is different for *small* and *large* deformations. In the former case the motivation was due to the fact that imaginary chemical potentials can be simulated by usual Monte Carlo algorithms, while those with real μ one cannot. Plotting lattice results for negative $\mu^2 < 0$ one can extrapolate to positive ones, e.g. by Taylor series: this strategy has been used in many lattice studies.

For large phase Roberge and Weiss predicted first order transitions at near $z = (2k + 1)\pi/Nc$, due to different N_c branches of the gluonic GPY potential. Of course, it is a perturbative argument expected to be true at large T only. And indeed, when dyons are numerous this transition ends, according to [16] it happens at $T_{RW} = 1.34(7)T_c = 208(5)\ MeV$.

Another form of deformed QCD is to select chemical potentials imaginary and proportional to T, so that fugacities of quarks become $exp(iz_f), f = 1..N_f$ with certain T-independent *periodicity phases*. Moving from conventional $z_f = \pi$ (quarks are fermions) to other values one should see multiple phase transitions, each time when one of z_f crosses Polyakov phases $\mu_i(T)$, as the fermion zero modes jump from one instanton-dyon to another. The 'ultimate" selection for $N_c = N_f$ theories was proposed in [19], with z_f suggested to be located "homogeneously", one inside each holonomy sector $[\mu_{i+1}, \mu_i]_{T<T_{deconfinement}}$. In Fig.4 we show our simulations for $N_c = N_f = 2$ QCD and Z_2QC_2D with one quark being fermion and one being boson. Indeed, the plots show drastic changes of both phase transitions. The deconfinement changes from crossover to strong first order, and chiral restoration is not seen at all [3]. Let us stress that multiple deformed worlds (such as $Z_{N_c}QCD$) are unphysical, qualitatively different from ours, thus separated by singularities. E.g. $Z_{N_c}QCD$ has flavor and chiral symmetries completely different (except at high T when all μ_i are near zero). Its chiral symmetry is split diagonally for all flavors $(U(1)_a)^{N_f-1}$, each broken *explicitly* by diquark t'Hooft operators for each dyon type. Obviously, this chiral symmetry breaking has no relation to *spontaneously broken* $SU(N_f)$ *chiral symmetry* occuring in our world due to multi-quark operators and only at finite dyon density.

Another take on unusual quark periodicity phases historically came from supersymmetry. Davis, Hollowood and Khose [18] considered $\mathcal{N}$=1 SYM theory on R^3S^1 with small circle and *bosonic* gluinos, calculating the quark condensate using instanton-dyons. In this case the gluino term in GPY potential changes sign, canceling the gluon term.

Unsal [20] went further, considering theories with *more than one flavor* of gluinos (adjoint fermions), $N_a > 1$. If so, the GPY potential is multiplied by $(1 - N_a)$ and for $N_a > 1$ get *inverted*, now favoring confinement at weak coupling (small circle or high T). It was suggested that with confinement present both at small and large $L = 1/T$, there would be *continuity* (no phase transitions) at any L. Yet lattice studies [17] have found *two* deconfined phases in between those two limits, preventing such continuity. Once again, one finds that considerations based on GPY potentials get invalid outside the weak coupling (high T) limit, where they belong.

Unsal et al wrote several more papers, studying phases with inverted GPY at weak coupling in similar settings. In [21] the $Z_{N_c}QCD$ is appended by heavy adjoint quarks with bosonic periodicity phase. The motivation is explained as follows: *"The idea of adiabatic continuity is to find a way to put an asymptotically-free gauge theory on* $R^{1,2}S^1$ *in such a way that its dependence on the spatial circle size L is smooth. If this condition is met, then one can get insight about the behavior of the theory for large L, where it is strongly coupled, by studying it for small L, where it is weakly coupled."*

Well, "adiabatic continuity" is just an unproven assumption, as no calculations at finite L (finite dyon density) were done or even attempted. Furthermore, one may doubt that, by adding to the

[3]Note that it is a somewhat special case. If the group be e.g. SU(3), and two sectors of M_1, M_2 dyons would have z_1, z_2 located in them, ultimately at high T these sectors shrink as $\mu_i(T)$ move toward zero. When they cross phases z_1, z_2 , the zero modes would return to L dyon and thus at infinite T limit would become similar to undeformed QCD, in which chiral symmetry is restored.

$Z_{N_c} QCD$ (rather unphysical by itself at any T!) even more unphysical construction inverting the GPY at high $T \gg M_{adj}$ will take it any closer to the real world. In general, there is no easy way around: only studies of QCD phases at finite T and density of topological objects, overcoming the GPY potential, can do it (as they being done on the lattice or in our simulations).

Summary: Semiclassical theory based on ensembles of instanton-dyons reproduces semi-quantitatively the main lattice finding about deconfinement and chiral phase transitions, in pure gauge theories, in QCD with light quarks. Deforming QCD by extra action with powers of Polyakov loop shift/modify the deconfinement transition. Deforming it via quark periodicity phases lead to phases with drastically different deconfinement and chiral transitions. While all these phases have multiple unphyisical properties, the existence elucidate the mechanisms driving QCD phase transitions. The key to them are “jumps" of the fermion zero modes, from one dyon type to another.

The dyon zero modes from lattice real-life QCD simulations preserve remarkably well their shapes on the lattice, even in the case of their strong overlaps. Millions of thermal gluons does not seem to affect them.

Comparing our results with lattice ones, one should note that reported dyon simulations are simple multiple integrals over dyon collective variables. So it is relatively easy to get to say $N_d \sim 300$ of dyons. Note that while lattice is of course based on first principles, its practical cost is such that one typically have $N_d \sim O(10)$, as e.g.seen from our Fig.2. If instanton-dyon ensembles are important for QCD phase transitions, as we suggest, such N_d are perhaps too small to get accurate description of their phases.

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